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Investigation of spontaneous magnetization of coupled 2×2 superconducting π ring array*

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We present the theoretical investigation of spontaneous magnetization of a coupled 2×2 π ring array. It is indicated by free energy calculation that the system has the lowest energy when the four π rings have the full antiparallel configuration. Furthermore, the numerical evaluation results show that the system which favours full antiparallel spontaneous magnetization is a quantum effect deriving from the phase cohering of the superconducting quantum wavefunctions in the four superconducting rings through the shared Josephson junctions.

Keywords: π ring array, Josephson junction, spontaneous magnetization
PACC: 7450, 7510H

1. Introduction

Josephson effects are the fundamental of superconducting electronics and the extensive research of Josephson junctions in low-temperature superconductor (LTSC) results in various weak current applications of superconductors.[1] High-temperature superconductor (HTSC) normal Josephson junctions were prepared successfully soon after the discovery of HTSC in the 1980s. Physicists found that HTSC could make Josephson junctions which have negative critical current under properly geometric configurations, due to the d wave symmetry of HTSC. This is equivalent to introducing a π phase shift to the normal Josephson junction and it was named π junction.[2] Recently another type of π junction was found in theory and experiment, whose composition is superconductor–ferromagnet–superconductor, and the superconductor here is LTSC.[3,4]

The π ring is a superconducting ring that consists of odd number of π junctions. It was first indicated by Sigrest and Rice [5] that the single-junction π ring will spontaneously magnetize when the inductance parameter $\beta = 2\pi LI_c/\Phi_0 > 1$, where L is the ring inductance, I_c the junction's critical current and Φ_0 the flux quantum. In a previous work,[6] we have systematically analysed the spontaneous magnetization of the multi-junction π ring in theory. Our results illustrate that the multi-junction π ring is different from the single-junction π ring: the system will spontaneously magnetize even when β approaches zero. The calculation also shows that the magnetic flux of the ring's spontaneous magnetization holds to $\Phi_0/2$ when the temperature is lower than the superconducting transition temperature. Until the temperature is very close to the superconducting transition temperature, the magnetic flux of the ring will reduce to zero rapidly. These results coincide with Tsuei's experimental observations.[7]

The investigation of π ring arrays has attracted considerable interest for both fundamental research and the potential applications. For example, they can be used to physically simulate the Ising model to study magnetic phenomena in antiferromagnets,[8,9] and to design a quantum computer based on flux-qubits.[10,11] Recently, Hilgenkamp *et al*[12] reported the fabrication of large-scale coupled π ring arrays based on $YBa_2Cu_3O_7$–Au–Nb ramp junctions, which makes it possible to study π ring arrays in experi-

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ment. We have studied the spontaneous magnetization of coupled two π rings in a recent work.[13] Our conclusion is that the system has a lower free energy when the two coupled π rings have inverse spontaneous magnetization configuration. Therefore, the flux polarity antiparallel is the spontaneous magnetization state of coupled one-dimensional π ring chain, which is consistent with the experimental observation of one-dimensional π ring chain with scanning superconducting quantum interference device (SQUID) by Hilgenkamp *et al.*[12] For further understanding the spontaneous magnetization of π ring arrays, we investigate the spontaneous magnetization of the simplest case of two-dimensional π ring arrays: 2×2 π ring array in this paper. Our conclusion is that the system has the lowest free energy when the four π rings have the full antiparallel configuration. In addition, the numerical evaluation results indicate that the spontaneous magnetization of the full antiparallel state in a coupled 2×2 π ring array is a quantum effect deriving from the phase coherence of the superconducting quantum wavefunctions in the four superconducting rings through shared Josephson junctions. We infer that the ground state of the coupled π ring array is a spontaneously magnetized full-antiparallel-configured two-dimensional antiferromagnet.

2. Free energy of the coupled 2×2 π ring array

The 2×2 π ring array is composed of two normal Josephson junctions (0 junction) and two π Josephson junctions (π junction) and they are connected by superconductor wires. Figure 1 shows the configuration of the four junctions: J_1, J_2 are 0 junctions, J_3, J_4 are π junctions, and two adjacent junctions make a π ring. For simplicity, we assume that all the junctions are small junctions and they have the same absolute critical current: $I_{c1} = I_{c2} = I_c$, $I_{c3} = I_{c4} = -I_c$, and we neglect the width of the superconductor wire. During the following derivation, we consider that the inductance of each π ring is L, the mutual inductance of the nearest neighbour rings is M_1=K_1L, and that of the next nearest neighbour rings is M_2=K_2L and $K_1 > K_2$.

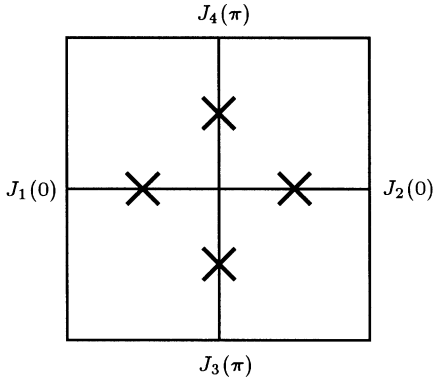


Fig.1. The configuration of 0 and π junctions in a 2×2π ring array, in which two adjacent junctions make a π ring.

The four π rings will spontaneously magnetize under zero magnetic field.[5] The spontaneous magnetization of the 2×2 π ring array can be classified by the state of the four π rings' spontaneous magnetization circulating current directions, and we will calculate their free energy.

2.1. Full antiparallel

The configuration of the full antiparallel 2×2 π ring array is shown in Fig.2(a), and the direction of each π ring's circulating current is shown in Fig.2(b). The direction of the circulating current in rings I, III is clockwise and that in rings II, IV is counter clockwise.

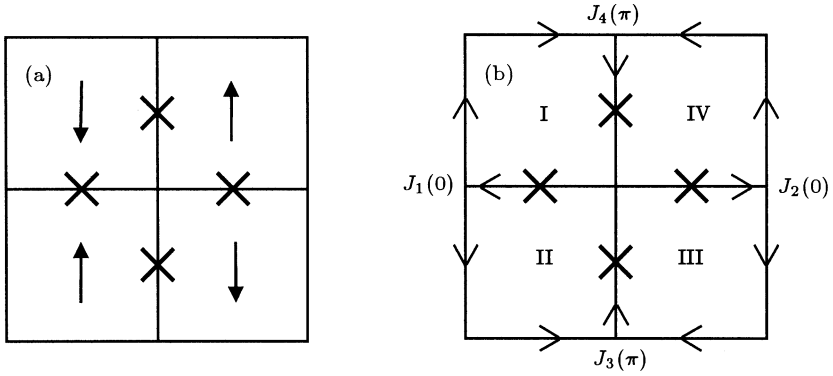


Fig.2. (a) The configuration of the 2×2 π ring array in which the flux polarity of spontaneous magnetization is full antiparallel; (b) the directions of the circulating currents in the four rings.

For ring III, if we assume its circulating current is I, then

$$\varphi_2 + \varphi_3 + \frac{2\pi}{\Phi_0}\Phi = 2n\pi, \quad (n = 0, \pm 1, \pm 2, \cdots), \quad (1)$$

where φ_2 and φ_3 are the gauge invariant phase difference of junctions J_2 and J_3 respectively; Φ is the flux in ring III. Because of the symmetry of the system's configuration, the other three rings have the same circulating current I, and thus

$$\Phi = LI + 2M_1 I - M_2 I = (1 + 2K_1 - K_2)LI, \quad (2)$$

where LI is the flux coming from the circulating current of ring III, $2M_1 I$ is the mutual inductance flux in ring III, which is generated from the circulating current of rings II and IV, and $M_2 I$ is the mutual inductance flux coming from ring I.

The current through J_2 can be written as $2I = I_c \sin\varphi_2$, and the current through J_3 is $2I = -I_c \sin\varphi_3$, so

$$\varphi_3 = \varphi_2 + \pi. \quad (3)$$

Introducing Eqs.(2) and (3) and the inductance parameter $\beta = 2\pi LI_c/\Phi_0$ into Eq.(1) yields

$$2\varphi_2 + \frac{1 + 2K_1 - K_2}{2}\beta \sin\varphi_2 = (2n-1)\pi. \quad (4)$$

Solving equations of $I = \frac{I_c}{2}\sin\varphi_2$ and $\beta = 2\pi LI_c/\Phi_0$, we can obtain the magnitude of the circulating current

$$I = \frac{\beta\Phi_0}{4\pi L}\sin\varphi_2. \quad (5)$$

The free energy of the system is

$$\begin{aligned} F =& 4 \times \frac{1}{2}LI^2 - 4 \times M_1 I^2 + 2 \times M_2 I^2 \\ &+ E_{J1} + E_{J2} + E_{J3} + E_{J4} \\ =& 2LI^2 - 4K_1 LI^2 + 2K_2 LI^2 \\ &- 2\frac{I_c\Phi_0}{2\pi}\cos\varphi_2 + 2\frac{I_c\Phi_0}{2\pi}\cos\varphi_3 \\ =& \frac{2}{L}\left(\frac{\Phi_0}{2\pi}\right)^2\left[(1 - 2K_1 + K_2)\frac{\beta^2}{4}\sin^2\varphi_2 - 2\beta\cos\varphi_2\right]. \end{aligned} \quad (6)$$

We define energy

$$F_0 = \frac{1}{2L}\left(\frac{\Phi_0}{2\pi}\right)^2. \quad (7)$$

The normalized free energy of the system can be written as

$$f = \frac{F}{F_0} = 4\left[(1 - 2K_1 + K_2)\frac{\beta^2}{4}\sin^2\varphi_2 - 2\beta\cos\varphi_2\right], \quad (8)$$

where φ_2 is determined by Eq.(4) for a given inductance parameter β.

2.2. Full parallel

Figure 3(a) shows the configuration of the 0 and π junctions in a 2×2 π ring array in which all the π rings have the same spontaneous magnetization direction. The circulating current directions shown in Fig.3(b) are all counter clockwise.

Because of the symmetry of the π ring array, the currents through all the four junctions are zero. So

$$\varphi_1 = \varphi_2 = 0.$$

For the lowest free energy, it should be

$$\varphi_3 = \varphi_4 = \pi. \quad (9)$$

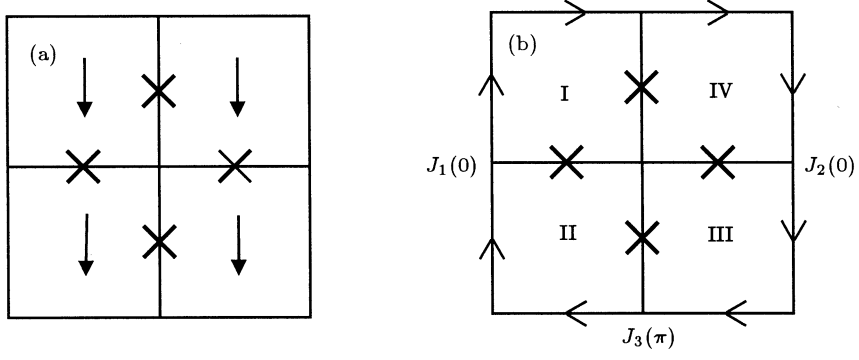


Fig.3. (a) The configuration of a 2×2 π ring array in which the flux polarity of spontaneous magnetization is full parallel; (b) the directions of circulating currents in the four rings.

For ring III, if we assume the circulating current is I, then

$$\varphi_2 + \varphi_3 + \frac{2\pi}{\Phi_0}(LI - 2K_1LI - K_2LI) = 2n\pi.$$

When n=1, the circulating current flux is

$$LI = \frac{\Phi_0}{2(1 - 2K_1 - K_2)}. \tag{10}$$

The free energy of the system is given by

$$\begin{aligned} F =& 4 \times \frac{1}{2}LI^2 + 4M_1I^2 + 2M_2I^2 \\ &+ E_{J1} + E_{J2} + E_{J3} + E_{J4} \\ =& \frac{2}{L}(1 + 2K_1 + K_2) \\ &\times \left(\frac{\Phi_0}{2}\right)^2 \frac{1}{(1 - 2K_1 - K_2)^2} - 4\frac{I_c\Phi_0}{2\pi} \\ =& \frac{2}{L}\left(\frac{\Phi_0}{2\pi}\right)^2 \left[\frac{1 + 2K_1 + K_2}{(1 - 2K_1 - K_2)^2}\pi^2 - 2\beta\right]. \end{aligned}$$

The normalized free energy of the system is

$$f = \frac{F}{F_0} = 4\left[\frac{1 + 2K_1 + K_2}{(1 - 2K_1 - K_2)^2}\pi^2 - 2\beta\right]. \tag{11}$$

2.3. Half parallel, half antiparallel

Figure 4(a) shows the configuration of the 0 and π junctions in a 2×2 π ring array in which the π rings' spontaneous magnetization directions are half parallel and half antiparallel.

If we assume the circulating current in ring III is I, the currents through all the four junctions are I because of the symmetry of the system. The Josephson current through junction J_2 is

$$2I = I_c \sin\varphi_2. \tag{12}$$

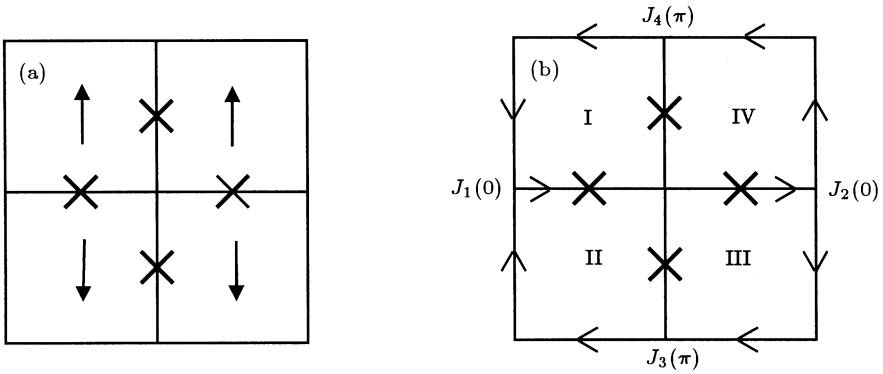


Fig.4. (a) The configuration of a 2×2 π ring array in which the flux polarity of spontaneous magnetization is half parallel, half antiparallel; (b) the directions of circulating currents in the four rings.

The current through J_3 is zero and the phase difference in J_3 should be π for the lowest free energy of the system.[13] So we have

$$\varphi_2 + \pi + \frac{2\pi}{\Phi_0}(LI + M_1I - M_1I + M_2I) = 2n\pi.$$

And so

$$\varphi_2 + (1 + K_2)\frac{\beta}{2}\sin\varphi_2 = (2n - 1)\pi. \tag{13}$$

Introducing inductance parameter β into Eq.(12), the circulating current flux is

$$LI = \frac{\Phi_0}{2\pi}\frac{\beta}{2}\sin\varphi_2. \tag{14}$$

The free energy of the system is given by

$$\begin{aligned} F =& 4 \times \frac{1}{2}LI^2 + 2M_1I^2 - 2M_1I^2 - 2M_2I^2 \\ &+ E_{J1} + E_{J2} + E_{J3} + E_{J4} \\ =& 2(1 - K_2)LI^2 - 2\frac{I_c\Phi_0}{2\pi}\cos\varphi_2 - 2\frac{I_c\Phi_0}{2\pi} \\ =& \frac{2}{L}\left(\frac{\Phi_0}{2\pi}\right)^2 \left[(1 - K_2)\left(\frac{\beta}{2}\right)^2 \sin^2\varphi_2 - \beta\cos\varphi_2 - \beta\right]. \end{aligned}$$

The normalized free energy is given by

$$f = 4\left[(1 - K_2)\left(\frac{\beta}{2}\right)^2 \sin^2\varphi_2 - \beta\cos\varphi_2 - \beta\right]. \tag{15}$$

3. Discussion

We classify the spontaneous magnetization of a 2×2 π coupled ring array to three states: full antiparallel, full parallel, half parallel, half antiparallel; we

have calculated their free energies respectively. Actually, in spite of these three topological symmetry situations there is the fourth state: three spontaneous magnetization directions upward and one direction downward. However, the circulating currents in the four rings cannot be equal forever in this situation because of the geometric asymmetry of the array. This can be confirmed in physics also: if we assume that all the rings have the same circulating current, a contradictory result will be produced when we list and solve the four Josephson junctions' current equations and the four π rings' single-valued wavefunctions. Therefore, this spontaneous magnetization state cannot exist.

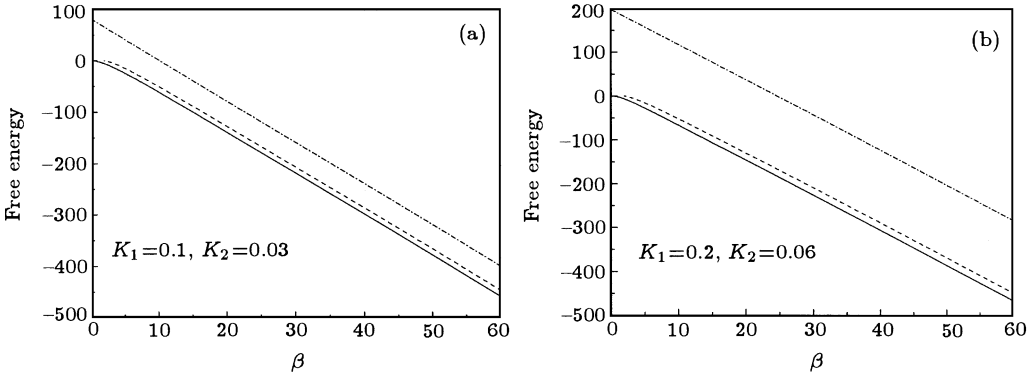


Fig.5. The variation of normalized free energy f with the conductance parameter β for different spontaneous magnetization states: full antiparallel (solid line), full parallel (dot-dashed line) and half parallel, half antiparallel (dashed line) in the coupled 2×2 π ring array. The mutual inductance parameters are K_1=0.1, K_2=0.03 (a) and K_1=0.2, K_2=0.06 (b).

The relationship between normalized free energy f and inductance parameter β of the full antiparallel, full parallel and half parallel half antiparallel is determined by Eqs.(4) and (8), (11), (13) and (15) respectively. We calculate numerically the free energies of these three situations to compare their magnitudes. During the calculation, we take n=1, and mutual inductance K_2=0.3K_1. Figure 5(a) shows the results when K_1=0.1, K_2=0.03 and Fig.5(b) shows the results when K_1=0.2, K_2=0.06. The results indicate that the full antiparallel has the lowest free energy. Furthermore, the larger the mutual inductance, the larger the magnitude of free energy difference for different spontaneous magnetization states, because the magnetic interaction energy of neighbour rings becomes larger.

The free energy of the coupled π ring array includes the self-magnetic energy of each ring, the magnetic interaction energy of neighbour rings and the Josephson coupling energy of each superconducting junction. Numerical evaluation indicates that the magnetic interaction energy is favourable to the state of full antiparallel spontaneous magnetization of the π ring array. The expression of free energy of the π ring array in Section 2 of this article illustrates that the Josephson junctions have different phase differences. When the mutual inductance is small enough, it can be neglected (K_1=K_2=0); the free energy difference of different spontaneous magnetization states only comes from the contribution of phase difference. Figure 6 shows the variation of the free energy for different states with the inductance parameter β, in which it still could be seen that the full antiparallel spontaneous magnetization state has the lowest free energy

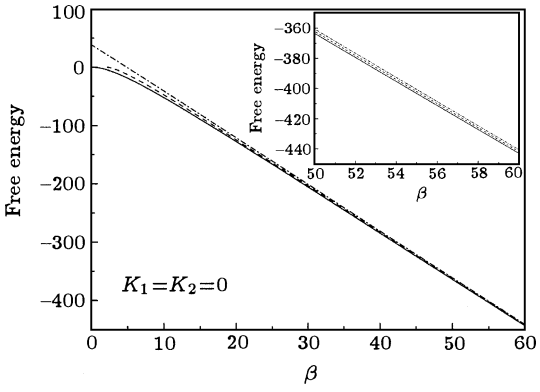


Fig.6. The variation of normalized free energy f with the inductance parameter β for different spontaneous magnetization states: full antiparallel (solid line), full parallel (dot-dashed line) and half parallel, half antiparallel (dashed line) in a coupled 2×2 π ring array when the mutual inductance is neglected. The inset is a magnification of the main picture for clearness.

in this situation. This indicates that the π ring array favours the full antiparallel spontaneous magnetization state which is due to the quantum effect coming from the phase coherence of the superconducting quantum wavefunctions in the superconducting rings through the shared Josephson junctions, not caused by the magnetic interaction energy of the four rings, although the existence of magnetic interaction energy favours the state of full antiparallel spontaneous magnetization.

4. Conclusion

In conclusion, we have deduced and numerically calculated the free energies of the spontaneous magnetization of a coupled 2×2 π ring array. It is indicated that the full antiparallel state has the lowest energy when the system has a defined inductance parameter β. In addition, the numerical evaluations show that the system favours the full antiparallel spontaneous magnetization state, which is due to the quantum effect deriving from the phase coherence of the superconducting quantum wavefunctions in the superconducting rings through the shared Josephson junctions. Furthermore, we can infer that the ground state of the coupled π ring array is a spontaneous magnetization full antiparallel configured two-dimensional antiferromagnet. The two-dimensional π ring array would construct an artificial ideal antiferromagnetic ordering state if it could be prepared experimentally, and the ring array could be used to simulate the physics in the two-dimensional antiferromagnet Ising model.

References

[1] Barone A and Paterno G 1982 *Physics and Applications of the Josephson Effect* (New York: Wiley Interscience)

[2] Geshkenbein V B, Larkin A I and Barone A 1987 *Phys. Rev.* B **36** 235

[3] Buzdin A I, Bulaevskii L N and Paniukov S V 1982 *JETP* **35** 178

[4] Guichard W, Aprili M, Bourgeois O, Kontos T, Lesueur J and Gandit P 2003 *Phys. Rev. Lett.* **90** 167001

[5] Sigrist M and Rice T M 1992 *J Phys. Soc. Japan* **61** 4283

[6] Deng P, Meng S C, Wang F R, Xie F X, Ma P and Dai Y D 2001 *Acta Phys. Sin.* **50** 2217 (in Chinese)

[7] Tsuei C C, Kirtley J R and Chi C C 1994 *Phys. Rev. Lett.* **73** 593

[8] Aeppli G and Chandra P 1997 *Science* **275** 177

[9] Davidovic D, Kumar S, Reich D H, Siegel J, Field S B, Tiberio R C, Hey R and Ploog K 1997 *Phys. Rev.* B **55** 6518

[10] Mooij J E, Orlando T P, Levitov L, Tian L, van der Wal C H and Lloyd S 1999 *Science* **285** 1036

[11] Loffe L B, Geshkenbein V B, Feigel'man M V, Fauchère A L, Blatter G 1999 *Nature* **398** 679

[12] Hilgenkamp H, Ariando, Smilde H H, Blank D H A, Rijnders G, Rogalla H, Kirtley J R and Tsuei C C 2003 *Nature* **422** 50

[13] Wang K, Ling J, Xie F X, Ma P, Yang T, Wang F R and Dai Y D 2003 *Acta Phys. Sin.* **52** 1509 (in Chinese)