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Momentum correlation of the final-state wavefunction for (e, 3e) collisions on helium*

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Based on our earlier paper, the momentum correlation of the four bodies in the final state is further considered for (e, 3e) processes on helium. A fivefold differential cross section (FDCS) for electron-impact double ionization of helium is calculated by use of the modified model for high incident energy (1–5.6keV). It has been found that the present results give a better description for the experimental data, as compared with the results of our earlier paper.

Keywords: double ionization, (e, 3e) reaction, momentum correlation, first Born approximation
PACC: 3480D, 3450H

1. Introduction

The study of atomic electron-impact double ionization, also called the (e, 3e) reaction, provides a powerful and straightforward tool for investigating the dynamics of interaction. Recently, there has been an increased activity in the study of the double ionization (DI) processes under photon or electron impact. We restrict this paper to the double ionization of atoms by electron impact. Nevertheless, these so-called (e, 3e) experiments are very difficult to perform because it is necessary to detect the three outgoing electrons in coincidence. The first (e, 3e) experiments were performed on argon [1] and on krypton.[2] These (e, 3e) experiments had a very poor energy resolution and were not able to distinguish the different states (np^4) of the residual ion.[3] New experiments using a multi-coincidence spectrometer have been performed recently on argon and on neon,[4] but the different states of the residual ion are not yet resolved. However, the most interesting experiments are those performed on helium. Helium is the simplest target for (e, 3e) experiments with no internal core and the residual He^{2+} ion is a bare nucleus. So there are no complications due to the multielectronic structure of the residual ion in the final channel, and this leads to a pure four-body problem in the final state. These difficult experiments have been performed recently [5,6] and it is an urgent task to investigate the theory of double ionization.

In our earlier paper,[7] hereafter referred to as I, we have calculated the fivefold differential cross section (FDCS) for electron-impact double ionization of helium in a coplanar energy-sharing geometry at an incident energy of $\simeq$ 5.6keV and under a small projectile's scattering angle of 0.45° . The two ejected electrons have been detected with equal energies E_b=E_c=10eV. In this case the following simplified assumptions are reasonable. These allow us to use plane waves for the description of both the incident and scattered electrons. However, the ejected electrons have been described by the 3C wavefunction.[8] Unfortunately, the 3C wavefunction is not in agreement with the measurements at low energies because the normalization factor corresponding to the repulsive electron–electron interaction goes exponentially to zero as the total energy above the threshold of the two electrons goes to zero. This exponential decrease in turn causes the magnitude of the cross section to decrease exponentially.[9,10]

However, the major limitation of the 3C wavefunction, we believe, lies in the fact that the influence on the strength of the interaction of any two particles by the presence of the third one has not been taken into account. In another paper of our ear-

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lier period, we have corrected this deficiency of 3C wavefunction by the introduction of effective Sommerfeld parameters in the two-body factors in the 3C wavefunction.[11−13] Such a modification represents a dynamic screening (DS) of the three two-body Coulomb interaction and hence the new wavefunction will be designated as DS3C. Results of the DS3C turned out to be in good agreement with experimental findings over a wide range of collision geometry.[14−21]

The goal of the present paper is to extend the work of I by use of the effective Sommerfeld parameters for any cases to modify the 3C wavefunction and use the present approach to calculate the FDCS for electron-impact double ionization of helium in both symmetric and asymmetric cases at high incident energies of 1.1 and 5.6 keV. It has been found that the present results give a better description for the experimental data, as compared with the results of I.

2. Theory

In this section we briefly sketch the theoretical model used to calculate the FDCS. Throughout this paper, for simplicity, atomic units are used. In particular, positive scattering and ejection angles are counted counter-clockwise, starting from the incident beam direction.

We consider events in which fast electrons having energy E_0 are incident on helium and are inelastically scattered with energy E_s into the solid angle $\mathrm{d}\Omega_s$ in the direction θ_s and eject two target electrons with low energies E_a and E_b into the solid angles $\mathrm{d}\Omega_a$ and $\mathrm{d}\Omega_b$ in the directions θ_a and θ_b respectively. The FDCS for this process is given by

$$\frac{\mathrm{d}^5\sigma}{\mathrm{d}\Omega_s\mathrm{d}\Omega_a\mathrm{d}\Omega_b\mathrm{d}E_a\mathrm{d}E_b} = (2\pi)^4\frac{\boldsymbol{k}_s\boldsymbol{k}_a\boldsymbol{k}_b}{k_0}\mid T_{fi}\mid^2. \quad (1)$$

The transition matrix element T_{fi} is given by

$$T_{fi} = \langle \Psi_{k_s,k_a,k_b}(\boldsymbol{r}_s,\boldsymbol{r}_a,\boldsymbol{r}_b)\mid V_i\mid \Phi(\boldsymbol{r}_s,\boldsymbol{r}_a,\boldsymbol{r}_b)\rangle, \quad (2)$$

where $\boldsymbol{r}_s$, $\boldsymbol{r}_a$, $\boldsymbol{r}_b$ are the position vectors of the projectile and the two initially bound electrons, respectively, and $\boldsymbol{k}_s$, $\boldsymbol{k}_a$, $\boldsymbol{k}_b$ are the corresponding conjugate momenta. The initial channel perturbation V_i in Eq.(2) is given by

$$V_i = -\frac{Z_T}{\boldsymbol{r}_s} + \frac{1}{\mid \boldsymbol{r}_s-\boldsymbol{r}_a\mid} + \frac{1}{\mid r_s-\boldsymbol{r}_b\mid}, \quad (3)$$

where Z_T=2 is the charge of the He^{2+} ion. $\mid\Psi_{k_s,k_a,k_b}(\boldsymbol{r}_s,\boldsymbol{r}_a,\boldsymbol{r}_b)\rangle$ and $\mid\Phi(\boldsymbol{r}_s,\boldsymbol{r}_a,\boldsymbol{r}_b)\rangle$ represent the wavefunctions describing the whole system in its final and initial states, respectively.

The wavefunction $\mid\Phi(\boldsymbol{r}_s,\boldsymbol{r}_a,\boldsymbol{r}_b)\rangle$ is an eigensolution to the Hamiltonian H_i with an eigenvalue E_i, where $E_i = E_0+\epsilon$ with $\epsilon\simeq 2.9073$(a.u.) being the binding energy of helium. The Hamiltonian H_i has the form

$$\hat{H}_i = \hat{H}_0 - \frac{Z_T}{\boldsymbol{r}_a} - \frac{Z_T}{\boldsymbol{r}_b} + \frac{1}{\mid \boldsymbol{r}_a-\boldsymbol{r}_b\mid}, \quad (4)$$

where $\hat{H}_0$ is the total kinetic energy operator. Equation (4) can be written as

$$\hat{H}_i\Phi(\boldsymbol{r}_s,\boldsymbol{r}_a,\boldsymbol{r}_b) = E_i\Phi(\boldsymbol{r}_s,\boldsymbol{r}_a,\boldsymbol{r}_b). \quad (5)$$

The present (e,3e) study has been made at an high incident energy, $E_0\simeq$1–5.6keV. So, for the initial state we choose the unperturbed representation

$$\Phi(\boldsymbol{r}_s,\boldsymbol{r}_a,\boldsymbol{r}_b) = (2\pi)^{-3/2}\mathrm{e}^{\mathrm{i}k_0\cdot r_s}\phi(\boldsymbol{r}_a,\boldsymbol{r}_b). \quad (6)$$

Substitution of Eq.(6) into Eq.(5) leads to the following set of equations:

$$-\frac{1}{2}\nabla^2_{\boldsymbol{r}_0}\Phi(\boldsymbol{r}_s,\boldsymbol{r}_a,\boldsymbol{r}_b) = E_0\Phi(\boldsymbol{r}_s,\boldsymbol{r}_a,\boldsymbol{r}_b) \quad (7)$$

and

$$\left(-\frac{1}{2}\nabla^2_{\boldsymbol{r}_a} - \frac{1}{2}\nabla^2_{\boldsymbol{r}_b} - \frac{Z_T}{\boldsymbol{r}_a} - \frac{Z_T}{\boldsymbol{r}_b} + \frac{1}{\mid \boldsymbol{r}_a-\boldsymbol{r}_b\mid}\right) \times \phi(\boldsymbol{r}_a,\boldsymbol{r}_b) = \epsilon\phi(\boldsymbol{r}_a,\boldsymbol{r}_b). \quad (8)$$

In order to consider the effect of initial state on FDCS, the ground state of He is represented by a Byron and Joachain (BJ) wavefunction,[22]

$$\phi(\boldsymbol{r}_a,\boldsymbol{r}_b) = \varphi(\boldsymbol{r}_a)\varphi(\boldsymbol{r}_b), \quad (9)$$

$$\varphi(r) = \sqrt{\frac{1}{4\pi}}\left(g_a\mathrm{e}^{-\lambda_a r} + g_b\mathrm{e}^{-\lambda_b r}\right); \quad (10)$$

or alternatively, a Silvermann and Platas (SP) wavefunction,[23]

$$\phi(\boldsymbol{r}_a,\boldsymbol{r}_b) = N_0\left[\mathrm{e}^{-(\eta_a r_a+\eta_b r_b)} + \mathrm{e}^{-(\eta_b r_a+\eta_a r_b)}\right], \quad (11)$$

where $\lambda_a = 1.41$, $\lambda_b = 2.61$, $g_a = 2.61$, $g_b = 2.08$, $\eta_a = 2.17$, $\eta_b = 1.21$ and $N_0 = 0.72$.

Then the final-state wavefunction Ψ_{k_s,k_a,k_b} ($\boldsymbol{r}_s$, $\boldsymbol{r}_a$, $\boldsymbol{r}_b$) should be an eigenfunction of the full four-body Hamiltonian with a bare nucleus and three outgoing electrons. The total Hamiltonian of the system can be written as

$$\hat{H} = -\frac{1}{2}\nabla^2_{\boldsymbol{r}_s} - \frac{1}{2}\nabla^2_{\boldsymbol{r}_a} - \frac{1}{2}\nabla^2_{\boldsymbol{r}_b} - \frac{Z_T}{\boldsymbol{r}_s} - \frac{Z_T}{\boldsymbol{r}_a} - \frac{Z_T}{\boldsymbol{r}_b} + \frac{1}{\mid \boldsymbol{r}_a-\boldsymbol{r}_b\mid} + \frac{1}{\mid \boldsymbol{r}_s-\boldsymbol{r}_a\mid} + \frac{1}{\mid \boldsymbol{r}_s-\boldsymbol{r}_b\mid}. \quad (12)$$

The final-state wavefunction satisfies the following Schrödinger equation:

$$\hat{H}\,\Psi_{k_s,k_a,k_b}(\boldsymbol{r}_s,\boldsymbol{r}_a,\boldsymbol{r}_b) = E_f\,\Psi_{k_s,k_a,k_b}(\boldsymbol{r}_s,\boldsymbol{r}_a,\boldsymbol{r}_b). \tag{13}$$

The state vector $\Psi_{k_s,k_a,k_b}(\boldsymbol{r}_s,\boldsymbol{r}_a,\boldsymbol{r}_b)$ is an eigenstate of $\hat{H}$ with an eigenenergy

$$E_f = \frac{k_s^2}{2} + \frac{k_a^2}{2} + \frac{k_b^2}{2} + \frac{k_{\rm ion}^2}{2M}, \tag{14}$$

where $k_{\rm ion}$ is the recoil momentum of He^{2+}, and M is the mass of the alpha particle.

The exact solution of the Schrödinger equation(13) is very difficult. One has to resort to approximate procedures to obtain useful expressions for $\Psi_{k_s,k_a,k_b}(\boldsymbol{r}_s,\boldsymbol{r}_a,\boldsymbol{r}_b)$.

Recently, a strategy has been proposed [24] to derive an approximate wavefunction for N interacting continuum particles. The resulting wavefunction is a product of $N(N-1)/2$ two-body Coulomb interacting waves. Each of them represents a specific, Coulomb interacting two-body subsystem. Unfortunately, in general, even a simple approximation is intractable for practical applications. Therefore, for the present case of two slow and one fast electrons moving in the double continuum of He^{2+}, it has been suggested to completely subsume the interaction of the fast electron with all other particles into effective electron–nucleus interactions of the slow electrons.

To this end we employ the same ideas as in Ref.[24]. Defining $\boldsymbol{r}_{ij} = \boldsymbol{r}_i - \boldsymbol{r}_j$ $(i,j \in \{s,a,b\})$, we write the total potential as

$$\begin{aligned} &-\frac{Z_T}{\boldsymbol{r}_s} - \frac{Z_T}{\boldsymbol{r}_a} - \frac{Z_T}{\boldsymbol{r}_b} + \frac{1}{\boldsymbol{r}_{sa}} + \frac{1}{\boldsymbol{r}_{sb}} + \frac{1}{\boldsymbol{r}_{ab}} \\ &\equiv -\frac{Z_a}{\boldsymbol{r}_a} - \frac{Z_b}{\boldsymbol{r}_b} + \frac{Z_{ab}}{\boldsymbol{r}_{ab}}, \end{aligned} \tag{15}$$

with Z_a, Z_b and Z_{ab} being functions yet to be determined. A way of specifying these functions is to linearly expand them in terms of the two-body interactions, as done in Ref.[24], and require Eq.[15] to be identically fulfilled. Therefore, the effective charges Z_a, Z_b and Z_{ab} are given by [5]

$$Z_a = Z_T + \frac{Z_T}{2}\left(\frac{k_a}{k_s}\right) - \frac{\boldsymbol{k}_a}{\mid \boldsymbol{k}_s - \boldsymbol{k}_a \mid}, \tag{16}$$

$$Z_b = Z_T + \frac{Z_T}{2}\left(\frac{k_b}{k_s}\right) - \frac{\boldsymbol{k}_b}{\mid \boldsymbol{k}_s - \boldsymbol{k}_b \mid}, \tag{17}$$

$$Z_{ab} = 1. \tag{18}$$

In this case, Eq.[12] can be reduced to

$$\hat{H} = -\frac{1}{2}\nabla^2_{\boldsymbol{r}_s} - \frac{1}{2}\nabla^2_{\boldsymbol{r}_a} - \frac{1}{2}\nabla^2_{\boldsymbol{r}_b} - \frac{Z_a}{\boldsymbol{r}_a} - \frac{Z_b}{\boldsymbol{r}_b} + \frac{Z_{ab}}{\boldsymbol{r}_{ab}}. \tag{19}$$

This paper is restricted to the high-energy domain where the incident and scattered electrons are always fast, and the ejected electrons leave the ion with small energies. These kinematical conditions enable us to use plane waves for the description of both the incident and the scattered electrons. Therefore, the eigenfunction $\Psi_{k_s,k_a,k_b}(\boldsymbol{r}_s,\boldsymbol{r}_a,\boldsymbol{r}_b)$ of $\hat{H}$ can be approximated by

$$\Psi_{k_s,k_a,k_b}(\boldsymbol{r}_s,\boldsymbol{r}_a,\boldsymbol{r}_b) = (2\pi)^{-3/2}\mathrm{e}^{\mathrm{i}k_s\cdot r_s}\,\Psi_{k_a,k_b}(\boldsymbol{r}_a,\boldsymbol{r}_b), \tag{20}$$

where $\Psi_{k_a,k_b}(\boldsymbol{r}_a,\boldsymbol{r}_b)$ will be described by 3C wavefunction in its general form

$$\Psi_{k_a,k_b}(\boldsymbol{r}_a,\boldsymbol{r}_b) = M\mathrm{e}^{\mathrm{i}k_a\cdot r_a}\mathrm{e}^{\mathrm{i}k_b\cdot r_b}\chi(\boldsymbol{r}_a,\boldsymbol{r}_b), \tag{21}$$

with

$$\begin{aligned} \chi(\boldsymbol{r}_a,\boldsymbol{r}_b) = &\prod_{j=a}^{b} {}_1F_1[\mathrm{i}\alpha_j;1;-\mathrm{i}(\boldsymbol{k}_j\boldsymbol{r}_j + k_j\cdot r_j)] \\ &\times {}_1F_1[\mathrm{i}\alpha_{ab};1;-\mathrm{i}(\boldsymbol{k}_{ab}\boldsymbol{r}_{ab} + k_{ab}\cdot r_{ab})] \end{aligned} \tag{22}$$

and

$$\begin{aligned} M = &\exp[-\pi(\alpha_a+\alpha_b+\alpha_{ab})/2]\Gamma(1-\mathrm{i}\alpha_a) \\ &\times \Gamma(1-\mathrm{i}\alpha_b)\Gamma(1-\mathrm{i}\alpha_{ab})/(2\pi)^3, \end{aligned} \tag{23}$$

where α is called Sommerfeld parameter, $\alpha_a = -Z_a/\boldsymbol{k}_a$, $\alpha_b = -Z_b/\boldsymbol{k}_b$ and $\alpha_{ab} = 1/(2\boldsymbol{k}_{ab})$ with

$$\boldsymbol{k}_{ab} = \frac{1}{2}(\boldsymbol{k}_a - \boldsymbol{k}_b). \tag{24}$$

The Sommerfeld parameter $\alpha = Z_aZ_b\mu_{ab}/\boldsymbol{k}_{ab}$ is a measure of the strength of the Coulomb interaction between two particles with charges Z_a and Z_b, reduced mass μ_{ab}, and relative momentum $\boldsymbol{k}_{ab} = \mu_{ab} \mid \boldsymbol{k}_a - \boldsymbol{k}_b \mid$ conjugate to $\boldsymbol{r}_{ab} = \mid \boldsymbol{r}_a - \boldsymbol{r}_b \mid$. Because the strength of interaction of any two particles is affected by the presence of the third particle, the new Sommerfeld parameters introduced here should be functions of all three relative momenta. The modification of the strength of a particular two-body Coulomb interaction depends on the momenta of the two particles relative to the third one, which represents a dynamic screening (DS) of the three two-body Coulomb interaction and hence the new wavefunction will be designated as DS3C.

Based on this consideration, in our earlier paper,[14] we have corrected this deficiency of 3C wavefunction by the introduction of effective Sommerfeld parameters for any geometry. The new Sommerfeld parameters are given by [14]

$$\beta_a = -\frac{Z_a - \frac{2k_{ab}k_a^2}{(k_a+k_b)^3}\left[\frac{3+\cos^2[4\xi(k_b)]}{4}\right]^2}{k_a}, \tag{25}$$

$$\beta_b = -\frac{Z_b - \frac{2k_{ab}k_b^2}{(k_a+k_b)^3}\left[\frac{3+\cos^2[4\xi(k_a)]}{4}\right]^2}{k_b}, \tag{26}$$

$$\beta_{ab} = \frac{Z_{ab} - \frac{4k_{ab}^2}{(k_a+k_b)^3}\left[k_a\left(\frac{3+\cos^2[4\xi(k_b)]}{4}\right) + k_b\left(\frac{3+\cos^2[4\xi(k_a)]}{4}\right)\right]^2}{2k_{ab}}, \tag{27}$$

where

$$\xi(k) = \cos^{-1}\left(\frac{k}{\sqrt{k_a^2+k_b^2}}\right). \tag{28}$$

In this work, we apply the present DS3C model to calculate the FDCS for electron-impact double ionization of helium for both symmetric and asymmetric cases at high incident energies of 1.1 and 5.6keV.

3. Results and discussion

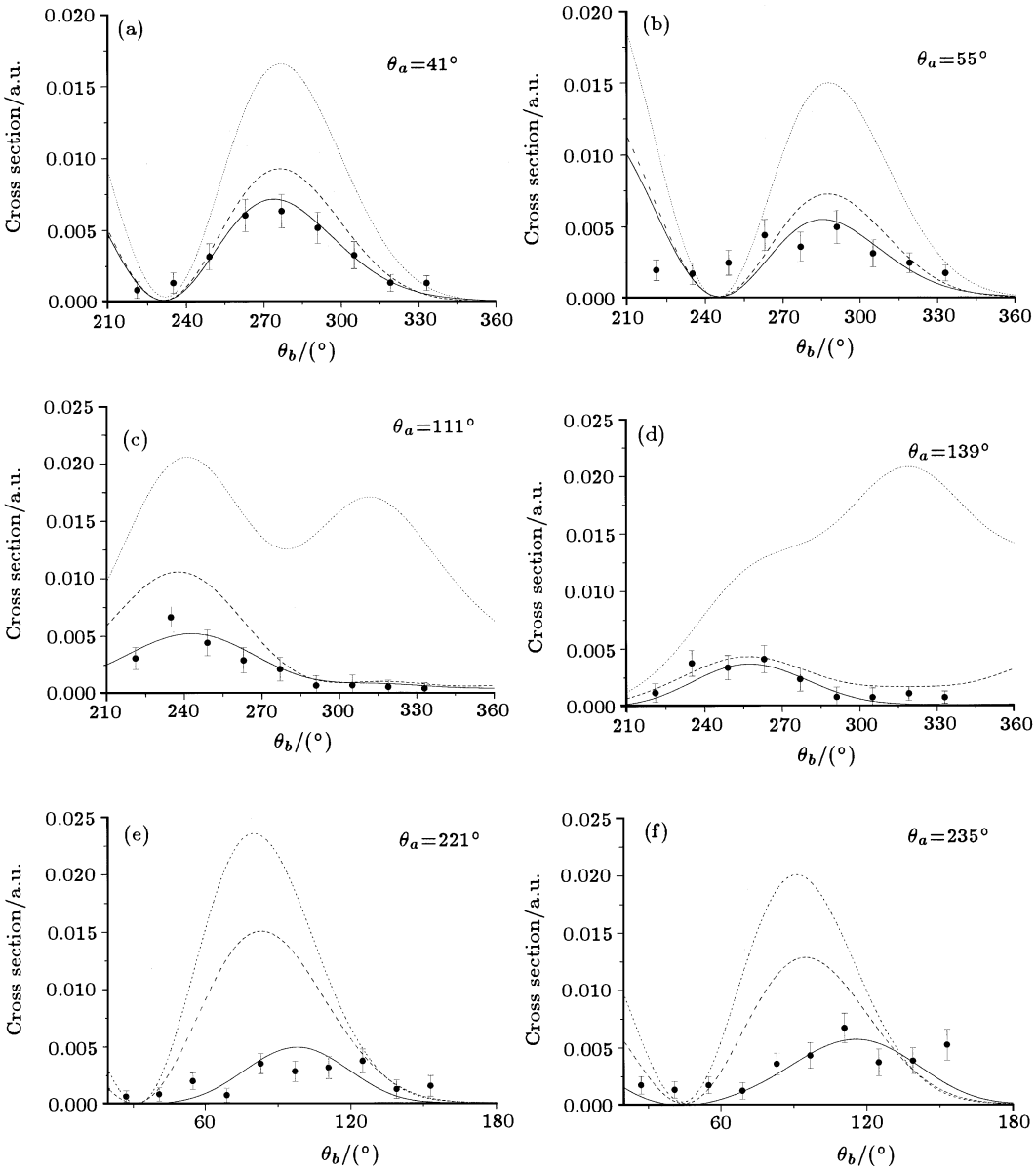
The calculated and measured FDCS are shown in Figs.1(a)–(f) and in Figs.2(a)–(i) according to the 'fixed ejected angle mode', that is, as angular distributions of one electron for fixed emission direction of the second one. In these plots and in the following discussion, we denote by θ_a and θ_b the fixed and the variable electron angles, respectively. Finally, Fig.3 presents the so-called symmetric geometry where both electrons emerge at equal angles on both sides of the incident beam, θ_a=$-\theta_b$. The fast electron is observed at an angle θ_s=+1.10°, not shown in the figures.

Figure 1 shows the variation of FDCS with the angle of ejection θ_b at an incident energy E_0=5599eV, scattering angle θ_s=0.45° and ejected electron energies E_a=E_b=10eV. One of the electrons is assumed to be ejected along the θ_a direction. The theoretical results are compared with the absolute experimental data of Lahmam-Bennani *et al*[5] in the angular range $41° \leq \theta_a \leq 235°$. The present study is divided into two parts. We compare the angular variation of FDCS obtained by using different initial-state wavefunctions with the experimental data, and then discuss the common features and differences in the fully differential cross sections for the electron-impact double ionization of He evaluated using the 3C and DS3C methods.

For the first part, to consider how the FDCS is affected by the initial state, two types of wavefunctions are used for the ground state of helium. It can be seen from Fig.1 that the absolute value of the cross sections turns out to be very sensitive to the initial states used. It can be seen that the wavefunction (11), which contains a partly radial correlation, yields cross sections (plotted in Fig.1 as broken curve) in better agreement with the experiment of Lahmam-Bennani *et al*[5] than those obtained using Eq.(10) (plotted in Fig.1 as dotted curve). Particularly, there is a small peak in Figs.1(c) and (d) at $\theta_b \simeq 315°$, where the experimental data point to a minimum. The BJ-type wavefunctions thus fail to reproduce the angular variation of FDCS in this kinematics.

As was shown by Lahmam-Bennani *et al*[5] the shape of the FDCS in the fixed-angle mode can be analysed in terms of the dipole selection rules which forbid the escape of the two equal-energy electrons in opposite directions. These constraints put strict limitations on the shape of the FDCS which should possess a two-peak structure with two nodes in the directions $\hat{\boldsymbol{k}}_b = \pm\hat{\boldsymbol{k}}_a$. Since the momentum transfer at the present kinematics is small, but still finite, the dipole selection rule is only approximate, and the minimum at $\hat{\boldsymbol{k}}_b = -\hat{\boldsymbol{k}}_a$ (θ_b = 221°(a), 235°(b), 291°(c), 319°(d), 41°(e), 55°(f)) is filled with a non-zero cross section, both in theory and experiment.

A similar two-peak structure is observed experimentally and is reproduced by the theoretical model. The obtained angular position of the peaks is similar in the 3C and DS3C calculations. However, the relative magnitudes of the peaks are quite different. Figure 1 shows that the present DS3C approach is in good agreement with the experiment, though small quantitative discrepancies remain. The 3C overestimate of cross sections in the peaks is corrected by the present DS3C.



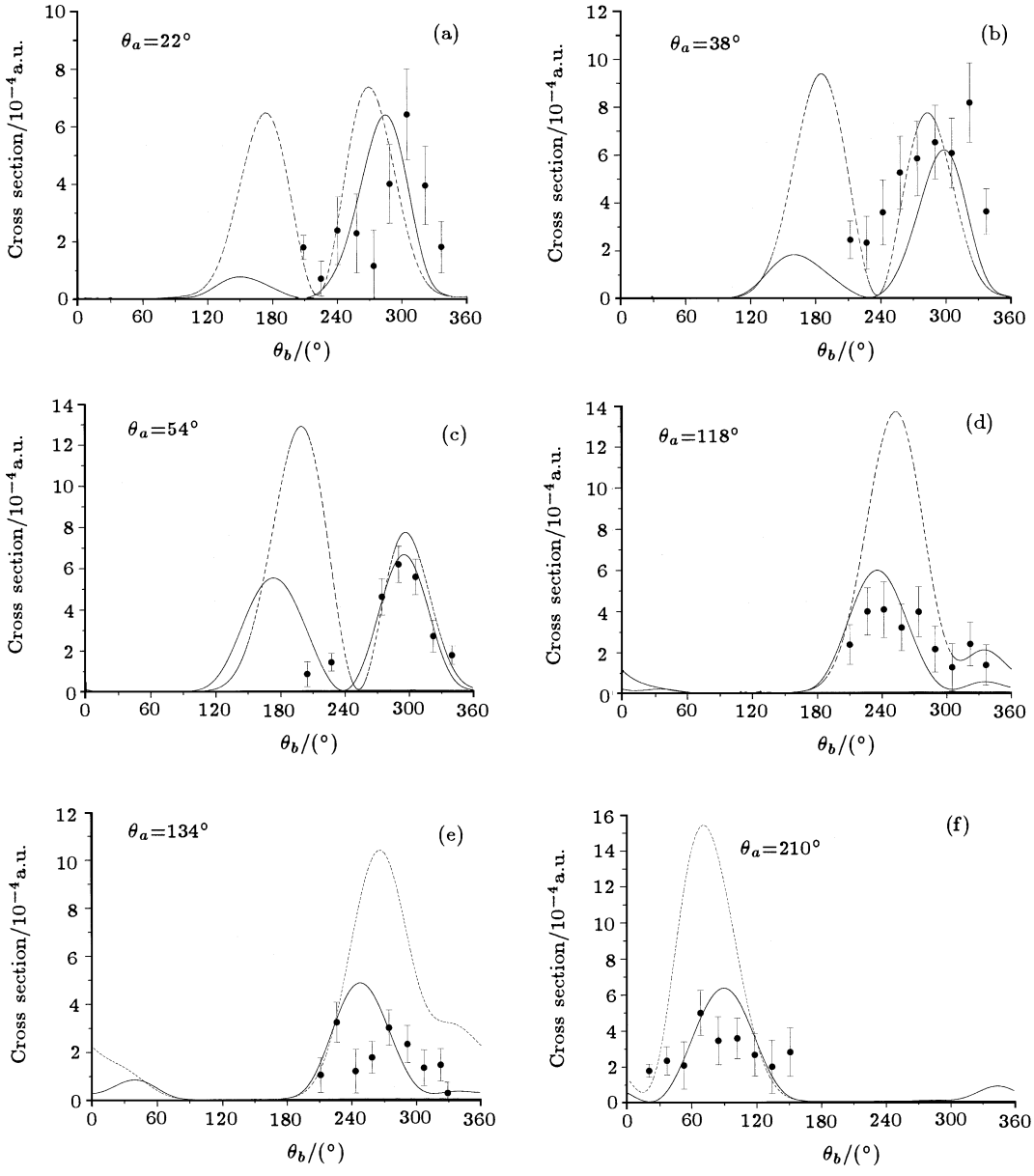
Figs.1. (a)–(f) FDCS for helium coplanar double ionization by electron impact at E_0=5599keV. The scattered electron is observed at a fixed angle, $\theta_s = 0.45°$, with an energy E_s=5500eV (corresponding to a fixed momentum transfer, K=0.24a.u., in the direction $\theta_k = 319°$). The two electrons are ejected at equal energies E_a=E_b=10eV. The angular distribution of one of the electrons (electron b) is scanned while the other ejected electron is detected under a fixed angle θ_a which is indicated in the figures. The full circles are the experimental measurements of Lahmam-Bennani *et al.*[5] Full curve denotes DS3C results; broken curve, 3C results with a SP initial-state wavefunction; dotted curve, 3C results with a BJ initial-state wavefunction.

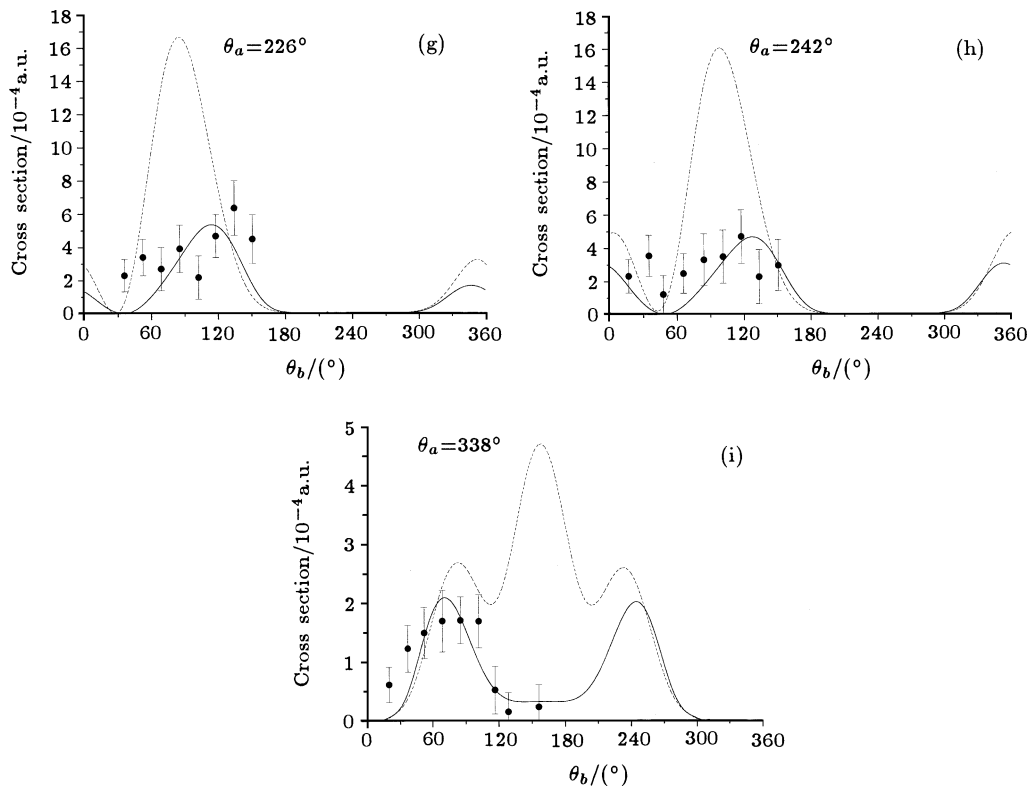
In Fig.2, the impact energy is E_0=1099eV. The scattered electron is observed at a fixed angle, $\theta_s = +1.10°$, with an energy E_s=1000eV (corresponding to a fixed momentum transfer, K=0.45a.u., in the direction $\theta_s = 338.4°$). The two ejected electrons have identical energies, E_a=E_b=10eV.

From Fig.2, one may compare the two theoretical results. They generally yield a two-peak structure for the FDCS angular distributions. The two peaks are separated on the one hand by a strict zero intensity for the parallel emission, due to the Coulomb repulsion between the two electrons with equal energies emerging in the same direction, and on the other hand, by a deep minimum of intensity for the antiparallel or back-to-back emission(θ_b = 202°(a), 218°(b), 234°(c), 298°(d), 314°(e), 30°(f), 46°(g), 62°(h), 158°(i)).

From Fig.2, it can be seen that the 3C overestimate of cross sections in the peaks is corrected by

the present DS3C. For θ_a = 22°(a), 38°(b), 54°(c), 118°(d), 242°(h) and 338°(i), the location of the peak predicted by the 3C is shifted by about 10°–40° from the experimental one. This shift of the peak is corrected by the present DS3C. Particularly, there is a peak in Fig.2(i) at $\theta_b \simeq 120°$–180° where the experimental data point to a minimum. The deficiency of 3C wavefunction is also corrected by the present DS3C. Besides all this, the agreement with the experimental data is not very good here. Considerable deviations between theory and experiment have been observed which have been attributed to the contributions of scattering processes whose description goes beyond the first Born approximation (FBA). This proposition seems reasonable in view of the relatively low energy of the incident electron. It might indicate the large role played by non-first Born processes, even though the incident energy is large (1.1keV) and the momentum transfer is small (0.45a.u.). Both the angular shift and the breaking of symmetry may be directly attributed to the second- or higher-order effects in the projectile-target interaction process. But, in general, the deviations from 'perfect' symmetry were smaller, as one would expect from the higher incident energy (5.6 keV) and smaller momentum transfer (0.24a.u.) used in Fig.1.





Figs.2. (a)–(i) As Fig.1, but for E_0=1099eV. The scattered electron is observed at a fixed angle, $\theta_s = 1.10°$, with an energy E_s=1000eV (corresponding to a fixed momentum transfer, K=0.45a.u., in the direction $\theta_k = 338.4°$). The full circles are the experimental measurements of Lahmam-Bennani *et al.*[6] Full curve denotes DS3C results; broken curve, 3C results.

In Fig.3, the data have been sorted for the two ejected electrons to emerge at equal but opposite angles. The present DS3C results together with those of convergent close-coupling (CCC) and the absolute measurements [6] at incident energy of E_0=1099eV in coplaner geometry, scattering angles of $\theta_a = -\theta_b$ and under equal energy sharing 10+10eV are displayed in Fig.3, respectively. The results of the 3C model without modification has also been plotted for comparison.

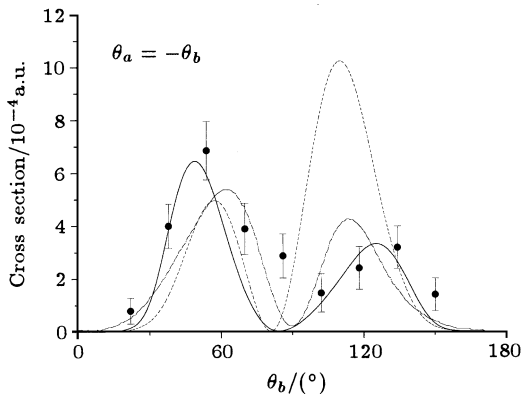


Fig.3. Same as Figs.2, but for $\theta_a = -\theta_b$. The full circles are the experimental measurements of Lahmam-Bennani *et al.*[6] Full curve, DS3C results; broken curve, 3C results; chain curve, CCC results.[6]

The CCC method developed recently has been widely applied to various problems in atomic physics and proved to be a successful reliable method for the description of electron scattering on helium and hydrogen-like targets at all projectile energies. However, this method does not have correct boundary conditions for ionization problems because it expands the total wavefunction using an orthogonal Laguerre basis, thereby treating the target continuum via a square-integrable representation.

In this representation, the present theoretical and experimental results show two peaks, one forward at about 60° and one backward at ~130° in the experiment and ~110° in the theories(3C and CCC). The location of the peak predicted by the 3C and CCC are shifted by about 20° from the experimental one. From Fig.3, it can be seen that the 3C theory overestimates of cross sections in the backward peak and the shift in the peak are corrected by the present DS3C.

4. Conclusion

The present DS3C model employs a correlated four-body final state and goes beyond the FBA via the introduction of effective charges and shows great improvement over the 3C in obtaining better magni-

tudes and shapes of cross sections compared with the experimental data. However, the variation of the effective charges is small, and the non-first Born effects due to this model should be weak.

The reason why the present DS3C approach is not in full agreement, especially at the incident energy of E_0=1099eV, with the experimental data may be traced to the fact that the second- or higher-order effects in the projectile-target interaction process should be weak. Generally, higher Born corrections have to be included in the theoretical models of the (e, 3e) reaction. This will be the direction of future work.

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