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Adaptive regulation of uncertain chaos with dynamic neural networks*

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A novel adaptive control for uncertain nonlinear chaotic system is presented. A dynamical neural networks is used to perform 'black box' identification. Based on the identifier, the state feedback control is employed to drive the unknown chaotic system toward the desired target. Simulations show the derived control via the neuro-identifier turns out to be very effective.

Keywords: chaos, adaptive control, identification, dynamical neural networks
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1. Introduction

Chaos is a relatively new branch of mathematical science that seeks to interpret complicated nonlinear phenomena. Controlling chaos has recently been focused on in the nonlinear dynamics literature.[1−7]

In general, the unknown chaos is a black box belonging to a given class of nonlinearities. Therefore, adaptive control method is required, and a nonmodel-based scheme is suitable. A neuro-controller is popular for regulating uncertain chaotic system. As powerful tools for learning highly nonlinear systems, due to their massive parallelism, very fast adaptability, and inherent approximation capabilities, neural networks have attracted a lot of research, especially in the field of identification and control.

In this paper, singular perturbation analysis is applied to study the stability and robustness of a dynamic neural network identifier. A dynamic state feedback control based on neuro-identifier is developed to direct the unknown chaotic system toward desired trajectories. The effectiveness of the suggested approach is illustrated by its implementation on the Duffing equation.

This paper is organized as follows. In Section 2 we describe the concept of identification chaos using neural networks. In Section 3 we present a dynamical neural networks identifier. In Section 4 we develop adaptive state feedback control and prove the convergence of error and boundedness of all signals in the closed loop. In Section 5 the scheme proposed is applied to identify and control chaotic system. Finally, conclusions are summarized in Section 6.

2. Statement of the problem

Consider the nonlinear system given by

$$\dot{x} = f(x,t) + G(x)u, \tag{1}$$

where $x \in R^n$ is the state vector of the system, control action $u \in \mu \subset R^n$ where μ is the class of admissible input, f is a smooth nonlinear function with unknown parametrics describing the system dynamics, G is a matrix with columns G_i, $i = 1,2,\ldots,n$, $G = [G_1, G_2, \ldots, G_n]$, where G_i is a smooth vector, and suppose also that all states of the system are available for measurement, and suppose:

A1: Given μ, for any finite initial condition, the system states are uniformly bounded.

A2: f, G are continuous and satisfy a local Lipschitz condition so that the solution $x(t)$ to Eq.(1) is unique.

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To identify the dynamical system (1), we take the following dynamic neural networks (DNNs):

$$\dot{x}_{nn} = Ax_{nn} + BW_{\sigma}(x) + BW_{n+1}\vartheta(x)u, \quad (2)$$

where $x_{nn} \in R^n$ is the state of the DNN, control input $u \in \mu \subset R^n$, A, B are $n \times n$ diagonal matrices with elements a_i, b_i, for all $i = 1, 2, \ldots, n$. $W \in R^{n\times n}$ is a matrix of weights, and W_{n+1} is a $n \times n$ diagonal matrix of weights having the form $W_{n+1} = \mathrm{diag}[w_{1,n+1}, w_{2,n+1}, \ldots, w_{n,n+1}]$. Vector $\sigma(x) \in R^n$ and ϑ are $n \times n$ diagonal matrices, with elements $\sigma(x_i), \vartheta(x_i)$ respectively, both being smooth, monotonically increasing functions.

3. Dynamic neuro-identifier

This section aims at finding a learning law that guarantees stability of the neural networks and convergence of its output and weights to desired values. Due to parametric uncertainties, we may assume that there exist weights W^*, W^*_{n+1} such that the system (1) is completely described by a neural network

$$\dot{x} = Ax + BW^*\sigma(x) + W^*_{n+1}\vartheta(x)u, \quad (3)$$

where all matrices are as defined previously. Define the error between the identifier states and the real system states as $e = x_{nn} - x$. Then from (2) and (3) we can obtain the error equation

$$\dot{e} = Ae + BW_A\sigma(x) + BW_B\vartheta(x)u, \quad (4)$$

where $W_A = W_W^*$, $W_B = W_{n+1} - W^*_{n+1}$.

The Lyapunov stability theory is applied to derive stable adaptive laws. Take the Lyapunov function candidate

$$\begin{aligned}&\nu(e, W_A, W_B)\\ =&\frac{1}{2}e^{\mathrm{T}}Pe + \frac{1}{2}\mathrm{tr}\{W_A^{\mathrm{T}}W_A\} + \frac{1}{2}\mathrm{tr}\{W_B^{\mathrm{T}}W_B\}, \quad (5)\end{aligned}$$

where $P > 0$ is chosen to satisfy the Lyapunov equation $A^{\mathrm{T}}P + PA = -I$.

Differentiating (5) along the solution of (4), we can easily obtain the learning law in an elementary form as

$$\begin{aligned}\dot{w}_{ij} &= -b_i p_i \sigma(x_j) e_i,\\ \dot{w}_{i,n+1} &= -b_i \vartheta(x_i) p_i u_i e_i, \quad (6)\end{aligned}$$

for all $i, j = 1, 2, \ldots, n$, which can guarantee $\dot{\nu} = -\frac{1}{2}\|e\|^2 \le 0$.

Generally speaking, the model of neural networks is of lower order than the actual system, in view of the unmodelled dynamics present in the system, thus we extend the theory of singular perturbations to the case where dynamic unknown plant can be completely represented by

$$\begin{aligned}\dot{x} =& Ax + BW^*\sigma(x) + W^*_{n+1}\vartheta(x)u\\ &+ \psi(x, W, W_{n+1})A_0^{-1}B_0W_0u + \psi(x, W, W_{n+1})\delta,\\ \lambda\dot{\delta} =& A_0\delta + B_0W_0u, \delta \in R^r, \quad (7)\end{aligned}$$

where δ is the state of the unmodelled dynamics and $\lambda > 0$ a small singular perturbation constant. Then from Eqs.(2) and (7), the error equation can be formulated as

$$\begin{aligned}\dot{e} =& Ae + BW_A\sigma(x) + BW_B\vartheta(x)u\\ &- \psi(x, W, W_{n+1})A_0^{-1}B_0W_0u + \psi(x, W, W_{n+1})\delta,\\ \lambda\dot{\delta} =& A_0\delta + B_0W_0u, \delta \in R^r, \quad (8)\end{aligned}$$

where $\psi(x, W, W_{n+1})A_0^{-1}B_0W_0u, \psi(x, W, W_{n+1})\delta$ in Eq.(8) are bounded and differential for all $w_A \in \Omega_{w_A}$ a ball in R^n, $w_B \in \Omega_{w_B}$ a ball in R^n and $x \in \Omega_x$ a ball in R^n.

Note that $\dot{\delta}$ is large since λ is small, hence the unmodelled dynamics are fast. For a singular perturbation from $\lambda > 0$ to $\lambda = 0$ we have $\delta = -A_0^{-1}B_0W_0u$. As is well known from singular perturbation theory, the state δ may be expressed as

$$\delta = \gamma(x, \rho) + \rho, \quad (9)$$

where $\gamma(x, \rho)$ is defined as the quasi-steady-state of δ and ρ due to its fast transient. Thus $\gamma(x, \rho) = -A_0^{-1}B_0W_0u$. Substituting (9) into (8), the singularly perturbed model can be written as

$$\begin{aligned}\dot{e} =& Ae + BW_A\sigma(x) + BW_B\vartheta(x)u\\ &- \psi(x, W, W_{n+1})\rho,\\ \lambda\dot{\rho} =& A_0\rho - \lambda\dot{\gamma}(e, W_A, W_B, \rho, u), \quad (10)\end{aligned}$$

where

$$\dot{\gamma}(e, W_A, W_B, \rho, u) = \frac{\partial\gamma}{\partial e}\dot{e} + \frac{\partial\gamma}{\partial W_A}\dot{W}_A + \frac{\partial\gamma}{\partial W_B}\dot{W}_B.$$

Lemma 1 The $\dot{\gamma}(e, W_A, W_B, \rho, u)$ is bounded by $\|\dot{\gamma}(e, W_A, W_B, \rho, u)\| \le \alpha_1\|e\| + \alpha_2\|\rho\|$ provided that the following properties hold:

$$\begin{aligned}&\left\|\gamma_W\dot{W}_A\right\| \le \beta_0\|e\|, \left\|\gamma_{W_{n+1}}\dot{W}_B\right\| \le \beta_1\|e\|,\\ &\|\gamma_e BW_B\vartheta(x)u\| \le \beta_2\|e\|, \|\gamma_e BW_A\sigma(x)\| \le \beta_3\|e\|,\\ &\|\gamma_e\varphi(x, W, W_{n+1})\| \le \alpha_2,\\ &\|\gamma_e Ae\| \le \beta_4\|e\|, \|\gamma_u\dot{u}\| \le \beta_5\|e\|,\end{aligned}$$

and

$$\alpha_1 = \beta_0 + \beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5.$$

Theorem 1 The equilibrium of the singularly perturbed model is asymptotically stable for all $\lambda \in \left(0, \frac{1}{\tau_1\tau_2 + 2\tau_3}\right)$ and an estimate of its region of attraction is

$$\Lambda = \{e, W_A, W_B, \rho : \nu(e, W_A, W_B, \rho) \leq \tau\},$$

where τ is the largest constant such that the set Λ is confined to $B_e \times B_w \times B_{w_{n+1}}$. Also, the following properties are guaranteed:

- $e, x_{nn}, \rho, W, W_{n+1} \in L_\infty, e, \rho \in L_2$.
- $\lim\limits_{t\to\infty} e(t) = 0, \lim\limits_{t\to\infty} \rho(t) = 0,$
  $\lim\limits_{t\to\infty} \dot{W}_A(t) = 0, \lim\limits_{t\to\infty} \dot{W}_B(t) = 0.$

Proofs of the above can be found in Ref.[8].

4. Adaptive tracking control

In this section we investigate a more general case where parametric and dynamic uncertainties are present. In the tracking problem we want the real system states to follow the state of a reference model. Let the reference model be described by $\dot{x}_m = \varphi(x_m, t), x_m \in R^n$. Define the error between the identifier states and the model states as

$$e_c = x_{nn} - x_m. \tag{11}$$

Thus, the control scheme is now represented by the following differential equations:

$$\begin{aligned} \dot{e} &= Ae + BW_A\sigma(x) + BW_B\vartheta(x)u - \varphi(x, W, W_{n+1})\rho, \\ \dot{e}_c &= Ae_c, \lambda\dot{\rho} = A_0\rho - \lambda\dot{\gamma}(e, W_A, W_B, \rho, u), \end{aligned} \tag{12}$$

$$u = -[BW_{n+1}\vartheta(x)]^{-1}[Ax_m + BW\sigma(x) - \varphi(x_m, t)].$$

To apply the control law u in (12), we have to assure the existence of $[BW_{n+1}\vartheta(x)]^{-1}$, namely $w_{i,n+1} \neq 0, \forall t \geq 0, \forall i = 1, 2, \ldots, n$. Hence, $W_{n+1}(t)$ is contained through the usage of a projection algorithm [9] to the set $\Omega = \{W_{n+1} : \|W_B\| \leq \omega_m\}$ where $\omega_m > 0$ is a constant. Therefore, the standard adaptive laws (6) can be modified in the form of matrix

$$\dot{W} = -EBPS_0,$$

$$\dot{W}_{n+1} = \begin{cases} -BP\vartheta UE \text{ if } W_{n+1} \in \Lambda \text{ or } \{\|W_B\| = \omega_m \text{ and } \mathrm{tr}\{-BP\vartheta UEW_B\} \leq 0\}, \\ -BP\vartheta UE + \mathrm{tr}\{BP\vartheta\sigma UW_B\}\left(\dfrac{1 + \|W_B\|}{\omega_m}\right)^2 W_B \text{ if } \{\|W_B\| = \omega_m \text{ and } \mathrm{tr}\{-BP\vartheta UEW_B\} > 0\}, \end{cases} \tag{13}$$

Where all matrices are defined as follows:

$$\begin{aligned} P &= \mathrm{diag}[p_1, p_2, \ldots, p_n], \\ B &= \mathrm{diag}[b_1, b_2, \ldots, b_n], \\ E &= \mathrm{diag}[e_1, e_2, \ldots, e_n], \\ U &= \mathrm{diag}[u_1, u_2, \ldots, u_n], \\ S_0 &= \begin{bmatrix} \sigma(x_1) & \cdots & \sigma(x_n) \\ \vdots & & \vdots \\ \sigma(x_1) & \cdots & \sigma(x_n) \end{bmatrix}. \end{aligned}$$

Theorem 2 The control scheme (12) is asymptotically stable for all $\lambda \in (0, \lambda_0)$, where $\lambda_0 = \frac{1}{2}\left(\frac{1}{2k_1k_2 + k_3}\right)$. Moreover, the learning law (13) guarantees the following properties:

- $e, e_c, \rho, x_{nn}, W_A, W_B \in L_\infty, e, e_c, \rho \in L_2$.
- $\lim\limits_{t\to\infty} e(t) = 0, \lim\limits_{t\to\infty} e_c(t) = 0, \lim\limits_{t\to\infty} \rho(t) = 0,$
  $\lim\limits_{t\to\infty} \dot{W}_A(t) = 0, \lim\limits_{t\to\infty} \dot{W}_B(t) = 0.$

Proof: Construct Lyapunov function candidate

$$\begin{aligned} \nu(e, W_A, W_B, \rho) =& \frac{1}{2}k_1e^{\mathrm{T}}Pe + \frac{1}{2}k_1e_c^{\mathrm{T}}Pe_c \\ &+ \frac{1}{2}k_2\rho^{\mathrm{T}}P_0\rho + \frac{1}{2}k_1\mathrm{tr}\{W_A^{\mathrm{T}}W_A\} \\ &+ \frac{1}{2}k_2\mathrm{tr}\{W_B^{\mathrm{T}}W_B\}, \end{aligned} \tag{14}$$

where $P, P_0 > 0$ are the solutions of the Lyapunov equation $A^{\mathrm{T}}P + PA = -I$, $A_0^{\mathrm{T}}P_0 + P_0A_0 = -I$. Then the derivative of (14) is

$$\begin{aligned} \dot{\nu} =& \frac{k_1}{2}(-e^{\mathrm{T}}e + \sigma^{\mathrm{T}}(x)W_A^{\mathrm{T}}BPe + u^{\mathrm{T}}\vartheta(x)W_BBPe \\ &- \rho^{\mathrm{T}}\varphi^{\mathrm{T}}(x, W, W_{n+1})Pe) + \frac{k_1}{2}((\sigma^{\mathrm{T}}(x)W_A^{\mathrm{T}}BPe)^{\mathrm{T}} \\ &+ (u^{\mathrm{T}}\vartheta(x)W_BBPe)^{\mathrm{T}} - (\rho^{\mathrm{T}}\varphi^{\mathrm{T}}(x, W, W_{n+1})Pe)^{\mathrm{T}}) \\ &- \frac{k_1}{2}e_c^{\mathrm{T}}e_c + k_1\mathrm{tr}\{\dot{W}_A^{\mathrm{T}}W_A\} + k_2\mathrm{tr}\{\dot{W}_B^{\mathrm{T}}W_B\} \\ &+ \frac{k_2}{2}(-\frac{1}{\lambda}\rho^{\mathrm{T}}\rho - (\dot{\gamma}^{\mathrm{T}}(e, W_A, W_B, \rho)P_0\rho \\ &- (\dot{\gamma}^{\mathrm{T}}(e, W_A, W_B, \rho)P_0\rho)^{\mathrm{T}}). \end{aligned}$$

Note that

$$\sigma^{\mathrm{T}}(x)W_A^{\mathrm{T}}BPe, \quad u^{\mathrm{T}}\vartheta(x)W_BBPe,$$
$$\rho^{\mathrm{T}}\varphi^{\mathrm{T}}(x,W,W_{n+1})Pe, \quad \dot{\gamma}^{\mathrm{T}}(e,W_A,W_B,\rho)P_0\rho$$

are scalars, consequently

$$\sigma^{\mathrm{T}}(x)W_A^{\mathrm{T}}BPe = (\sigma^{\mathrm{T}}(x)W_A^{\mathrm{T}}BPe)^{\mathrm{T}},$$
$$u^{\mathrm{T}}\vartheta(x)W_BBPe = (u^{\mathrm{T}}\vartheta(x)W_BBPe)^{\mathrm{T}},$$

and

$$\rho^{\mathrm{T}}\varphi^{\mathrm{T}}(x,W,W_{n+1})Pe = (\rho^{\mathrm{T}}\varphi^{\mathrm{T}}(x,W,W_{n+1})Pe)^{\mathrm{T}},$$
$$\dot{\gamma}^{\mathrm{T}}(e,W_A,W_B,\rho)P_0\rho = (\dot{\gamma}^{\mathrm{T}}(e,W_A,W_B,\rho)P_0\rho)^{\mathrm{T}}.$$

Thus $\dot{\nu}$ becomes

$$\begin{aligned}\dot{\nu} =& \frac{k_1}{2}(-\|e\|^2 + 2\sigma^{\mathrm{T}}(x)W_ABPe + 2u^{\mathrm{T}}\vartheta(x)W_BBPe \\ &- 2\rho^{\mathrm{T}}\varphi^{\mathrm{T}}(x,W,W_{n+1})Pe) - \frac{k_1}{2}\|e_c\|^2 \\ &+ k_1\mathrm{tr}\{\dot{W}_A^{\mathrm{T}}W_A\} + k_2\mathrm{tr}\{\dot{W}_B^{\mathrm{T}}W_B\} \\ &+ \frac{k_2}{2}\left(-\frac{1}{\lambda}\|\rho\|^2 - 2\rho^{\mathrm{T}}P_0\dot{\gamma}^{\mathrm{T}}(e,W_A,W_B,\rho)\right). \quad (15)\end{aligned}$$

Using the learning laws obtained previously, (15) turns out to be

$$\begin{aligned}\dot{\nu} =& -\frac{k_1}{2}\|e\|^2 - \frac{k_1}{2}\|e_c\|^2 - \frac{k_2}{2\lambda}\|\rho\|^2 \\ &- k_1\rho^{\mathrm{T}}\varphi^{\mathrm{T}}(x,W,W_{n+1})Pe \\ &- k_2\dot{\gamma}^{\mathrm{T}}(e,W_A,W_B,\rho)P_0\rho \\ &+ I_n\mathrm{tr}\{\mathrm{tr}\{BP\vartheta UEW_B\}\left(\frac{1+\|W_B\|}{\omega_m}\right)^2 W_B^{\mathrm{T}}W_B \\ \le& -\frac{k_1}{2}\|e\|^2 - \frac{k_1}{2}\|e_c\|^2 - \frac{k_2}{2\lambda}\|\rho\|^2 \\ &+ k_1\|\rho\|\,\|\varphi(x,W,W_{n+1})\|\,\|P\|\,\|e\| \\ &+ k_2\|\dot{\gamma}(e,W_A,W_B,\rho)\|\,\|P_0\|\,\|\rho\| \\ &+ I_n\mathrm{tr}\{BP\vartheta UEW_B\}\left(\frac{1+\omega_m}{\omega_m}\right)^2\|W_B\|^2.\end{aligned}$$

According to Lemma 1 we establish

$$\begin{aligned}\dot{\nu} \le& -\frac{k_1}{2}\|e\|^2 - \frac{k_1}{2}\|e_c\|^2 - \frac{k_2}{2\lambda}\|\rho\|^2 \\ &+ 2k_1k_2\|\rho\|\,\|e\| + k_2k_3\|\rho\|^2 \\ &+ I_n\mathrm{tr}\{BP\vartheta UEW_B\}(1+\omega_m)^2,\end{aligned}$$

provided that $\alpha_1\|P_0\| \le k_1$, $\|\varphi(x,W,W_{n+1})\|\,\|P\| \le k_2$, $\alpha_2\|P_0\| \le k_3$ which finally takes the form

$$\dot{\nu} \le -\begin{bmatrix}\|e\| & \|\rho\| & \|e_c\|\end{bmatrix}\begin{bmatrix}\frac{k_1}{2} & -k_1k_2 & 0\\ -k_1k_2 & k_2\left(\frac{1}{2\lambda}-k_3\right) & 0\\ 0 & 0 & \frac{k_1}{2}\end{bmatrix}\begin{bmatrix}\|e\|\\ \|\rho\|\\ \|e_c\|\end{bmatrix} + I_n\mathrm{tr}\{BP\vartheta UEW_B\}(1+\omega_m)^2. \quad (16)$$

The 3×3 matrix in (16) is positive definite when $\lambda \le \frac{1}{2}\left(\frac{1}{2k_1k_2+k_3}\right)$. Since $\dot{\nu} \le 0$ we conclude that $\nu \in L_\infty$, which implies e, e_c, ρ, W_A, $W_B \in L_\infty$. In addition, $x_{nn} = e + x$, $W = W_A + W^*$, $W_{n+1} = W_B + W_{n+1}^*$ are also bounded. Consider that ν is non-increasing function of time and bounded, the $\lim_{t\to\infty}\nu = \nu_\infty$ exists. Therefore by integrating $\dot{\nu}$ from 0 to ∞, we obtain

$$\begin{aligned}&\frac{k_1}{2}\int_0^\infty \|e\|^2\mathrm{d}t + k_2\left(\frac{1}{2\lambda}-k_3\right)\int_0^\infty\|\rho\|^2\mathrm{d}t \\ &+ \frac{k_1}{2}\int_0^\infty\|e_c\|^2\mathrm{d}t - 2k_1k_2\int_0^\infty\|e\|\,\|\rho\|\,\mathrm{d}t \\ &- I_n(1+\omega_m)^2\int_0^\infty\mathrm{tr}\{BP\vartheta UEW_B\}\mathrm{d}t \\ &\le [\nu_0 - \nu_\infty] < \infty,\end{aligned}$$

which implies that e, e_c, $\rho \in L_2$. On the basis of (12), since u, A_0, $\dot{\gamma}$ (e, W_A, W_B, ρ), $\sigma(x)$, $\vartheta(x)$ are bounded, $\dot{e}$, $\dot{e}_c$, $\dot{\rho} \in L_\infty$. Since e, e_c, $\rho \in L_2 \cap L_\infty$ using Barbalat's lemma [10] we conclude that $\lim_{t\to\infty} e(t) = 0$, $\lim_{t\to\infty} e_c(t) = 0$, $\lim_{t\to\infty}\rho = 0$.

Considering the boundedness of $u, \sigma(x), \vartheta(x)$ and the convergence of $e(t)$ to zero, we conclude that $\dot{W}_A$, $\dot{W}_B$ also converges to zero.

5. Simulation results

In this section, a typical chaotic system is chosen to demonstrate the abilities of the neuro-identifier and adaptive tracking controller based on the identification estimations. The Duffing equation describes a specific nonlinear circuit or the hardening spring effect observed in many mechanical problems and can be written as

$$\begin{cases}\dot{x}_1 = x_2 + u_1,\\ \dot{x}_2 = -c_1x_1 - c_2x_1^3 - cx_2 + q\cos(wt) + u_2,\end{cases} \quad (17)$$

where c, c_1, c_2, q, w are constants and $u = [u_1, u_2]^{\mathrm{T}}$ is control input. It is known that the solution of (17)

exhibits almost periodic and chaotic behaviour in the uncontrolled case if we select c=0.4, c_1=1.1, c_2=1, q=2.1, w=1.8. To identify system (17), we use the neural

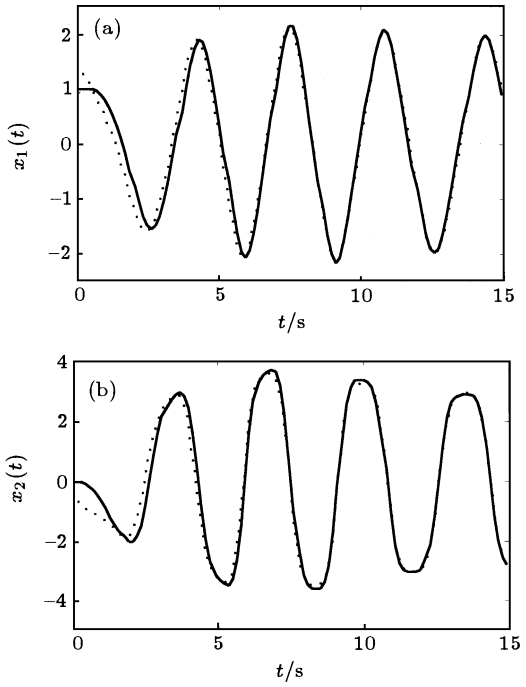


Fig.1. Identification curves of the system state x_1, x_2.

network as in (2), where $A = \mathrm{diag}(-6,-6)$, $B = \mathrm{diag}(12,12)$. The elements of $\sigma(x), \vartheta(x)$ are represented by sigmoids of the form $\sigma(x_i) = \dfrac{k}{1+\mathrm{e}^{-lx_i}}$, $\vartheta(x_i) = \dfrac{k}{1+\mathrm{e}^{-lx_i}} + \varepsilon$ for all $i = 1, 2, \ldots, n$, where k, l represent the bound (k) and slope (l) of sigmoid's curvature and $\varepsilon > 0$ a constant that shifts the sigmoid, such that $\vartheta(x) > 0$. These parameters are selected as k=3, l=1.6, ε=0.1, with initial value $x_0 = [1,0]^{\mathrm{T}}$. The identification results are shown in Fig.1. The solid lines correspond to the state of the uncontrolled system and the dashed lines are the states of the neural network identifier. Then we implement the control law given by (12) based on the identification estimates. The nonlinear reference model to be followed is selected as a circle $x_m = (\cos(t), \sin(t))$. The trajectory tracking results are shown in Fig.2. The solid lines are the given reference trajectories and the dashed lines are the controlled trajectories. Simulations show that good identification results provide a small enough tracking error.

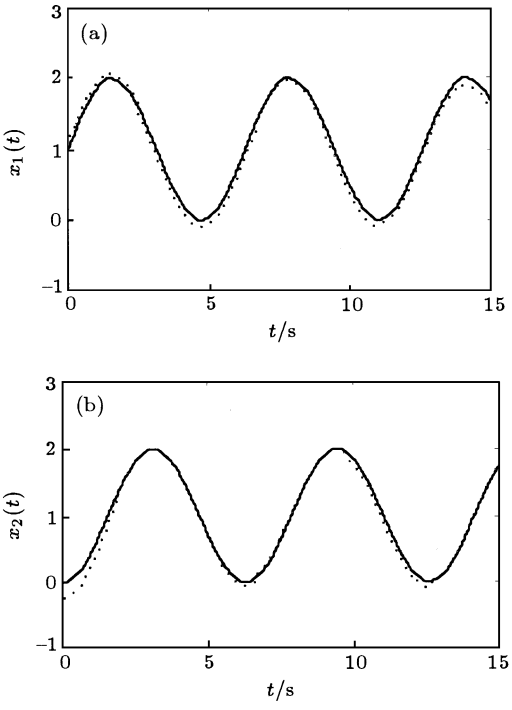


Fig.2. Tracking control of the Duffing system.

6. Conclusions

This paper studies a new adaptive control of unknown chaotic system. A dynamical neural networks is employed to conduct 'black box' identification. Based on the identification estimates, the state feedback control is developed to appropriately regulate chaotic states, and simulation results are presented to illustrate the suggested scheme.

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