Vol 13 No 4, April 2004
1009-1963/2004/13(04)/0469-04

Chinese Physics

© 2004 Chin. Phys. Soc.
and IOP Publishing Ltd

Non-factorizable contributions in $D^0 \to \pi^+\pi^-$ decay *

Wu Xiang-Yao(吴向尧),[a)b)†] Yin Xin-Guo(尹新国),[a)] and Guo Yi-Qing(郭义庆)[c)]

[a)] *Department of Physics, Huaibei Coal Industry Teacher's College, Huaibei* 235000, *China*
[b)] *Institute of Physics, Nankai University, Tianjin* 300071, *China*
[c)] *Institute of High Energy Physics, Beijing* 100039, *China*

(Received 6 June 2003; revised manuscript received 11 November 2003)

We have analyzed the $D^0 \to \pi^+\pi^-$ decay with α_s corrections in quantum chromodynamic (QCD) factorization and with the soft-gluon corrections in the light cone QCD sum rules. The soft-gluon effects are firstly calculated in the decay channel. The results show that once the factorization contributions, the α_s corrections, and the soft-gluon effects are all considered, we can satisfactorily explain the experimental data in the decay channel.

Keywords: D meson decay, soft-gluon effects, branching ratio
PACC: 1440L, 1235C, 1150L

1. Introduction

The decays of charmed particles can provide useful information about strong interactions and understanding of the quark mixing sector of the standard model (SM) and also the search for the sources of CP violation. However, the hadronic two-body weak decays of D mesons involve nonperturbative strong interactions and spoil the simplicity of the short distance behaviour of weak interactions. Therefore, a simplified approach in which the amplitudes of these processes are given by a factorizable short distance current–current effective Hamiltonian is not expected to work well. Various approaches have been employed to include long distance effects. The most commonly and very frequently used prescription, motivated by $\frac{1}{N_c}$ arguments,[1] is to apply generalized factorization.[2,3] This phenomenological treatment works reasonably well in Cabibbo-favoured D decays,[3] but it fails in the Cabibbo-suppressed $D\to \pi\pi$ and $D\to K\bar{K}$ decays.[4–7]

It is necessary to study D meson two-body weak decays beyond the factorization approach. Recently, Beneke *et al*[8] gave a next-to-leading order (NLO) calculation of $B \to \pi\pi, K\pi$ in the heavy quark limit. In this limit, the hadronic matrix elements for $B \to \pi\pi$ can be expanded into the powers of α_s and $\frac{\Lambda_{QCD}}{m_b}$ as follows:

$$\langle\pi\pi|O_i|B\rangle = \langle\pi|j_1|B\rangle\langle\pi|j_2|0\rangle \times \left[1 + \sum r_n \alpha_s^n + O\left(\frac{\Lambda_{QCD}}{m_b}\right)\right], \quad (1)$$

where O_i are some local four-quark operators in the weak effective Hamiltonian and $j_{1,2}$ are bilinear quark currents. They pointed out that in the heavy quark limit the radiative corrections in the order of α_s can be calculated with the perturbative QCD (PQCD) method. In $D \to \pi\pi$ decay, the momentum transfer square is $q^2 = 1.7\mathrm{GeV}^2$, and the radiative corrections of hard-gluon can also be calculated with PQCD approach. So, Eq.(1) is suitable for $D\to \pi\pi$ decay also. In Eq.(1), the power correct term $O\left(\frac{\Lambda_{QCD}}{m_b}\right)$, which includes soft-gluon effects, final state interaction and the soft effects, cannot be calculated in QCD factorization and the PQCD method. For the B meson two-body decay, this term is small, but it is large and cannot be neglected in the $D\to \pi\pi$ decay. Recently, Khodjamirian[9] has presented a new method to calculate the hadronic matrix elements of nonleptonic B meson decays within the framework of the light cone QCD sum rules, where the nonfactorizable soft-gluon contributions can be effectively dealt with. Obviously, this approach can be applied to the $D\to \pi\pi$ decay.

*Project supported by the Natural Science Foundation of Department of Education, Anhui Province, China (Grant No 2001kj200zc).
†E-mailing address: phymath@etang.com

2. $D^0 \to \pi^+\pi^-$ in QCD factorization

The low-energy effective Hamiltonian for $D^0 \to \pi^+\pi^-$ can be expressed as follows:

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{ud}V_{cd}^*[(C_1(\mu)O_1(\mu) + C_2(\mu)O_2(\mu)] + {\rm h.c.}, \quad (2)$$

where $C_i(\mu)$ are Wilson coefficients which have been evaluated to NLO. The four-quark operators $O_{1,2}$ are

$$O_1 = (\bar{u}d)_{V-A}(\bar{d}c)_{V-A},$$
$$O_2 = (\bar{u}_\alpha d_\beta)_{V-A}(\bar{d}_\beta c_\alpha)_{V-A}, \quad (3)$$

where α and β are the $SU(3)$ colour indices and the Wilson coefficients evaluated at $\mu = m_{\rm c}$ scale are [10]

$$C_1 = 1.274, \quad C_2 = -0.501. \quad (4)$$

In the following, we calculate the nonfactorizable effects in $D^0 \to \pi^+\pi^-$ with the QCD factorization approach. This method is similar to that for $B \to \pi\pi, \pi K$ decay; see Ref.[8] for details. As in Ref.[8], we obtain the QCD coefficients a_i (i=1,2) on NLO in the naive dimension regularization (NDR) scheme. The coefficients a_i are given as

$$a_1 = C_1 + \frac{C_2}{N} + \frac{\alpha_{\rm s}}{4\pi}\frac{C_{\rm F}}{N}C_2F,$$
$$a_2 = C_2 + \frac{C_1}{N} + \frac{\alpha_{\rm s}}{4\pi}\frac{C_{\rm F}}{N}C_1F. \quad (5)$$

Here N=3 (f=4) is the number of colours (flavours), and $C_{\rm F} = \dfrac{N^2-1}{2N}$ is the factor of colour. The functions in Eq.(5) can be found in Ref.[8], which are

$$F = -12\ln\frac{\mu}{m_{\rm c}} - 18 + f_{\rm I} + f_{\rm II},$$
$$f_{\rm I} = \int_0^1 g(x)\phi_\pi(x){\rm d}x, \quad (6)$$

with the hard-scattering function

$$g(x) = 3\frac{1-2x}{1-x}\ln x - 3{\rm i}\pi.$$

The hard spectator scattering contribution is given by

$$f_{\rm II} = \frac{4\pi^2}{N}\frac{f_\pi f_{\rm D}}{F^{{\rm D}\to\pi}(0)m_{\rm D}^2} \times \int_0^1 \frac{\phi_{\rm D}(\xi)}{\xi}{\rm d}\xi\left[\int_0^1 \frac{\phi_\pi(x)}{x}{\rm d}x\right]^2, \quad (7)$$

where $f_\pi(f_{\rm D})$ is the pion (D meson) decay constant, $m_{\rm D}$ the D meson mass, $F^{{\rm D}\to\pi}(0)$ the $D \to \pi$ form factor at zero momentum transfer, and ξ the light-cone momentum fraction of the spectator in the D meson. $f_{\rm II}$ depends on the wavefunction $\phi_{\rm D}$ through the integral $\int_0^1 {\rm d}\xi\phi_{\rm D}(\xi)/\xi = m_{\rm D}/\Lambda_{\rm QCD} = 17.16$. Using $f_\pi = 131$MeV, $f_{\rm D} = 235$MeV, $F^{{\rm D}\to\pi}(0) = 0.692$, $\alpha_{\rm s} = 0.353$, $m_{\rm D} = 1.865$GeV, and the asymptotic wavefunction $\phi_\pi = 6x(1-x)$, we find

$$a_1 = 0.9001 - 0.0622{\rm i}. \quad (8)$$

3. The soft-gluon effect in $D^0 \to \pi^+\pi^-$

To estimate the soft-gluon corrections for $D^0 \to \pi^+\pi^-$ decays, it is useful to rewrite the effective Hamiltonian with the help of the Fierz transformation. For example, applying the Fierz transformation to the operator $O_2 = (\overline{d}\Gamma_\mu d)(\bar{u}\Gamma_\mu c)$, we have the effective Hamiltonian relevant to the tree operators,

$$\mathcal{H}_{\rm eff} = \frac{G_{\rm F}}{\sqrt{2}}\left[V_{ud}V_{cd}^*\left(C_1(\mu) + \frac{C_2(\mu)}{3}\right)O_1(\mu) + 2C_2(\mu)\widetilde{O}_1(\mu)\right], \quad (9)$$

where

$$\widetilde{O}_1 = \left(\overline{d}\Gamma_\mu\frac{\lambda^a}{2}c\right)\left(\overline{u}\Gamma^\mu\frac{\lambda^a}{2}d\right). \quad (10)$$

In the above $\Gamma_\mu = \gamma_\mu(1-\gamma_5)$, ${\rm Tr}(\lambda^a\lambda^b) = 2\delta^{ab}$, and

$$O_2 = \frac{1}{3}O_1 + 2\widetilde{O}_1. \quad (11)$$

First of all, we calculate the nonfactorizable matrix elements induced by the operator $\widetilde{O}_1$. As a starting object for the derivation of light cone QCD sum rules (LCSR) we choose the following vacuum-pion correlation function:

$$F_\alpha^{(\widetilde{O}_1)}(p,q,k) = -\int {\rm d}^4x{\rm e}^{-{\rm i}(p-q)x}\int {\rm d}^4y{\rm e}^{{\rm i}(p-k)} \times\langle 0|T\{j_{\alpha5}^{(\pi)}(y)\widetilde{O}_1(0)j_5^{\rm D}(x)\}|\pi^-(q)\rangle, \quad (12)$$

where $j_{\alpha5}^{(\pi)} = \overline{u}\gamma_\alpha\gamma_5 d$ and $j_5^{({\rm D})} = m_c\overline{c}{\rm i}\gamma_5 u$ are the quark currents interpolating π and D mesons, respectively. The decomposition of the correlation function (12) into independent momenta is straightforward and contains four invariant amplitudes:

$$F_\alpha^{(\widetilde{O}_1)} = (p-k)_\alpha F^{(\widetilde{O}_1)} + q_\alpha \tilde{F}_1^{(\widetilde{O}_1)} + k_\alpha \tilde{F}_2^{(\widetilde{O}_1)} + \epsilon_{\alpha\beta\lambda\rho}q^\beta p^\lambda k^\rho \tilde{F}_3^{(\widetilde{O}_1)}. \quad (13)$$

In what follows only the amplitude $F^{(\widetilde{O}_1)}$ is relevant. The correlation function is calculated in QCD by expanding the T-product of three operators, two currents and $\widetilde{O}_1$, near the light-cone $x^2 \sim y^2 \sim (x-y)^2 \sim$

0. For this expansion to be valid, the kinematical region should be chosen:

$$q^2 = p^2 = k^2 = 0,$$
$$|(p-k)^2| \sim |(p-q)^2| \sim |P^2| \gg \Lambda^2_{\rm QCD}, \quad (14)$$

where $P \equiv p-k-q$. Following the standard procedure for QCD sum rule calculation,

$$\begin{aligned} &A^{(\widetilde{O}_1)}(\mathrm{D}^0 \to \pi^+\pi^-) \\ =&\langle \pi^+(p)\langle \pi^-(-q)|\widetilde{O}_1|D(p-q)\rangle \\ =&\frac{-\mathrm{i}}{\pi^2 f_\pi f_{\rm D} m_{\rm D}{}^2}\int_0^{s_0^\pi} \mathrm{d}s \mathrm{e}^{\frac{-s}{M^2}} \int_{m_{\rm c}^2}^{\bar{R}(s,m_{\rm c}^2,m_{\rm D}^2,s_0^{\rm D})} \mathrm{d}s' \\ &\times \mathrm{e}^{\frac{m_{\rm D}^2-s'}{M'^2}} \mathrm{Im}_{s'}\mathrm{Im}_s F^{(\widetilde{O}_1)}_{\rm QCD}(s,s',m_{\rm D}^2), \end{aligned} \quad (15)$$

where s_0^π and $s_0^{\rm D}$ are the effective threshold parameters.

A straightforward calculation shows that only the twist-3 wavefunction $\varphi_{3\pi}(\alpha_i)$ and the twist-4 ones $\varphi_\parallel(\alpha_i), \varphi_\perp(\alpha_i)$, whose definitions can be found in Ref.[9], contribute to the invariant function $F^{(\widetilde{O}_1)}$. The results are

$$F^{(\widetilde{O}_1)}_{\rm QCD} = F^{(\widetilde{O}_1)}_{\rm tw3} + F^{(\widetilde{O}_1)}_{\rm tw4}, \quad (16)$$

where $F^{(\widetilde{O}_1)}_{\rm tw3}$ and $F^{(\widetilde{O}_1)}_{\rm tw4}$ can be found in Refs.[8,10], with a replacement $m_{\rm c} \to m_{\rm b}$. Finally, the LCSR for the $\mathrm{D}^0 \to \pi^+\pi^-$ matrix element of the operator $\widetilde{O}_1$ from the soft-gluon exchange is obtained by applying the duality approximation and Borel transformation to the D channel. The result can be written as

$$\begin{aligned} &A^{(\widetilde{O}_1)}(\mathrm{D}^0 \to \pi^+\pi^-) \\ =&\mathrm{i}m_{\rm D}^2\left(\frac{1}{4\pi^2 f_\pi}\int_0^{s_0^\pi}\mathrm{d}s\mathrm{e}^{-\frac{s}{M^2}}\right) \\ &\times\left(\frac{m_{\rm c}^2}{2f_{\rm D}m_{\rm D}^4}\int_{u_0^{\rm D}}^1 \frac{\mathrm{d}u}{u}\mathrm{e}^{\frac{m_{\rm D}^2}{M'^2}-\frac{m_{\rm c}^2}{uM'^2}}\right. \\ &\times\left[\frac{m_{\rm c}f_{3\pi}}{u}\int_0^u\frac{\mathrm{d}v}{v}\varphi_{3\pi}(1-u,u-v,v)\right. \\ &+f_\pi\int_0^u\frac{\mathrm{d}v}{v}\left[3\widetilde{\varphi}_\perp(1-u,u-v,v)\right. \\ &-\left.\left(\frac{m_{\rm c}^2}{uM'^2}-1\right)\frac{\Phi_1(1-u,v)}{u}\right] \\ &+\left.\left.f_\pi\left(\frac{m_{\rm c}^2}{uM'^2}-2\right)\frac{\Phi_2(u)}{u^2}\right]\right), \end{aligned} \quad (17)$$

where $u_0^{\rm D} = m_{\rm c}^2/s_0^{\rm D}$, with the following definitions being introduced:

$$\begin{aligned} \frac{\partial\Phi_1(w,v)}{\partial v} &= \widetilde{\varphi}_\perp(w,1-w-v,v)+\widetilde{\varphi}_\parallel(w,1-w-v,v), \\ \frac{\partial\Phi_2(v)}{\partial v} &= \Phi_1(1-v,v). \end{aligned} \quad (18)$$

The asymptotic forms of the pion distribution amplitudes in Eq.(17) are given by [9]

$$\begin{aligned} \varphi_{3\pi}(\alpha_i) &= 360\alpha_1\alpha_2\alpha_3^2, \\ \widetilde{\varphi}_\perp(\alpha_i) &= 10\delta^2\alpha_3^2(1-\alpha_3), \\ \widetilde{\varphi}_\parallel(\alpha_i) &= -40\delta^2\alpha_1\alpha_2\alpha_3. \end{aligned} \quad (19)$$

Equation (17) is similar to that in Refs.[8,10] only with some replacements. In the following, we write the decay amplitudes about $\mathrm{D}^0 \to \pi^+\pi^-$, which include factorization and nonfactorization parts:

$$\begin{aligned} M_{f+\alpha_{\rm s}}(\mathrm{D}^0 \to \pi^+\pi^-) =&\mathrm{i}\frac{G_{\rm F}}{\sqrt{2}}f_\pi F^{\mathrm{D}\to\pi}(0) \\ &\times(m_{\rm D}^2-m_\pi^2)V_{ud}V_{cd}^* a_1, \end{aligned} \quad (20)$$

$$M_{nfg}(\mathrm{D}^0 \to \pi^+\pi^-) = \sqrt{2}G_{\rm F}V_{ud}V_{cd}^* A C_2, \quad (21)$$

where the amplitude $M_{f+\alpha_{\rm s}} = M_f + M_{\alpha_{\rm s}}$ represents the sum of factorization and nonfactorization parts from the hard-gluon exchanges; M_{nfg} is the amplitude of the nonfactorization part from the soft-gluon exchanges; A is the hadronic matrix element in Eq.(17). The total amplitude M is the sum of $M_{f+\alpha_{\rm s}}$ and M_{nfg} in the $\mathrm{D}^0 \to \pi^+\pi^-$ decay channel.

4. Numerical calculation

In numerical calculations we take $f_\pi = 132$MeV, $s_0^\pi = 0.7\mathrm{GeV}^2$ and $M^2 = 6-15\mathrm{GeV}^2$ [9] for the parameters of the pion channel, and $f_{\rm D} = 235$MeV, $m_{\rm c} = 1.3$GeV, $s_0^{\rm D} = 6\mathrm{GeV}^2$, $\mu_c = \sqrt{m_{\rm D}^2 - m_{\rm c}^2} \approx 1.3$GeV, M'^2=8–18GeV^2, $f_{3\pi}(\mu_c) = 0.0032\mathrm{GeV}^2$, $\delta^2(\mu_{\rm c}) = 0.19\mathrm{GeV}^2$,[12] $\tau(\mathrm{D}^0) = 4.11\times10^{-13}$s. In the D rest frame, the two-body decay width is

$$\Gamma(\mathrm{D}^0 \to \pi^+\pi^-) = \frac{1}{8\pi}|M(\mathrm{D}^0 \to \pi^+\pi^-)|^2\frac{|P|}{m_{\rm D}^2}, \quad (22)$$

where the momentum of the π meson is given by

$$|P| = \frac{[(m_{\rm D}^2(m_{\rm D}^2-4m_\pi^2)]^{\frac{1}{2}}}{2m_{\rm D}}, \quad (23)$$

The corresponding branching ratio is given by

$$\mathrm{Br}(\mathrm{D}^0 \to \pi^+\pi^-) = \frac{\Gamma(\mathrm{D}^0 \to \pi^+\pi^-)}{\Gamma_{\rm tot}}. \quad (24)$$

With the above parameters and formulae, we can obtain the branching ratios and compare the branching ratio of $\mathrm{D}^0 \to \pi^+\pi^-$ decay obtained in some approaches with that of the experiment.

Table 1. The branching ratios of $D^0 \to \pi^+\pi^-$ decay obtained in some approaches together with the experimental result.

Decay mode	NF	QCDF	QCDF+SGE	Experiment
$D^0 \to \pi^+\pi^-$	1.86×10^{-3}	1.69×10^{-3}	1.55×10^{-3}	$(1.52 \pm 0.09) \times 10^{-3}$

The data are presented in Table 1, where the second column gives the naive factorization results, the third column the QCD factorization results, the fourth column the QCD factorization and soft-gluon exchange results, and the final column the experimental data.

5. Summary

From Table 1, We find that the prediction of naive factorization is far greater than the experimental result, and the QCD factorization result approaches the experimental data. However, in the QCD factorization method, if we calculate the power correction term $O(\Lambda_{\rm QCD}/m_{\rm c})$ then the results are in agreement with the experimental data. We also find that the power term $O(\Lambda_{\rm QCD}/m_{\rm c})$ comes mainly from the soft-gluon effects, and the final state interaction contribution is small in $D^0 \to \pi^+\pi^-$ decay. From the calculated results, we find that the QCD factorization method can be applied to $D^0 \to \pi^+\pi^-$ decay, once the power term $O(\Lambda_{\rm QCD}/m_{\rm c})$ is included.

From Refs.[9,11], we can find that the soft-gluon contributions are different in different $B \to \pi\pi$ decay channels. For example, in $B^0 \to \pi^+\pi^-$ decay, the soft-gluon effects are smaller than the hard-gluon contributions because of the small Wilson coefficient C_2 in the decay amplitude. However, the contributions of the soft gluon and the hard gluon are on the same order in $B^0 \to \pi^0\pi^0$ decay since the Wilson coefficient C_1 is large in this decay amplitude. In $D \to \pi\pi$ decay, we can obtain a similar result. As shown in Table 1, the soft-gluon and the hard-gluon contributions are on the same order in $D \to \pi^+\pi^-$ decay, and neither of them can be neglected in this decay channel.

References

[1] Buras A J, Gerard J M and Ruckl R 1986 *Nucl. Phys.* B **268** 16
Blok B Y and Shifman M 1987 *Sov. J. Nucl. Phys.* **45** 135

[2] Deshpande N, Gronau M and Sutherland D 1980 *Phys. Lett.* B **90** 431

[3] Bauer M, Stech B and Wirbel M 1987 *Phys.* C **34** 103

[4] Kamel A N and Pham T N 1994 *Phys. Rev.* D **50** 1832

[5] Yao Y S, Zou J and Shao B 2003 *Chin. Phys.* **12** 649

[6] Yin W and Liang J Q 2003 *Chin. Phys.* **12** 599

[7] Kou S P 2001 *Chin. Phys.* **10** 398

[8] Beneke M, Buchalla G, Neubert M and Sachralda C T 1999 *Phys. Rev. Lett.* **83** 1914
Beneke M, Buchalla G, Neubert M and Sachralda C T 2000 *Nucl. Phys.* B **591** 313

[9] Khodjamirian A 2001 *Nucl. Phys.* B **605** 558

[10] Gronau M 1998 *Phys. Rev. Lett.* **83** 4005

[11] Wu X Y, Li Z H, Cui J Y and Huang T 2002 *Chin. Phys. Lett.* **19** 1596

[12] Belyaev V M, Braum V M, Khodjamirian A and Ruckl R 1995 *Phys. Rev.* D **51** 6177