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Mathematical modelling of gain-switched RF-excited CO_2 waveguide laser*

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The detailed mathematical models for the evolution of light pulses in RF-excited CO_2 waveguide lasers are derived. Explicit expressions for the pulse characteristics in RF-excited CO_2 waveguide lasers are obtained. The effects of losses and unsaturated gain on output power are calculated.

Keywords: mathematical modelling, gain-switched, RF excited, CO_2 waveguide lasers
PACC: 4255D, 0520D

1. Introduction

Due to their interesting physical and chemical parameters, CO_2 lasers have become widely utilized in scientific and industrial applications. However, many of the processes for determining the dynamic transitions between vibrational energy levels of molecules of the gas mixture CO_2–N_2–He are still not fully understood.[1,2] For instance, there are many factors that may cause a decrease in the output power, such as nonsystematical electrical discharge and dissociation of CO_2 to CO molecules.[3] The degree of dissociation could reach 80%–90% according to experimental data, particularly in sealed systems.[4] Letalick *et al* made a theoretical analysis for the kinetic processes of a Q-switched CO_2 laser.[5] They considered mainly the processes of rotational relaxation but neglected some processes such as dissociation of CO_2, electron collision excitation and energy-transfer of some vibrational levels.[6]

In addition to the dissociation phenomena, many reactions in the gas mixture could produce different molecular gases such as NO_2, O_2, N_2O, NO, CO_2^+, CO_2^-, ..., etc.

In previous studies of the output power CO_2 lasers, there are several models describing the dynamic emission of CO_2 laser: i.e. four-, five- and six-temperature models.[7–9] The purpose of the present work is to develop new analytic models for the laser output including a time-dependent pump rate and spontaneous relaxation.

These solutions are reduced to the simplest possible mathematical forms and then applied to specific laser systems.

2. Theoretical analysis

There are various ways to approach the problem of light propagation in CO_2 laser systems, and the details depend on the type of approximations that can be made. In the majority of practical applications the coherence time is short compared with any other time of interest and the behaviour is governed by familiar rate equations.[10] Thus the coupled pair of rate equations governing the frequency-dependent population densities of the upper and lower laser states in a medium with inhomogeneous broadening is [11]

$$\begin{aligned}&\frac{\partial n_2(y,z,t)}{\partial t}\\ &=s_2(y,z,t)\\ &\quad-n_2(y,z,t)\left(A_2+\frac{2B_0}{\pi\Delta\upsilon_{\rm h}}\int_{-\infty}^{\infty}\frac{I(y_l,z,t)}{1+(y-y_l)^2}{\rm d}y_l\right)\\ &\quad+n_1(y,z,t)\frac{2B_0}{\pi\Delta\upsilon_{\rm h}}\int_{-\infty}^{\infty}\frac{I(y_l,z,t)}{1+(y-y_l)^2}{\rm d}y_l,\end{aligned}\tag{1}$$

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$$\frac{\partial n_1(y,z,t)}{\partial t} = s_1(y,z,t) + n_2(y,z,t) \times \left(A_{21} + \frac{2B_0}{\pi\Delta\upsilon_{\rm h}} \int\limits_{-\infty}^{\infty} \frac{I(y_l,z,t)}{1+(y-y_l)^2}{\rm d}y_l\right) - n_1(y,z,t)\left(A_1 + \frac{2B_0}{\pi\Delta\upsilon_{\rm h}} \int\limits_{-\infty}^{\infty} \frac{I(y_l,z,t)}{1+(y-y_l)^2}{\rm d}y_l\right). \quad (2)$$

Here $s_1\ (y,z,t)$ and $s_2\ (y,z,t)$ are space- and time-dependent pumping rates, $y_l = 2(\upsilon_{\rm i}-\upsilon_0)/\Delta\upsilon_{\rm h}$ is a frequency parameter normalized in units of the homogeneous linewidth. $\Delta\upsilon_{\rm h}, I(y_l,z,t)$ is the average spectral density at the frequency y_l, and the significance of the Einstein A and B coefficients is apparent. If the lower state decays quickly compared with the pulse length, it follows from Eqs.(1) and (2) that the inversion density $n = n_2 - n_1$ is governed by

$$\frac{\partial n(y,z,t)}{\partial t} = s(y,z,t) - n(y,z,t)\frac{2B_0}{\pi\Delta\upsilon_{\rm h}} \int\limits_{-\infty}^{\infty} \frac{I(y_l,z,t)}{1+(y-y_l)^2}{\rm d}y_l - \frac{n(y,z,t)}{\tau_2}. \quad (3)$$

In a CO_2 laser medium, the growth of intensity is proportional to the population inversion, and the governing equation is

$$\frac{\partial I(y_l,z,t)}{\partial z} + \frac{1}{\vartheta_g}\cdot\frac{\partial I(y_l,z,t)}{\partial t} = h\upsilon_l \frac{2B_0}{\pi\Delta\upsilon_{\rm h}}[I(y_l,z,t)+\eta] \times \int\limits_{-\infty}^{\infty} \frac{n(y,z,t)}{1+(y-y_l)^2}{\rm d}y - \gamma I(y_l,z,t), \quad (4)$$

where ϑ_g is the group velocity. The cavity losses are lumped into the parameter $\gamma_{\rm c}=(2l\gamma - \ln R_1R_2)/T$, where γ represents distributed losses, l is the amplifier length, T is the round trip time ($T = 2L/c$), L is the cavity length, c is the velocity of light, and R_1 and R_2 are the mirror reflectivities.

Then Eqs.(3) and (4) can be integrated over the frequency to obtain the following expressions:

$$\frac{\partial N(z,t)}{\partial t} = S(z,t) - sI(z,t)N(z,t) - \frac{N(z,t)}{\tau_2}, \quad (5)$$

$$\frac{\partial I(z,t)}{\partial t} + \frac{1}{\vartheta_g}\cdot\frac{\partial I(z,t)}{\partial t} = h\upsilon_l s[I(z,t)+I_0]N(z,t) - \gamma I(z,t), \quad (6)$$

where a new inversion density

$$N(z,t) = \int\limits_{-\infty}^{\infty} n(y,z,t){\rm d}y,$$

intensity

$$I(z,t) = \int\limits_{-\infty}^{\infty} I(y_l,z,t){\rm d}y_l,$$

pump

$$S(z,t) = \int\limits_{-\infty}^{\infty} S(y,z,t){\rm d}y,$$

saturation parameter $s = \dfrac{2B_0}{\pi\Delta\upsilon_{\rm h}}$, and I_0 is the noise intensity (spontaneous emission input); $I_0 = \pi\eta$, η is the noise parameter $\eta = h\upsilon_l\Delta\upsilon_{\rm h}/2A$, where A is the mode area and the homogeneous pressure-broadened linewidth of the transition at a temperature of 300K for 10P20 line (λ=10.6μm) of CO_2 laser is given in Ref.[12]: $\Delta\upsilon_{\rm h} = 7.58\times10^6(\psi_{\rm CO_2}+0.73\psi_{\rm N_2}+0.6\psi_{\rm He})\cdot P$, where ψ_x is the partial fraction of gas x, and P is the pressure in Pa.

Equation (5) is a linear first-order differential equation with nonconstant coefficients. The solution of such equations is well known and the result is given by [13]

$$N(z,t) = \exp\left[-s\int\limits_{-\infty}^{t} I(z,t'){\rm d}t' - \frac{t}{\tau_2}\right] \times \int\limits_{-\infty}^{t} S(z,t')\exp\left[s\int\limits_{-\infty}^{t'} I(z,t''){\rm d}t'' + \frac{t'}{\tau_2}\right]{\rm d}t'. \quad (7)$$

The inversion at $t = -\infty$ has been assumed to be zero. Substituting Eq.(7) in Eq.(6), we obtain

$$\frac{\partial I(z,t)}{\partial t} + \frac{1}{\vartheta_{\rm g}}\cdot\frac{\partial I(z,t)}{\partial t} = h\upsilon_l s[I(z,t)+I_0)]\exp\left(-s\int\limits_{-\infty}^{t} I(z,t'){\rm d}t' - \frac{t}{\tau_2}\right) \times \int\limits_{-\infty}^{t} S(z,t')\exp\left(s\int\limits_{-\infty}^{t'} I(z,t''){\rm d}t'' + \frac{t'}{\tau_2}\right){\rm d}t' - \gamma I(z,t). \quad (8)$$

As a first check on the practical applicability of Eq.(8), one may consider the determination of the unsaturated gain. The single most important pumping function in practice is the z-independent exponential,

$$S(z,t) = S_0\exp\left(-\frac{1}{a\tau_3}\right), \quad (0<t<t_0); \qquad = 0, \quad (t<0). \quad (9)$$

Equation (8) is the basic working equation for homogeneously broadened lasers. After some basic algebra, one obtains

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t} = \frac{2l}{T} h\upsilon_l s(I(t)+I_0) \times \exp\left(-2s\int_{-\infty}^{t} I(t')\mathrm{d}t' - \frac{t}{\tau_2}\right) \times \int_{-\infty}^{t} S(t')\exp\left(2s\int_{-\infty}^{t'} I(t'')\mathrm{d}t'' + \frac{t'}{\tau_2}\right)\mathrm{d}t' - \gamma_c I(t), \quad (10)$$

where the gain is represented by the coefficient $g_c = \frac{2lh\upsilon_l s S_0 \tau_3}{T}$, and τ_3 is the relaxation time associated with the transfer of energy between the symmetric mode of N_2 and the asymmetric mode of CO_2. In the limit $\tau_3 \to 0$ Eq.(10) can be simplified to

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t} = \frac{2lS_0 h\upsilon_l s\tau_3}{T}(I(t)+I_0) \times \exp\left(-2s\int_{0}^{t} I(t')\mathrm{d}t' - \frac{t}{\tau_2}\right) - \gamma_c I(t), \quad (11)$$

where h is the Planck constant, s is the saturation parameter, $s = \frac{c^2}{4\pi^2 h\upsilon_l^3 \tau_2 \Delta\upsilon_h}$, and τ_2 is the relaxation time of the bending mode of CO_2, obtained as a function of pressure and temperature. The general formula is

$$\tau_{ij} = [N_{CO_2} k_{ijCO_2} + N_{N_2} k_{ijN_2} + N_{He} k_{ijHe}]^{-1},$$

and the kinetic rate constants, the k_{ij}, used in this work are listed in Table 1.

Table 1. Kinetic rate constants at T=300°K.

	CO_2	N_2	He
τ_{43}	0	6×10^{-13}	0
τ_3	5.0×10^{-15}	1.6×10^{-15}	1.1×10^{-15}
τ_{20}	6.0×10^{-15}	1.0×10^{-15}	1.0×10^{-13}
τ_{12}	4.5×10^{-11}	0	0
τ_{10}^{a}	...	...	...

[a] No data : set at 4.5 τ_{20}.

The equation in the mathematical approach was solved numerically, since the derivative at any instant of time predicts the intensity at a later time according to $I(t+\Delta t) = I(t) + (\mathrm{d}I/\mathrm{d}t)\Delta\mathrm{t}$. This approach has been used to describe the process of dynamic emission in the electro-optically Q-switched laser, and the result is shown in Fig.1. It is evident that the stimulated emission takes the form of a short pulse when the gain g_c exceeds the loss γ_c.

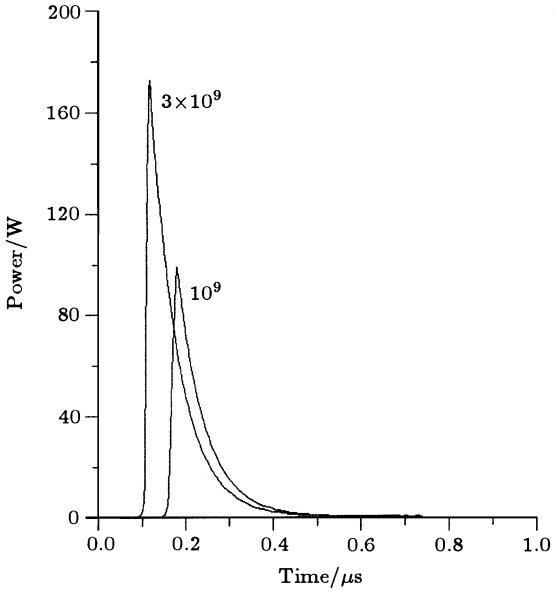


Fig.1. Output power characteristics of the CO_2 laser for 1:1:3 gas mixture of CO_2:N_2:He as a function of initial unsaturated gain.

The laser output power as a function of losses for a 1:1:3 gas mixture of CO_2:N_2:He, with the gain g_c=$2\times10^9 s^{-1}$ and the loss coefficient γ_c=$1.4\times10^8 s^{-1}$ is plotted in Fig.2. It shows that the output power is reduced with the decrease of the losses, and the tail of the pulse changes accordingly. With increasing pumping (larger g_c) the output pulse becomes higher and narrower as observed in practice. Also, the delay between the discharge and the pulse maximum is reduced as the gain is increased.

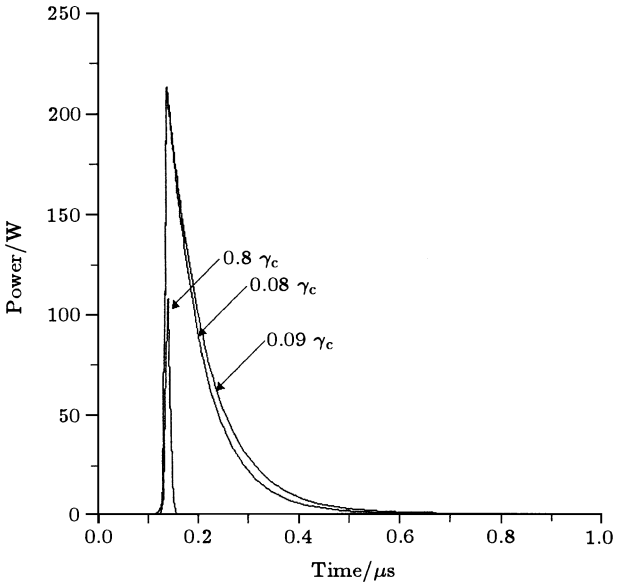


Fig.2. Output power of the CO_2 laser as a function of loss.

The intensity within the cavity builds up rapidly after switching (within a short transit time) with a simultaneous decrease in the inversion, so that a portion of the CO_2 vibrational energy is transformed into photon energy. This happens because the transfer from N_2 to CO_2 is not fast enough to maintain the inversion at its original level. The gain consequently saturates (decreases) to below threshold and the intensity begins to decrease at a rate not greater than the passive cavity decay rate. If the replenishing rate for the upper-level CO_2 is not great enough, then the intensity decreases to a value at which the loss of upper-level CO_2 via stimulated emission becomes relatively slow. Concurrently the inversion begins to rebuild, and when the threshold is reached, the intensity again increases, and the process repeats. If the replenishment rate for the upper-level CO_2 is sufficiently fast, then no minimum is observed, and the pulse shape after the peak is a monotonic decay to the steady value set by this rate.

3. Discussion

A straightforward mathematical formalism has been developed for analysing the properties of practical gain-switched laser oscillators.

The effects of loss on peak power are profound. These features are explained as follows: as losses increase, the gain cannot reach its primary peak value, hence the power is reduced. To reach its new peak value in the presence of the same level of losses, the gain needs more time, i.e. the pulse is delayed, which means that the pulse takes a longer time to build up from noise. The trailing edge of the gain is affected also by the losses, hence the pulse is compressed.

References

[1] Pate L C 1964 *Phys. Rev. Lett.* **13** 617
[2] Tian Z S *et al* 2001 *Opt. Quant. Electron.* **34** 331
[3] Soukieh M *et al* 1988 *Optics Laser Technol.* **30** 451
[4] McArthu B A and Tulip J 1988 *Rev. Sci. Instrum.* **59** 712
[5] Letalick D *et al* 1989 *Opt. Eng.* **28** 172
[6] LI J and Wang R 1995 *Chin. J. Laser* **22** 195 (in Chinese)
[7] Tian Z S *et al* 2001 *Acta Phys. Sin.* **50** 2369 (in Chinese)
[8] Tian Z S *et al* 2001 *Appl. Opt.* **41** 3033
[9] Smith K and Thomson R M 1978 *Computer Modeling of Gas Lasers* (New York: Plenum) p57
[10] Wagner W G and Lengyel B A 1963 *Appl. Phys.* **34** 2040
[11] Casperson L W 1975 *Appl. Opt.* **14** 299
[12] Abrams R L 1974 *Appl. Phys. Lett.* **25** 609
[13] Abramowitz M and Stegun I A 1964 *Handbook of Mathematical Functions* (New York: Dover)