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Mode-locking behaviour: ac response of a driven disordered vortex lattice*

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Using random pinning force simulations, we study numerically the alternating current (ac) response of the driven disordered vortex lattice. In the presence of a superimposed ac force, mode-locking steps are observed in the velocity–direct current (dc) force characteristics at high dc forces, where a smectic order occurs. The step width shows damped oscillations with the strength of interaction between vortices, in good agreement with recent experiments.

Keywords: vortex lattice, mode locking
PACC: 7460G, 0570

Recently, the non-equilibrium dynamics of vortex matter, particularly the vortex lattice in superconductors, has aroused intensive interest. Much attention has been focused on the driven disordered vortex lattice due to its direct relevance to typical transport measurement. The direct current (dc) driven disordered vortex lattice has been studied extensively.[1−8] It is established that, for all but very weak pinning, the vortex lattice is plastically deformed and the onset of vortex motion takes place through channels. This result goes beyond the applicability of the collective pinning theory for current–voltage (I–V) characteristics.[9] An actual dynamic phase and a moving crystal for the vortex lattice was first proposed by Koshelev and Vinokur,[10] and has been numerically demonstrated.[5,8] Their idea is that the random pinning noise diminishes with increasing vortex velocity, allowing a high-velocity reordering.

Most recently, the alternating current (ac) response of vortex lattice has been widely studied.[11−13] Probing the dc I–V characteristics in the presence of superimposed rf currents has become a powerful tool to obtain time averaged, small scale information on the moving structure.[14−16]

In 1971, Fiory [17] observed steps in the I–V characteristics of ac driven superconducting aluminium films. In the last year, Kokubo *et al* [15] observed mode-locking steps in the I–V curves of vortex matter driven through mesoscopic channels. In fact, mode-locking steps have also been found in several systems with many degrees of freedom, such as charge density waves,[18,19] spin density waves,[20] and superconductors with periodic pinning arrays.[21]

Fiory attributed his observation to a quantum interference effect between the applied ac and the local-supercurrent oscillations generated by the moving vortex lattice.[17,22] However, Kolton *et al.* [14] pointed out that the mode-locking steps are correlated with the nature of temporal order in the absence of ac drive.

In this paper, we study the ac response of two-dimensional (2D) vortex lattice with randomly distributed point-like pinning centres. The object is to investigate the mode-locking behaviour with increasing strength of vortex–vortex interaction. The moving equation of vortices is described by [11,12]

$$\eta\frac{\mathrm{d}\boldsymbol{R}_i}{\mathrm{d}t} = -\sum_{i\neq j}\nabla_i U_{\mathrm{v}}(\boldsymbol{R}_i-\boldsymbol{R}_j) - \sum_{j'}\nabla_i U_{\mathrm{p}}(\boldsymbol{R}_i-\boldsymbol{r}_{j'}) + \boldsymbol{F}_{\mathrm{dc}} + \boldsymbol{F}_{\mathrm{ac}}\cos(\omega t), \tag{1}$$

where η is the viscosity coefficient, $\boldsymbol{R}_{i,j}$ and $\boldsymbol{r}_{j'}$ are the

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vortex and pin coordinates. $U_{\rm v}(\boldsymbol{R})$, $U_{\rm p}(\boldsymbol{R}-\boldsymbol{r})$, $\boldsymbol{F}_{\rm dc}$, $\boldsymbol{F}_{\rm ac}$ and ω are the vortex–vortex interaction potential, the vortex–pin interaction potential, the driving dc force, the amplitude and frequency of superimposed ac force, respectively.

It is known that the sample thickness d is much smaller than the magnetic penetration length λ ($d \ll \lambda$) in 2D thin films. Therefore, the interaction potential between vortices can be regarded as the long-range and repulsive logarithmic [8,11,12]

$$U_{\rm v}(\boldsymbol{R}) = A_{\rm v}\left(\frac{R^2}{R_{\rm v}^2}-1\right)\ln\left(\frac{R}{R_{\rm v}}\right). \qquad (2)$$

The vortex–pin interaction is modelled by an attracting Gaussian potential [8,23]

$$U_{\rm p}(\boldsymbol{r}) = -\frac{F_{\rm p}}{2}r_{\rm p}\exp\left(-\frac{r^2}{r_{\rm p}^2}\right). \qquad (3)$$

We fix our force and time scales by putting the parameters F_p and η equal to unity. All the lengths will be measured with respect to a_0 $\left(\frac{\sqrt{3}}{2}a_0^2 = n_{\rm v}^{-1},\right.$ where $n_{\rm v}$ is the vortex density$\left.\right)$. $A_{\rm v}$ is the dimensionless prefactor of vortex–vortex interaction potential. $R_{\rm v}$ and the size of pinning centres $r_{\rm p}$ are chosen to be $R_{\rm v} = 2.8a_0$ and $r_{\rm p} = 0.1a_0$, respectively. $N_{\rm v}$(=100) vortices are initially placed in a perfect triangular lattice subject to periodic triangular boundary conditions, and $N_{\rm p}$(=800) point-like pinning centres are randomly distributed (dense pinning is typically realized in experimental samples). The direction of superimposed ac force is fixed to that of the driving dc force. We slowly increase $F_{\rm dc}$ along the horizontal symmetry axis (x-axis) and measure the average velocity $V_{\rm d}=\frac{1}{N_{\rm v}}\sum_{i=1}^{N_{\rm v}} V_i\cdot\hat{x}$ and its differential ${\rm d}V_{\rm d}/{\rm d}F_{\rm dc}$. A time integration step of Δt=0.001 is used and averages are evaluated during 2×10^5 steps after 1×10^5 steps for equilibrium.[12]

Figure 1 shows a $V_{\rm d}$–$F_{\rm dc}$ curve for $F_{\rm ac}$=0. From Fig.1 we can see that there exists a critical pinning force $F_{\rm c}$ ($F_{\rm c}\approx 0.82$) below which the lattice is pinned and the velocity is generated mainly by rearrangement of topological defects, producing so small an advance of the pinned lattice that can be neglected. When the driving force $F_{\rm dc}$ is increased above $F_{\rm c}$, plastic flow occurs and vortices flow in channels. In this regime, the velocity–force curve is strongly nonlinear. When $F_{\rm dc}$ is further increased above $F_{\rm s}$ ($F_{\rm s}\approx 2.4$), smectic flow takes place and all the vortices move in a seemingly isotropic channel network with maximum interconnectivity. The orientational order begins to appear in the moving direction in this region.

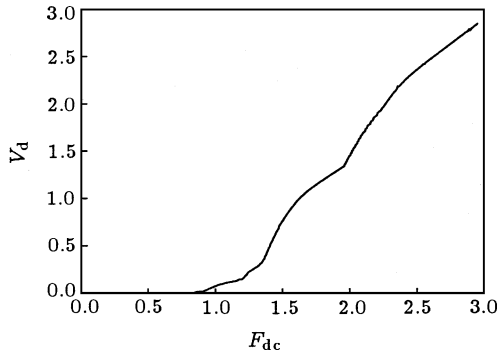


Fig.1. $V_{\rm d}$–$F_{\rm dc}$ curve for $F_{\rm ac}$=0 and $A_{\rm v}$=0.0007.

To see clearly the above features, we examine the structure factor [7]

$$S(\boldsymbol{k}) = \left\langle \left| \left(\frac{1}{N_{\rm v}}\sum_{i=1}\exp[{\rm i}\boldsymbol{k}\cdot\boldsymbol{R}_i(t)]\right|^2 \right\rangle \qquad (4)$$

in different regimes. Fig.2 presents the structure factors at two different values of $F_{\rm dc}$. From Fig.2(a), we can see that, when $F_{\rm dc} < F_{\rm c}$ there are no Bragg peaks in the structure factor and only one peak is found in the centre ($k_x = k_y = 0$). However, at high enough driving forces ($F_{\rm dc} \geq 2.4$), the order takes place as shown in Fig.2(b) where we find two Bragg peaks along the k_x axis. The phenomenon indicates that the smectic order occurs and the positional correlation along the flow direction begins to arise.

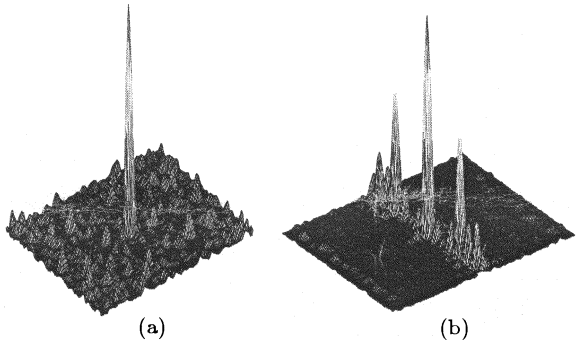


Fig.2. The structure factor $S(\boldsymbol{k})$ for $F_{\rm ac}$=3.5. (a) $F_{\rm dc}$=0.5. (b) $F_{\rm dc}$=2.7.

When an ac force is superimposed, mode-locking steps appear in the $V_{\rm d}$–$F_{\rm dc}$ curve. When a vortex lattice moves coherently at the average velocity through a weak pinning potential, a microscopically periodic velocity modulation is induced.[24] In the presence of ac force with frequency ω these modulations get

mode-locked, giving rise to interference plateaus in the curves of velocity–force dependence.[15] The step velocity and width vary with the amplitude and frequency of ac force, which have been studied in detail in Ref.[11].

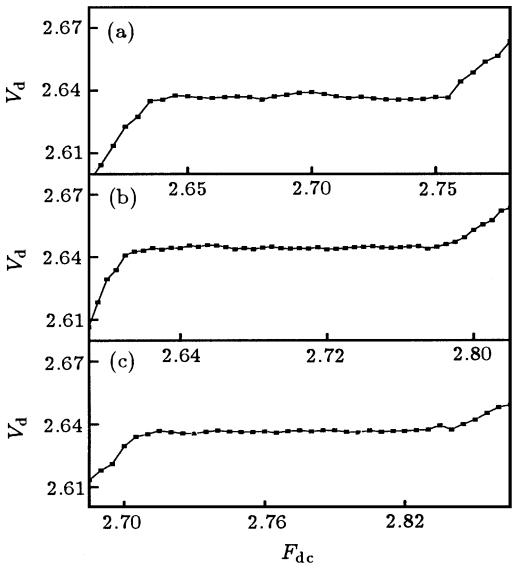


Fig.3. V_d–F_{dc} curve for F_{ac}=3.5. (a) A_v=0.0005. (b) A_v=0.001. (c) A_v=0.002.

We now investigate the mode-locking behaviour with different vortex–vortex interacting strengths (through changing the dimensionless prefactor A_v) for a given ac force (F_{ac}=3.5 and ω=5.0). With a given ac force, a mode-locking step is found in each simulated V_d–F_{dc} curve at high dc forces. Figs.3(a)–(c) show the steps for different values of A_v. We find that the step velocity does not vary basically with A_v, but the step width changes. Detailed analysis tells us that the step width ΔF_{dc} shows damped oscillation with A_v, as shown in Fig.4. This is in good agreement with recent experiments on vortex matter driven through mesoscopic channels.[15] The reason is as follows. For small values of A_v, the pinning disorder can compete with the vortex interaction, and the microscopically periodic velocity modulation maintains, thus the step width is generally large. Moreover, in this case, the mode-locking behaving is much influenced by the vortex–vortex interaction, resulting in a rapid variation of the step width with A_v. When A_v is increased above a certain value, the vortex–vortex interaction dominates and the pinning force is small compared with the interaction between vortices. The coherent motion of vortices is little modulated by the very weak pinning potential. Therefore, the step width is small and varies slowly with A_v.

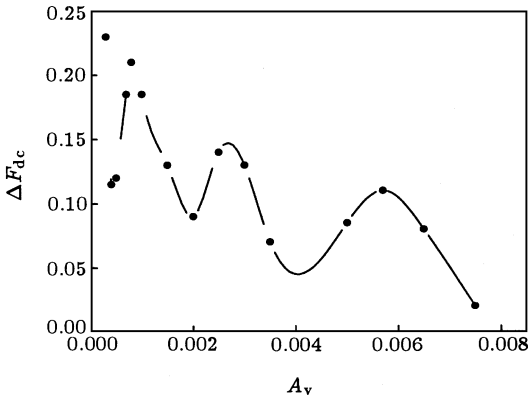


Fig.4. The step width ΔF_{dc} vs A_v.

In summary, we have studied numerically the ac response of a driven disordered vortex lattice. In the presence of a superimposed ac force, mode-locking steps have been observed in the velocity–dc force characteristics at high dc forces, where we found the smectic order. The step velocity does not vary with the strength of interaction between vortices, while the step width shows damped oscillations, in good agreement with recent experiments.

References

[1] Brand E H 1983 *Phys. Rev. Lett.* **50** 1599
Jensen H J, Brechet Y and Brass A 1989 *J. Low Temp. Phys.* **74** 293

[2] Shi A C and Berlinsky A J 1991 *Phys. Rev. Lett.* **67** 1926

[3] Ryu S, Hellerqvist M, Doniach S, Kapitulnik A and Stroud D 1996 *Phys. Rev. Lett.* **77** 5114

[4] Cha M C and Fertig H A 1998 *Phys. Rev. Lett.* **80** 3851

[5] Olson C J, Reichhardt C and Nori F 1998 *Phys. Rev. Lett.* **81** 3757

[6] Cao Y G and Jiao Z K 1999 *Physica* C **321** 177

[7] Kolton A B, Dominguez D and Jensen N G 1999 *Phys. Rev. Lett.* **83** 3061

[8] Cao Y G, Jiao Z K and Ying H P 2000 *Phys. Rev.* B **62** 4163
Cao Y G and Jiao Z K 2000 *Physica* C **334** 283

[9] Larkin A I and Ovchinnikov Yu N 1979 *J. Low Temp. Phys.* **34** 409

[10] Koshelev A E and Vinokur V M 1994 *Phys. Rev. Lett.* **73** 3580

[11] Cao Y G and Jiao Z K 2003 *Physica* C **387** 341

[12] Cao Y G and Jiao Z K 2003 *Commun. Theor. Phys.* **39** 112
[13] Cao Y G and Jiao Z K 2000 *Chin. Phys.* **9** 199
[14] Kolton A B, Dominguez D and Jensen N G 2001 *Phys. Rev. Lett.* **86** 4112
[15] Kokubo N, Besseling R, Vinokur V M and Kes P H 2002 *Phys. Rev. Lett.* **88** 247004
[16] Van Look L, Rosseel E, Van Bael M J, Temst K, Moshchalkov M M and Bruynseraede Y 1999 *Phys. Rev.* B **60** R6998
[17] Fiory A T 1971 *Phys. Rev. Lett.* **27** 501
[18] Middleton A A, Biham O, Littlewood P B and Sibani P 1992 *Phys. Rev. Lett.* **68** 1586
[19] Higgin M J, Middlenton A A and Bhattacharya S 1993 *Phys. Rev. Lett.* **70** 3786
[20] Barthel E, Kriza G, Quirion G, Wzietek P, Jérome D, Christensen J B, Jϕrgensen M and Bechgaard K 1993 *Phys. Rev. Lett.* **71** 2825
[21] Reichhardt C 2000 *Phys. Rev.* B **61** R11914
[22] Fiory A T 1973 *Phys. Rev.* B **7** 1881
[23] Brandt E H 1983 *J. Low Temp. Phys.* **53** 71
[24] Troyanovski A M, Aarts J and Kes P H 1999 *Nature* (London) **399** 665