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A theory of nonlinear AC response in coated composites*

Wei En-Bo(魏恩泊)[a)†], Tian Ji-Wei(田纪伟)[b)], and Song Jin-Bao(宋金宝)[a)]

[a)] *Institute of Oceanology, Chinese Academy of Sciences, Qingdao* 266071, *China*

[b)] *Physical Oceanography Laboratory, Ocean University of China, Qingdao* 266003, *China*

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A method for determining effective dielectric responses of Kerr-like coated nonlinear composites under the alternating current (AC) electric field is proposed by using perturbation approach. As an example, we have investigated the composite with coated cylindrical inclusions randomly embedded in a host under an external sinusoidal field with finite frequency ω. The local field and potential of composites in general consists of components with all harmonic frequencies. The effective nonlinear AC responses at all harmonics are induced by the coated nonlinear composites because of the nonlinear constitutive relation. Moreover, we have derived the formulae of effective nonlinear AC responses at the fundamental frequency and the third harmonic in the dilute limit.

Keywords: nonlinear composites, alternating current, effective dielectric response
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1. Introduction

Nonlinear responses of composite media have been studied recently because of their applications in engineering.[1−6] The effective responses of composites are dependent not only on their physical properties and volume fraction of inclusions, but also on the particle shape and the applied field.[7−10] In particular, if an external alternating current (AC) electric field is applied to the nonlinear composites, there are many components of local field at harmonics when the constitutive relations of composites are nonlinear.[11] The local fields at finite frequencies affect the nonlinear susceptibilities and the nonlinear optical response.[12] A number of authors have developed theories for investigating the effective nonlinear AC response of nonlinear composites in view of the interest of applications to engineering design. For example, Levy *et al*[4] have considered the second harmonic generation and the induced third harmonic generation in spherical nonlinear composite. Hui *et al*[11] derived the effective nonlinear dielectric susceptibility tensor at higher order harmonics. Gu *et al*[12−14] have derived the nonlinear effective response of cubic weakly nonlinear composites at harmonic frequencies of external AC electric field. In this work, we develop the perturbation method to study the effective nonlinear dielectric constant of the coated nonlinear composites in an external AC field.

We consider homogeneous and isotropic composites, where coated cylindrical particles are randomly embedded in a host medium. The constitutive relations between electric displacement D and electric field E are Kerr-like cubic nonlinear:

$$D_p = \varepsilon_p E + \chi_p |E|^2 E, \quad \text{in } \Omega_p, \quad p = \text{i, s, h}, \qquad (1)$$

where the subscripts p=i,s,h denote the quantities in inclusion core particle (i), coated shell layer material (s) and host (h) regions, respectively. The quantity Ω_p is the region occupied by the type p material. We assume that the real linear dielectric constant ε_p and the real nonlinear susceptibility χ_p are independent of the frequencies of applied field. This assumption is valid for low frequency processes under the quasi-electrostatic field equation $\nabla \times E = 0$. The corresponding governing equation satisfies $\nabla \cdot D = 0$. The boundary conditions of the coated composites are the

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†Corresponding author: E-mail: ebwei@ms.qdio.ac.cn

continuity of the potentials and the normal components of the electric displacement on the surface of the core, coated layer and host. At infinity the potential must match that of the external applied field.

2. Perturbation method for AC process

For weakly nonlinear composites with coated cylindrical inclusion, let us consider the concentric cylindrical particle with radius of core a and the coated shell radius b. A simple sinusoidal field with finite frequency ω is applied to nonlinear composites along the $\hat{x}$-direction:

$$E_{app} = E_a \sin(\omega t), \tag{2}$$

where $E_a = \varepsilon_a \hat{x}$. In this case, the applied field induces sinusoidal components at harmonic frequencies because of the cubic nonlinearity of the constitutive relation. In different regions, the electric potentials at a point x are written as

$$\begin{aligned}\Phi_p(x,t) =& \Phi_1^p(x)\sin(\omega t) + \Phi_3^p(x)\sin(3\omega t)\\ &+ \Phi_5^p(x)\sin(5\omega t) + \cdots,\\ &\text{in} \quad \Omega_p, \quad p = \text{i, s, h.}\end{aligned} \tag{3}$$

In order to develop the perturbation to deal with the weakly nonlinear composites,[15] we choose the nonlinear susceptibility $\chi_{\rm h}$ of the host medium as the perturbation parameter. The potentials $\Phi_n^p(x)$ can be expanded with the parameter $\chi_{\rm h}$,

$$\begin{aligned}&\Phi_n^p(x) = \Phi_n^{0p}(x) + \chi_{\rm h}\Phi_n^{1p}(x) + \chi_{\rm h}^2\Phi_n^{2p}(x) + \cdots,\\ &\text{in} \quad \Omega_p, \quad p = \text{i, s, h}, \quad n = 1,3,5,\cdots.\end{aligned} \tag{4}$$

With the equation $E(x,t) = -\nabla\Phi(x,t)$, we get the perturbation expansion of the electric displacement D in whole regions by substituting the potentials (3) and (4) into the Eq.(1):

$$\begin{aligned}&D_n^p(x) = D_n^{0p}(x) + \chi_{\rm h}D_n^{1p}(x) + \chi_{\rm h}^2 D_n^{2p}(x) + \cdots,\\ &n = 1,3,5,\cdots, \quad p = \text{i, s, h},\end{aligned} \tag{5}$$

where $D_n^{kp}(x)$ can be expressed by potentials $\Phi_m^{lp}(x)$. From the governing equation $\nabla\cdot D(x,t) = 0$, we obtain a set of equations by comparing the power of the perturbation parameter:

$$\begin{aligned}&\nabla\cdot D_n^{kp}(x) = 0, \quad \text{in} \quad \Omega_p,\\ &k = 0,1,2,\cdots, \quad n = 1,3,5,\cdots, \quad p = \text{i, s, h}.\end{aligned} \tag{6}$$

The boundary conditions of the potentials and the electric displacements are as follows.

At the interfaces between the inclusion core and inside layer of the coated shell, $\partial\Omega_{\rm i}$,

$$\begin{aligned}&\Phi_n^{k\rm i}(x) = \Phi_n^{k\rm s}(x), \quad \hat{n}\cdot D_n^{k\rm i}(x) = \hat{n}\cdot D_n^{k\rm s},\\ &\text{on} \quad \partial\Omega_{\rm i}, \quad n = 1,3,5,\cdots, \quad k = 0,1,2,\cdots,\end{aligned} \tag{7}$$

where $\hat{n}$ is the surface normal of the core.

At the interface between the outside layer of the coated shell and host medium, $\partial\Omega_{\rm s}$,

$$\begin{aligned}&\Phi_n^{k\rm h}(x) = \Phi_n^{k\rm s}(x), \quad \hat{n}\cdot D_n^{k\rm h}(x) = \hat{n}\cdot D_n^{k\rm s},\\ &\text{on} \quad \partial\Omega_{\rm s}, \quad n = 1,3,5,\cdots, \quad k = 0,1,2,\cdots.\end{aligned} \tag{8}$$

At infinity the potential is written as

$$\begin{aligned}&\partial\Phi_n^{k\rm h}/\partial x|_{x=\infty} = -\varepsilon_a(\text{or zero}), \quad \text{if } k = 0\\ &\text{and} \quad n = 1 \text{ (or other case for } k,n).\end{aligned} \tag{9}$$

3. Potentials in coated cylindrical composites

From Eqs.(5) and (6), we can obtain the governing equation of the zero-order potentials at basic frequency,

$$\varepsilon_p\nabla^2\Phi_1^{0p}(x) = 0, \text{ in } \Omega_p, \ p = \text{i, s, h}. \tag{10}$$

With the boundary conditions (7), (8) and (9), we derive the zero-order potentials of the core, coated layer and host regions using Eq.(10). At the fundamental frequency, the zero-order potentials are

$$\Phi_1^{0\rm i}(x) = A_0(a_0+1)\rho\varepsilon_a\cos\theta, \quad \rho \le a, \tag{11}$$

$$\Phi_1^{0\rm s}(x) = A_0(a_0\rho + a^2\rho^{-1})\varepsilon_a\cos\theta, \quad a \le \rho \le b, \tag{12}$$

$$\Phi_1^{0\rm h}(x) = (-\rho + B_0b^2\rho^{-1})\varepsilon_a\cos\theta, \quad \rho \ge b, \tag{13}$$

where

$$\begin{aligned}a_0 =& (\varepsilon_{\rm i}+\varepsilon_{\rm s})/(\varepsilon_{\rm s}-\varepsilon_{\rm i}),\\ A_0 =& 2(\varepsilon_{\rm i}-\varepsilon_{\rm s})\varepsilon_{\rm h}\Big/\Big[(\varepsilon_{\rm s}+\varepsilon_{\rm h})(\varepsilon_{\rm s}+\varepsilon_{\rm i})\\ &-\left(\frac{a}{b}\right)^2(\varepsilon_{\rm s}-\varepsilon_{\rm h})(\varepsilon_{\rm s}-\varepsilon_{\rm i})\Big],\\ B_0 =& A_0\left(a_0+\left(\frac{a}{b}\right)^2\right)+1.\end{aligned}$$

For the boundary-value problem of the zero-order potentials at third harmonic 3ω, the zero-order potentials $\Phi_3^{0p}(x)$ in composite regions are zero because of the governing equation without source. With the

zero-order potentials vanishing at the third harmonics, the governing equations of the first-order potentials at fundamental frequency and at the third harmonic are, respectively, rewritten as

$$\begin{aligned}&\varepsilon_p\nabla^2\Phi_1^{1p}(x)\\&=-\beta_p\nabla\cdot\left[\frac{3}{4}\nabla\Phi_1^{0p}(x)(\nabla\Phi_1^{0p}(x))^2\right],\end{aligned}\tag{14}$$

and

$$\begin{aligned}&\varepsilon_p\nabla^2\Phi_3^{1p}(x)\\&=-\beta_p\nabla\cdot\left[-\frac{1}{4}\nabla\Phi_1^{0p}(x)(\nabla\Phi_1^{0p}(x))^2\right],\end{aligned}\tag{15}$$

where $\beta_p=\chi_p/\chi_{\rm h}$, $p=$ i, s, h. In the core, coating layer and host regions, the general solutions of potentials at fundamental frequency and the third harmonic can be exactly solved with the boundary conditions. The solutions are given as follows:

$$\begin{aligned}\Phi_1^{1{\rm i}}(x)=&[A_1^{1{\rm i}}\rho\cos(\theta)+A_3^{1{\rm i}}\rho^3\cos(3\theta)]\varepsilon_a^3,\quad \rho\le a,\\\Phi_1^{1{\rm s}}(x)=&(A_1^{1{\rm s}}\rho+C_1^{1{\rm s}}\rho^{-1}+A_{\rm s}a^4\rho^{-3}\\&+B_{\rm s}a^6\rho^{-5})\varepsilon_a^3\cos(\theta)\\&+(A_3^{1{\rm s}}\rho^3+C_3^{1{\rm s}}\rho^{-3}+D_{\rm s}\rho^{-1})\varepsilon_a^3\cos(3\theta),\\&a\le\rho\le b,\\\Phi_1^{1h}(x)=&(C_1^{1{\rm h}}\rho^{-1}+A_{\rm h}\rho^{-3}+B_{\rm h}\rho^{-5})\varepsilon_a^3\cos(\theta)\\&+(C_3^{1{\rm h}}\rho^{-3}+D_{\rm h}\rho^{-1})\varepsilon_a^3\cos(3\theta),\quad \rho\ge b,\end{aligned}$$

$$\begin{aligned}\Phi_3^{1{\rm i}}(x)=&[A_1^{3{\rm i}}\rho\cos(\theta)+A_3^{3{\rm i}}\rho^3\cos(3\theta)]\varepsilon_a^3,\quad \rho\le a,\\\Phi_3^{1{\rm s}}(x)=&(A_1^{3{\rm s}}\rho+C_1^{3{\rm s}}\rho^{-1}+A_3^{\rm s}a^4\rho^{-3}\\&+B_3^{\rm s}a^6\rho^{-5})\varepsilon_a^3\cos(\theta)\\&+(A_3^{3{\rm s}}\rho^3+C_3^{3{\rm s}}\rho^{-3}+D_3^{\rm s}\rho^{-1})\varepsilon_a^3\cos(3\theta),\\&a\le\rho\le b,\\\Phi_3^{1{\rm h}}(x)=&(C_1^{3{\rm h}}\rho^{-1}+A_3^{\rm h}\rho^{-3}+B_3^{\rm h}\rho^{-5})\varepsilon_a^3\cos(\theta)\\&+(C_3^{3{\rm h}}\rho^{-3}+D_3^{\rm h}\rho^{-1})\varepsilon_a^3\cos(3\theta),\quad \rho\ge b.\end{aligned}$$

In this work we investigate the effective nonlinear AC response of coated nonlinear composites. Thus we only give the coefficients of the potentials, which are related to the effective response:

$$\begin{aligned}A_{\rm s}&=\frac{3}{4}\beta_{\rm s}A_0^3a_0/\varepsilon_{\rm s},\\B_{\rm s}&=-\frac{1}{8}\beta_{\rm s}A_0^3/\varepsilon_{\rm s},\end{aligned}$$

$$\begin{aligned}A_1^{1{\rm i}}=&F_5/(\varepsilon_{\rm s}+\varepsilon_{\rm h})+\beta_{\rm s}A_0^3\\&\times\left[F_6/(\varepsilon_{\rm s}+\varepsilon_{\rm h})+\frac{3}{4}a_0/\varepsilon_{\rm s}-1/(8\varepsilon_{\rm s})\right]\\&+\left(\frac{\varepsilon_{\rm s}-\varepsilon_{\rm h}}{\varepsilon_{\rm s}+\varepsilon_{\rm h}}\left(\frac{a}{b}\right)^2+1\right)\\&\times\left[A_0^4\frac{\varepsilon_{\rm s}+\varepsilon_{\rm h}}{2\varepsilon_{\rm h}(\varepsilon_{\rm i}-\varepsilon_{\rm h})}(\beta_{\rm s}F_1-\frac{3}{4}\beta_{\rm i}(a_0+1)^3\right.\\&-\frac{\varepsilon_{\rm h}+\varepsilon_{\rm i}}{\varepsilon_{\rm s}+\varepsilon_{\rm h}}\beta_{\rm s}F_4)+A_0\frac{\varepsilon_{\rm s}+\varepsilon_{\rm h}}{2\varepsilon_{\rm h}(\varepsilon_{\rm i}-\varepsilon_{\rm h})}\\&\times\left.\left(F_2-\frac{\varepsilon_{\rm h}+\varepsilon_{\rm i}}{\varepsilon_{\rm s}+\varepsilon_{\rm h}}F_3\right)\right],\\A_1^{1{\rm s}}=&\frac{\varepsilon_{\rm s}-\varepsilon_{\rm h}}{\varepsilon_{\rm s}+\varepsilon_{\rm h}}\left(\frac{a}{b}\right)^2\left[A_0^4\frac{\varepsilon_{\rm s}+\varepsilon_{\rm h}}{2\varepsilon_{\rm h}(\varepsilon_{\rm i}-\varepsilon_{\rm s})}\right.\\&\times\left(F_1\beta_{\rm s}-\frac{3}{4}\beta_{\rm i}(a_0+1)^3-\frac{\varepsilon_{\rm h}+\varepsilon_{\rm i}}{\varepsilon_{\rm h}+\varepsilon_{\rm s}}\beta_{\rm s}F_4\right)\\&+A_0\frac{\varepsilon_{\rm s}+\varepsilon_{\rm h}}{2\varepsilon_{\rm h}(\varepsilon_{\rm i}-\varepsilon_{\rm s})}\left.\left(F_2-\frac{\varepsilon_{\rm h}+\varepsilon_{\rm i}}{\varepsilon_{\rm h}+\varepsilon_{\rm s}}F_3\right)\right]\\&+(F_5+\beta_{\rm s}A_0^3F_6)/(\varepsilon_{\rm s}+\varepsilon_{\rm h}),\\A_3^{\rm s}=&-\frac{1}{4}\beta_{\rm s}A_0^3a_0/\varepsilon_{\rm s},\\B_3^{\rm s}=&\frac{1}{24}\beta_{\rm s}A_0^3/\varepsilon_{\rm s},\\A_1^{3{\rm i}}=&-\frac{1}{3}A_1^{1{\rm i}},\\A_1^{3{\rm s}}=&-\frac{1}{3}A_1^{1{\rm s}},\end{aligned}$$

where

$$\begin{aligned}F_1=&\frac{1}{8}\left[\left(\frac{a}{b}\right)^6-1\right]+\frac{3}{4}a_0\left[\left(\frac{a}{b}\right)^4-1\right]\\&+\frac{3}{2}a_0^2\left[\left(\frac{a}{b}\right)^2-1\right]\\&+\left[\frac{3}{4}a_0\left(\left(\frac{a}{b}\right)^4-1\right)+\frac{1}{8}\left(1-\left(\frac{a}{b}\right)^6\right)\right]\varepsilon_{\rm i}/\varepsilon_{\rm s},\\F_2=&\frac{1}{8}B_0^3(\varepsilon_{\rm i}/\varepsilon_{\rm h}-1)+\frac{3}{4}B_0^2(\varepsilon_{\rm i}/\varepsilon_{\rm h}+1)-\frac{3}{2}B_0-\frac{3}{4},\\F_3=&\frac{1}{8}B_0^3(\varepsilon_{\rm s}/\varepsilon_{\rm h}-1)+\frac{3}{4}B_0^2(\varepsilon_{\rm s}/\varepsilon_{\rm h}+1)-\frac{3}{2}B_0-\frac{3}{4},\\F_4=&-\frac{3}{4}a_0^3+\frac{3}{2}a_0^2\left(\frac{a}{b}\right)^2+\frac{3}{2}a_0\left(\frac{a}{b}\right)^4,\\F_5=&-\frac{1}{4}B_0^3-\frac{3}{2}B_0-\frac{3}{4},\\F_6=&\frac{1}{8}\left(\frac{a}{b}\right)^6(1+\varepsilon_{\rm h}/\varepsilon_{\rm s})-\frac{3}{4}a_0\left(\frac{a}{b}\right)^4(\varepsilon_{\rm h}-\varepsilon_{\rm s})/\varepsilon_{\rm s}\\&+\frac{3}{2}a_0^2\left(\frac{a}{b}\right)^2-\frac{3}{4}a_0^3.\end{aligned}$$

4. Effective response in dilute limit

We derive the effective nonlinear AC response of cylindrical coated nonlinear media using the general formulae previously proposed for DC response in the dilute limit:[15]

$$\begin{aligned}&\frac{1}{V}\int_V[D(x,t)-(\varepsilon_{\rm h}E(x,t)\\&+\chi_{\rm h}|E(x,t)|^2E(x,t))]{\rm d}V\\=&\bar{D}(t)-[\varepsilon_{\rm h}\bar{E}(t)+\chi_{\rm h}|\bar{E}(t)|^2\bar{E}(t)],\end{aligned}\tag{16}$$

where V denotes the volume of composites. $\bar{D}(t)$ and $\bar{E}(t)$ are the spatial averages of electric displacement and electric field, respectively. At low concentration, since the average field of the composites can be simply regarded as the external field,[16] and the integration in the left hand side of Eq.(16) is zero in the host region, Eq.(16) is simplified as the form

$$\begin{aligned}&\frac{1}{V}\int_{\Omega_{\rm i}+\Omega_{\rm s}}[D(x,t)-(\varepsilon_{\rm h}E(x,t)\\&+\chi_{\rm h}|E(x,t)|^2E(x,t))]{\rm d}V\\=&\bar{D}(t)-\left[\varepsilon_{\rm h}\bar{E}+\frac{3}{4}\chi_{\rm h}|\bar{E}|^2\bar{E}\right]\sin(\omega t)\\&+\frac{1}{4}\chi_{\rm h}|\bar{E}|^2\bar{E}\sin(3\omega t),\end{aligned}\tag{17}$$

where $\bar{E}=E_a$. Since the constitutive relation is cubic nonlinear, under external AC electric field, the electric displacement includes harmonic components. We give a reasonable definition of effective AC response,

$$\begin{aligned}\bar{D}(t)=&\bar{D}_\omega\sin(\omega t)+\bar{D}_{3\omega}\sin(3\omega t)+\cdots\\=&[\varepsilon^*_\omega E_a+\chi^*_\omega|E_a|^2E_a+\cdots]\sin(\omega t)\\&+[\varepsilon^*_{3\omega}E_a+\chi^*_{3\omega}|E_a|^2E_a+\cdots]\sin(3\omega t)+\cdots\end{aligned}\tag{18}$$

where ε^*_ω and χ^*_ω are the linear and nonlinear effective dielectric constants at fundamental frequency, respectively. At the third harmonics, the linear and the nonlinear dielectric constants are denoted by $\varepsilon^*_{3\omega}$ and $\chi^*_{3\omega}$. Using the equation $E=-\nabla\Phi$ and substituting the potentials of the core and coated shell regions into the left-hand side of Eq.(17), we can derive the effective dielectric response at the fundamental frequency and the third harmonic respectively:

$$\begin{aligned}\varepsilon^*_\omega=&\varepsilon_{\rm h}+f_{\rm i}(\varepsilon_{\rm h}-\varepsilon_{\rm i})A_0(a_0+1)+f_{\rm s}(\varepsilon_{\rm h}-\varepsilon_{\rm s})A_0a_0,\\\chi^*_\omega=&\frac{3}{4}\chi_{\rm h}+\frac{3}{4}f_{\rm i}(\chi_{\rm h}-\chi_{\rm i})A_0^3(a_0+1)^3\\&+\frac{3}{4}f_{\rm s}(\chi_{\rm h}-\chi_{\rm s})A_0^3\left(a_0^3+2a_0\left(\frac{a}{b}\right)^2\right)\\&+f_{\rm i}(\varepsilon_{\rm h}-\varepsilon_{\rm i})\chi_{\rm h}A_1^{\rm 1i}+(\varepsilon_{\rm h}-\varepsilon_{\rm s})\chi_{\rm h}\\&\times\left[f_{\rm s}A_1^{\rm 1s}-f_{\rm s}A_{\rm s}\left(\frac{a}{b}\right)^2-f_{\rm i}B_{\rm s}\left(1-\left(\frac{a}{b}\right)^4\right)\right],\\\varepsilon^*_{3\omega}=&0,\\\chi^*_{3\omega}=&-\frac{1}{4}\chi_{\rm h}-\frac{1}{4}f_{\rm i}(\chi_{\rm h}-\chi_{\rm i})A_0^3(a_0+1)^3\\&-\frac{1}{4}f_{\rm s}(\chi_{\rm h}-\chi_{\rm s})A_0^3\left(a_0^3+2a_0\left(\frac{a}{b}\right)^2\right)\\&+f_{\rm i}(\varepsilon_{\rm h}-\varepsilon_{\rm i})\chi_{\rm h}A_1^{\rm 3i}+(\varepsilon_{\rm h}-\varepsilon_{\rm s})\chi_{\rm h}\\&\times\left[f_{\rm s}A_1^{\rm 3s}-f_{\rm s}A_3^{\rm s}\left(\frac{a}{b}\right)^2-f_{\rm i}B_3^{\rm s}\left(1-\left(\frac{a}{b}\right)^4\right)\right],\end{aligned}$$

where $f_{\rm i}$ and $f_{\rm s}$ are the volume fraction of the inclusion core and the coated shell, respectively. The quantity $\left(\frac{a}{b}\right)^2$ can be expressed by the volume fraction, $\left(\frac{a}{b}\right)^2=f_{\rm i}/(f_{\rm i}+f_{\rm s})$.

Considering the coefficients appearing in the effective response, we obtain a simple relationship between the effective nonlinear response at fundamental frequency and the third harmonic, $\chi^*_{3\omega}=-\frac{1}{3}\chi^*_\omega$. This relation can be used to estimate nonlinear effective response at the third harmonic provided the nonlinear effective response at fundamental frequency is available. In addition, we can derive the effective nonlinear AC response of the cylinder nonlinear composites without the coated shell from our results. In this case we assume that $\varepsilon_{\rm s}=\varepsilon_{\rm h}$ and $\chi_{\rm s}=\chi_{\rm h}$. Under this condition, the effective AC responses of the cylinder nonlinear composites are

$$\begin{aligned}\varepsilon^*_\omega=&\varepsilon_{\rm h}+f_{\rm i}(\varepsilon_{\rm h}-\varepsilon_{\rm i})A_0(a_0+1),\\\chi^*_\omega=&\frac{3}{4}\chi_{\rm h}+\frac{3}{4}f_{\rm i}(\chi_{\rm h}-\chi_{\rm i})A_0^3(a_0+1)^3\\&+f_{\rm i}(\varepsilon_{\rm h}-\varepsilon_{\rm i})\chi_{\rm h}A_1^{\rm 1i},\\\chi^*_{3\omega}=&-\frac{1}{4}\chi_{\rm h}-\frac{1}{4}f_{\rm i}(\chi_{\rm h}-\chi_{\rm i})A_0^3(a_0+1)^3\\&+f_{\rm i}(\varepsilon_{\rm h}-\varepsilon_{\rm i})\chi_{\rm h}A_1^{\rm 3i},\end{aligned}$$

where $A_0(a_0+1)=-c$, $A_1^{\rm 1i}=-b_3$ and $A_1^{\rm 3i}=-b_7$ if $\varepsilon_{\rm s}=\varepsilon_{\rm h}$ and $\chi_{\rm s}=\chi_{\rm h}$. The coefficients c, b_3 and b_7 are given in Ref.[12]. Thus this result is reduced to that of Ref.[12].

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