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Energy loss of a fast-electron beam due to the excitation of collective oscillation in hot plasma*

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Energy loss due to a fast-electron beam interacting with the hot plasma at a high density is analysed theoretically. By splitting the particle density fluctuations into the individual part due to the random thermal motion of the individual electrons and the collective part due to plasma–wave excitation, we are concerned with the collective interaction of the relativistic plasma electrons resulting from the Coulomb interactions. Consequently, we derive the frequency of the hot plasma and the "Debye length" with the modification of the relativistic effect. And finally we calculate the energy loss of a fast-electron beam due to the excitation of collective oscillation in the hot plasma.

Keywords: fast-electron beam, excitation of collective oscillation, hot plasma
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1. Introduction

During the last 10–15 years, the chirped-pulse amplification (CPA) technique has brought about a rapid development in ultra-short and ultra-intense lasers.[1] The physics of intense laser–plasma interaction has consequently received considerable theoretical and experimental attention.[2] However, though the intense laser–target interaction may produce hot or super-hot plasma, some studies still just take the dynamic parameters of the plasma as the non-relativistic ones, and in addition, take the electrons as non-relativistic particles.[3−5] In fact, the interaction of the intense laser with a target can produce energetic particles, such as MeV electrons.[6] Thus, the so-called super-hot relativistic electron in MeV energy arouses much interest.

The mass m of the hot plasma electron does not simply equal the rest mass $m_{\rm e}$ of the electron. It satisfies the well-known relativistic relation $m = m_{\rm e}/(1 - v^2/c^2)^{1/2}$. We should use the relativistic theory to redefine the motion of the hot plasma electron. Therefore, we first study the way in which the interaction between the relativistic electrons in the hot plasma brings about the organized behaviour and the collective oscillation.

The energy losses (such as those due to the ionization and the radiation, and those due to the interaction with free electrons) during the fast particle passing through a bulk of matter have been studied in many ways.[7−10] Some of the energy losses are calculated in terms of relativistic correction. But some of them are non-relativistic. Thus, in this paper, we study the energy loss of a fast-electron beam with a fixed beam density n_b due to the excitation of the collective oscillation in the hot plasma with a high density n_p.

2. Interaction between relativistic electrons in hot plasma

We begin by assuming the Coulomb interaction quantitatively plays an essentially important role in the problem. The Coulomb potential between the ith and jth electrons may be expanded as a Fourier series in a box of unit volume under periodic boundary conditions:

$$\begin{aligned} U(|\boldsymbol{x}_i - \boldsymbol{x}_j|) =& e^2/|\boldsymbol{x}_i - \boldsymbol{x}_j| \\ =& 4\pi e^2 \sum_k (1/k^2) {\rm e}^{{\rm i}k\cdot(x_i - x_j)}, \end{aligned} \quad (1)$$

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and hence, the Coulomb force will take the form

$$\begin{aligned}\boldsymbol{F}_i =& -\frac{\mathrm{d}U(|\boldsymbol{x}_i-\boldsymbol{x}_j|)}{\mathrm{d}\boldsymbol{x}_i}\\ =& -4\pi e^2\mathrm{i}\sum_{j\neq i,k\neq 0}(\boldsymbol{k}/k^2)\mathrm{e}^{\mathrm{i}k\cdot(x_i-x_j)}.\end{aligned}\tag{2}$$

On the other hand, we can write the expression for the force in the relativistic theory framework as follows:

$$\begin{aligned}\boldsymbol{F} =&\frac{\mathrm{d}\boldsymbol{P}}{\mathrm{d}t}=\frac{\mathrm{d}}{\mathrm{d}t}[m_\mathrm{e}\boldsymbol{v}/(1-v^2/c^2)^{1/2}]\\ =&m_\mathrm{e}\left[\gamma\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t}+\frac{\boldsymbol{v}}{c^2}\gamma^3\left(\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t}\cdot\boldsymbol{v}\right)\right]\\ =&m_\mathrm{e}\gamma\left\{\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t}+\frac{\gamma^2}{c^2}\left[(\boldsymbol{v}\cdot\boldsymbol{v})\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t}+\left(\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t}\times\boldsymbol{v}\right)\times\boldsymbol{v}\right]\right\}\\ =&m_\mathrm{e}\gamma\left[\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t}+\frac{v^2}{c^2}\gamma^2\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t}+\frac{\gamma^2}{c^2}\left(\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t}\times\boldsymbol{v}\right)\times\boldsymbol{v}\right],\end{aligned}\tag{3}$$

where γ is the Lorentz factor $\gamma=(1-v^2/c^2)^{-1/2}$; m_e, $\boldsymbol{v}$, $\boldsymbol{P}$, and $\boldsymbol{F}$ denote the rest mass, the velocity and the momentum of the electron, and the force acting on the electron, respectively.

Here we should pay some attention to the third term in the right-hand side of Eq.(3): $\left|\left(\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t}\times\boldsymbol{v}\right)\times\boldsymbol{v}\right|=\frac{\mathrm{d}v}{\mathrm{d}t}v^2\sin\theta$, where θ is the angle between $\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t}$ and $\boldsymbol{v}$. When the energy of the electron is high enough to cause the direction of the acceleration $\left(\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t}\right)$ to essentially approach to the direction of the velocity $(\boldsymbol{v})$, the value of $\sin\theta$ is extremely small and can be ignored. In fact, the system we discuss here is the hot plasma with a high energy. Therefore, such an assumption is reasonable. So, the third term is relatively small in comparison with the second term. At the same time, in order to avoid the complexity in calculation, we neglect the third term in the right-hand side of Eq.(3). Hence, we have the acceleration in the following form:

$$\begin{aligned}\dot{\boldsymbol{v}} =&\boldsymbol{F}/\left[m_\mathrm{e}\gamma\left(1+\frac{v^2}{c^2}\gamma^2\right)\right]\\ =&\boldsymbol{F}/[m_\mathrm{e}\gamma(1+\beta^2\gamma^2)],\end{aligned}\tag{4}$$

with the Lorentz parameters $\beta=\frac{v}{c}$, $\gamma=(1-\beta^2)^{-1/2}$.

These relations are important for us to study the behaviour of the interaction of the relativistic electrons in the hot plasma.

By virtue of Eqs.(2) and (4), we may take the following form for the equation of motion of the ith electron:

$$\dot{\boldsymbol{v}}_i=-[4\pi e^2\mathrm{i}/m_\mathrm{e}\gamma(1+\beta^2\gamma^2)]\sum_{j\neq i,k\neq 0}(\boldsymbol{k}/k^2)\mathrm{e}^{\mathrm{i}k\cdot(x_i-x_j)}.\tag{5}$$

The equations of motion (5) are very difficult to solve. Many years ago, Pines and Bohm [11] took some approach to deal with this kind of equation. In this paper, we follow a similar approach but take into account the relativistic effects just mentioned above.

First, we assume that we are dealing with point particles, so that the particle density in the box of unit volume may be given as follows:

$$\rho(\boldsymbol{x})=\sum_i\delta(\boldsymbol{x}-\boldsymbol{x}_i).\tag{6}$$

It is more convenient to work with the Fourier components of the density, and they are given by

$$\rho_k=\int\mathrm{d}\boldsymbol{x}\rho(\boldsymbol{x})\mathrm{e}^{-\mathrm{i}k\cdot x}=\sum_i\mathrm{e}^{-\mathrm{i}k\cdot x_i},\tag{7}$$

where ρ_0 denotes the mean electron density n, and ρ_k with $k\neq 0$ describe the fluctuations about the mean density. In this case, Eq.(5) may be rewritten in the following form:

$$\dot{\boldsymbol{v}}_i=-[4\pi e^2\mathrm{i}/m_\mathrm{e}\gamma(1+\beta^2\gamma^2)]\sum_k(\boldsymbol{k}/k^2)\rho_k\mathrm{e}^{\mathrm{i}k\cdot x_i}.\tag{8}$$

Here we note that ρ_k determines the force acting on each particle.

By differentiating Eq.(7) with respect to time t and using Eq.(5) for $\dot{\boldsymbol{v}}_i$, we obtain

$$\begin{aligned}\ddot{\rho}_k =& -\sum_i(\boldsymbol{k}\cdot\boldsymbol{v}_i)^2\mathrm{e}^{-\mathrm{i}k\cdot x_i}\\ &-\sum_{ij,k'\neq 0}[4\pi e^2/m_\mathrm{e}\gamma(1+\beta^2\gamma^2)(k')^2]\boldsymbol{k}\\ &\cdot\boldsymbol{k}'\{\exp[\mathrm{i}(\boldsymbol{k}'-\boldsymbol{k})\cdot\boldsymbol{x}_i]\}\mathrm{e}^{-\mathrm{i}k'\cdot x_j}.\end{aligned}\tag{9}$$

One may split the sum over $\boldsymbol{k}'$ into two parts. The first part, with $\boldsymbol{k}'=\boldsymbol{k}$, is independent of the coordinate $\boldsymbol{x}_i$, therefore the sum over i yields n, the total number of particles in the box of unit volume. The second part, with $\boldsymbol{k}'\neq\boldsymbol{k}$, contains phase factors $\exp\mathrm{i}(\boldsymbol{k}'-\boldsymbol{k})\cdot\boldsymbol{x}_i$. Since there are a very large number of particles distributed very nearly in random positions, these terms tend to average out to zero. As a first approximation, one may neglect such terms. With the random phase approximation, we then obtain

$$\begin{aligned}\ddot{\rho}_k =& -\sum_i(\boldsymbol{k}\cdot\boldsymbol{v}_i)^2\mathrm{e}^{-\mathrm{i}k\cdot x_i}\\ &-[4\pi ne^2/m_\mathrm{e}\gamma(1+\beta^2\gamma^2)]\sum_i\mathrm{e}^{-\mathrm{i}k\cdot x_i}.\end{aligned}\tag{10}$$

The first term in the right-hand side of Eq.(10) arises simply from the random thermal motion of individual electrons. The second term represents the effect of electrons interactions.

In order to study the collective behaviour of the relativistic electrons in the hot plasma, we discuss as follows according to the value of $\boldsymbol{k}$:

a) For the sufficiently small $\boldsymbol{k}$ case

Obviously, the first term can be neglected in comparison with the second. And hence, we have

$$\ddot{\rho}_k + [4\pi ne^2/m_e\gamma(1+\beta^2\gamma^2)]\rho_k = 0, \tag{11}$$

with

$$\omega_p = [4\pi ne^2/m_e\gamma(1+\beta^2\gamma^2)]^{1/2}. \tag{12}$$

As a result of the Coulomb interaction, the relativistic electron density oscillates with the "hot plasma frequency" ω_p. From definition (12) of ω_p, we can see that it contains a relativistic modified factor $\gamma(1+\beta^2\gamma^2)$. That is essentially different from the non-relativistic one. The plasma frequency ω_p becomes smaller with the increase of the velocity of the electron.

As a rough criterion for a collective description, we have

$$[4\pi ne^2/m_e\gamma(1+\beta^2\gamma^2)] \gg \langle(\boldsymbol{k}\cdot\boldsymbol{v}_i)^2\rangle_{\mathrm{AV}}. \tag{13}$$

This means that for most particles the collective term arising from the Coulomb forces in Eq.(10) is much greater than the individual term arising from the random thermal motion. Therefore, we note that the strong force of interaction and high particle density favour the collective behaviour, while high random thermal velocities oppose it.

For an isotropic velocity distribution, Eq.(13) becomes

$$k^2 \ll (12\pi ne^2/m_e\gamma(1+\beta^2\gamma^2)\langle\boldsymbol{v}_i^2\rangle_{\mathrm{AV}}) = \lambda_{\mathrm{D}}^{-2}, \tag{14}$$

where λ_{D} is the so called "Debye length",

$$\lambda_{\mathrm{D}}^2 = \frac{1}{3}[(\langle v_i^2\rangle_{\mathrm{AV}})/\omega_p^2]. \tag{15}$$

Obviously, the increase of the Lorentz factor γ, as well as the decrease of the plasma frequency ω_p, leads to the increase of the "Debye length". From Eq.(10) and expression (14), one can see that the organized behaviour is much more important when the distance is greater than λ_{D}, while the individual behaviour is favourable to the case with distances shorter than λ_{D}.

b) For the intermediate $\boldsymbol{k}$ case

From Eq.(10) one may notice that the electron density fluctuations ρ_k do not oscillate harmonically for appreciable values of k, and therefore some functions of the electron coordinates do not constitute a satisfactory set of collective coordinates. In this case, we divide ρ_k into two parts:

$$\rho_k = a_k q_k + \eta_k. \tag{16}$$

The first part $a_k q_k$ is a collective part of ρ_k; the second part η_k has no collective behaviour.

In order to find some functions that do oscillate harmonically for nonzero k, without considering the effects of random thermal motion, we define q_k as follows:

$$q_k = \sum_i \{1/[\omega^2 - (\boldsymbol{k}\cdot\boldsymbol{v}_i)^2]\}\mathrm{e}^{-\mathrm{i}k\cdot x_i}, \tag{17}$$

then according to expression (16), η_k may be expressed as

$$\eta_k = \sum_i \{1 - a_k/[\omega^2 - (\boldsymbol{k}\cdot\boldsymbol{v}_i)^2]\}\mathrm{e}^{-\mathrm{i}k\cdot x_i}. \tag{18}$$

If a_k is arbitrary, then $\ddot{\eta}_k$, just as $\ddot{\rho}_k$ in Eq.(10), may have two types of terms. The first one is free of $\dot{\boldsymbol{v}}_i$, which means the absence of inter-particle forces; the second one describes the organized behaviour of the system.

In order to ensure that η_k has no collective component, we choose $a_k = \omega_p^2$ (for detailed calculations see Appendix A). Consequently, if we include the coefficient ω_p^2 in the basic definition of η_k and q_k, we have

$$\eta_k = \sum_i \frac{\omega^2 - \omega_p^2 - (\boldsymbol{k}\cdot\boldsymbol{v}_i)^2}{\omega^2 - (\boldsymbol{k}\cdot\boldsymbol{v}_i)^2}\mathrm{e}^{-\mathrm{i}k\cdot x_i}, \tag{19}$$

$$q_k = \sum_i \{\omega_p^2/[\omega^2 - (\boldsymbol{k}\cdot\boldsymbol{v}_i)^2]\}\mathrm{e}^{-\mathrm{i}k\cdot x_i}. \tag{20}$$

As η_k has no collective behaviour, q_k oscillates harmonically with frequency ω:

$$\ddot{q}_k + \omega^2 q_k = 0, \tag{21}$$

if we have the following condition to ensure it (details are presented in Appendix B):

$$[4\pi e^2/m_e\gamma(1+\beta^2\gamma^2)]\sum_j 1/(\omega - \boldsymbol{k}\cdot\boldsymbol{v}_j)^2 = 1. \tag{22}$$

We can calculate ω by a numerical solution of Eq.(22). But for convenience, we reduce Eq.(22) to the following approximate dispersion relation:

$$\omega^2 = \omega_p^2 + k^2\langle v^2\rangle_{\mathrm{AV}}. \tag{23}$$

One may note that if k is small enough, Eq.(23) is reduced to the earlier dispersion relation (12).

3. Energy loss due to collective interaction

The energy losses of a fast particle in the matter are mainly determined by the ionization and excitation of the electrons in atoms.[10] Ionization leads to the increase of the free electron density, which causes additional energy losses of the fast particle due to direct interactions with these electrons, as well as the plasma–wave excitation. Here, we only pay attention to the energy loss of a fast-electron beam due to the excitation of collective oscillation in the hot plasma.

Now we study the collective response of the hot plasma to a fast electron moving through the system with velocity $\boldsymbol{v}_0$. By Fourier analysis, we describe the charge density of the fast electron as

$$-e\rho_s = -e\delta(\boldsymbol{x} - \boldsymbol{v}_0 t) = -e\sum_k \mathrm{e}^{\mathrm{i}k\cdot(x-v_0 t)}. \quad (24)$$

The force on the ith particle comes from two sources: one is the other particles in the system, the other is the fast electron.

According to Eqs.(8) and (24), the external particle changes $\dot{\boldsymbol{v}}_i$ by

$$\dot{\boldsymbol{v}}_i^{(s)} = -[\mathrm{i}4\pi e^2/m_\mathrm{e}\gamma(1+\beta^2\gamma^2)] \times \sum_k [\boldsymbol{k}/(k)^2]\mathrm{e}^{\mathrm{i}k\cdot(x_i - v_0 t)}. \quad (25)$$

The response of the Fourier component of the collective part of the charge density to the field of the fast electron is given by

$$q_k = -[\omega_p^2/[\omega^2 - (\boldsymbol{k}\cdot\boldsymbol{v}_0)^2]\mathrm{e}^{-\mathrm{i}k\cdot v_0 t}. \quad (26)$$

If a fast electron moving with a velocity $\boldsymbol{v}_0$ causes the denominator of Eq.(26) to vanish for some $k < k_\mathrm{D}$, we have

$$(\boldsymbol{k}\cdot\boldsymbol{v}_0)^2 = \omega^2 \approx \omega_p^2 + k^2\langle v^2\rangle_\mathrm{AV}. \quad (27)$$

Taking k_z in the direction of $\boldsymbol{v}_0$, from Eq.(27) we have

$$k_z^2[v_0^2 - \langle v^2\rangle_\mathrm{AV}] = \omega_p^2 + (k_x^2 + k_y^2)\langle v^2\rangle_\mathrm{AV}. \quad (28)$$

Obviously, the particles moving with a speed less than the mean thermal speed of the hot plasma electrons will not excite collective oscillations.

Now, we are interested in the collective part of the total charge density. In order to solve the boundary condition problem, we consider the following coordinate function $q(x)$ obtained from q_k in Eq.(26):

$$q(x) = \sum_{k<k_\mathrm{D}} q_k \mathrm{e}^{\mathrm{i}k\cdot x} = \sum_{k<k_\mathrm{D}} \{-\omega_p^2/[\omega^2 - (\boldsymbol{k}\cdot\boldsymbol{v}_0)^2]\}\mathrm{e}^{\mathrm{i}k\cdot(x-v_0 t)}. \quad (29)$$

If we choose k_z in the direction of $\boldsymbol{v}_0$, and use the dispersion relation (23), then we obtain

$$q(x) = \sum_{k<k_\mathrm{D}} \frac{-\omega_p^2 \exp\{\mathrm{i}[k_x x + k_y y + k_z(z - v_0 t)]\}}{\omega_p^2 + (k_x^2 + k_y^2)\langle v^2\rangle_\mathrm{AV} + k_z^2[\langle v^2\rangle_\mathrm{AV} - v_0^2]}. \quad (30)$$

The denominator in Eq.(30) vanishes for $k_z = k_z^s$, where

$$k_z^s = \pm\{[\omega_p^2 + (k_x^2 + k_y^2)\langle v^2\rangle_\mathrm{AV}]/[v_0^2 - \langle v^2\rangle_\mathrm{AV}]\}^{1/2}. \quad (31)$$

In order to calculate Eq.(30) for $q(x)$, one needs to simplify this expression. Firstly, in the case of v_0 sufficiently larger than $\langle v^2\rangle_\mathrm{AV}^{1/2}$, it is mathematically convenient to extend the limits from $\pm(k_\mathrm{D}^2 - k_x^2 - k_y^2)^{1/2}$ to $\pm\infty$ in the summation over k_z, although an error is introduced by such an extension; but fortunately, this error is negligible. So almost all values of k_z^s will lie within the limits $\pm(k_\mathrm{D}^2 - k_x^2 - k_y^2)^{1/2}$, and the denominator of Eq.(30) will become large for the values of k_z outside these limits. Secondly, we change the summation over k to an integration over k. Thirdly, to ensure that there is no disturbance ahead of the particle, we require that $q(x)$ must vanish for $z > v_0 t$, that is to say, $q(x)$ should correspond to a wave moving behind the particle (for $z < v_0 t$). By integrating over the complex k-plane and choosing a suitable path, the above boundary conditions may be introduced. The contour of the integration in the lower half-plane (for $z < v_0 t$) contains two singularities. By virtue of the well-known Residue Theorem, one has

$$q(x) = -\frac{\omega_p^2}{4\pi^2[v_0^2 - \langle v^2\rangle_\mathrm{AV}]^{1/2}} \int\int \mathrm{d}k_x \mathrm{d}k_y \times \left\{\frac{\{\sin[k_z^s(z - v_0 t)]\}\mathrm{e}^{\mathrm{i}(k_x x + k_y y)}}{[\omega_p^2 + (k_x^2 + k_y^2)\langle v^2\rangle_\mathrm{AV}]^{1/2}}\right\}. \quad (32)$$

What we are mainly interested in is the energy transferred by the incident fast electron to the wake. It is determined by the reaction of the electric field $\boldsymbol{\varepsilon}$ of the wake with the emitting particle located at $\boldsymbol{x} = \boldsymbol{v}_0 t (x = 0, y = 0, z = v_0 t)$. Only the z component (ε_z) needs to be considered. It is finite for $z < v_0 t$, while ε_z vanishes for $z > v_0 t$. The correct value of ε_z at $z = v_0 t$ is the sum taken by approaching this

point from both sides. Thus we take $\left(\frac{1}{2}\right)\varepsilon_z(v_0t)$ as the value when approaching from the left. This shows that

$$\varepsilon_z(v_0t) = -\frac{e\omega_p^2}{2\pi}\int\int \mathrm{d}k_x\mathrm{d}k_y\left\{\frac{1}{\omega_p^2+(k_x^2+k_y^2)v_0^2}\right\}. \tag{33}$$

We use polar coordinates to deal with the above integration and take the following expression:

$$(k_x^2+k_y^2)_{\max} \cong \left(\frac{2}{3}\right)k_{\mathrm{D}}^2 = 2\omega_p^2/\langle v^2\rangle_{\mathrm{AV}}. \tag{34}$$

Therefore, the force exerted on the particle caused by the wake is

$$\begin{aligned} F_z =& -e\varepsilon_z(v_0t) \\ =&[e^2\omega_p^2/2v_0^2]\ln\{1+2v_0^2/[\langle v^2\rangle_{\mathrm{AV}}]\}. \end{aligned} \tag{35}$$

The rate of energy loss per unit distance $\mathrm{d}E/\mathrm{d}z$ is

$$\begin{aligned} \mathrm{d}E/\mathrm{d}z =& F_z \\ =&\left[\frac{e^2\omega_p^2}{2v_0^2}\right]\ln\left[1+\left(\frac{2}{3}\right)\left(\frac{v_0^2}{\lambda_{\mathrm{D}}^2\omega_p^2}\right)\right], \end{aligned} \tag{36}$$

where

$$\omega_p = (4\pi ne^2/m_{\mathrm{e}}\gamma(1+\beta^2\gamma^2))^{1/2}$$

and

$$\lambda_{\mathrm{D}}^2 = \frac{1}{3}[(\langle v_i^2\rangle_{\mathrm{AV}})/\omega_p^2].$$

4. Conclusion

In conclusion, we have obtained the frequency of the hot plasma and the so-called "Debye length" with relativistic modification by studying the Coulomb interaction between relativistic electrons in the hot plasma. In this case, the increase of the Lorentz factor γ, as well as that of the energy of the electrons, leads to the decrease of the plasma frequency ω_p and the increase of the "Debye length". The lowering of the plasma frequency causes the relativistic electrons to behave collectively much easier. We believe that it is reasonable to assume that the acceleration direction $\left(\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t}\right)$ of the relativistic electron approaches the direction of its velocity $(\boldsymbol{v})$ when the energy of the electrons is sufficiently high.

Our study has been carried out under the condition that the fast-electron beam is diluted with a mean electron inter-distance larger by several orders of magnitude than that in the hot plasma. So the beam density n_b should not be too big. As usual, one takes for example $n_b/n_p \leq 10^{-4}$ (n_p is the plasma density). That is to say, the beam density is much smaller than the plasma density. But considering that the self-interaction is rather weak in the relativistic electron beam, we may take $n_b/n_p \leq 10^{-2}$ (or even 10^{-1}). In this case, the beam–plasma interaction can be treated as if each electron in the beam was interacting independently with the plasma and is then reducible to that of a linear superposition of isolated charges.

Furthermore, the expression of the rate of energy loss per unit distance in Eq.(36) is similar to that obtained by Pines and Bohm.[11] The differences are the definition of the plasma frequency and the "Debye length". If the system we discuss here is the conventional plasma, Eq.(36) is reduced to the earlier result of Pines and Bohm.

Finally, we only pay attention to the collective oscillation of relativistic electrons in the hot plasma. Actually, the organized behaviour and the individual thermal motion are present simultaneously. We will discuss the energy loss due to two-body collision as well as the collective excitation in details in another paper.

Appendix A

$$\eta_k = \sum_i \{1 - a_k/[\omega^2 - (\boldsymbol{k}\cdot\boldsymbol{v}_i)^2]\}\mathrm{e}^{-\mathrm{i}k\cdot x_i}. \tag{A1}$$

If a_k is arbitrary, then $\ddot{\eta}_k$ just as $\ddot{\rho}_k$ in Eq.(10) may have two types of terms. The first one is free of $\dot{\boldsymbol{v}}_i$, which means the absence of inter-particle forces:

$$\ddot{\eta}_k^{(1)} = \sum_i -(\boldsymbol{k}\cdot\boldsymbol{v}_i)^2\{1 - a_k/[\omega^2 - (\boldsymbol{k}\cdot\boldsymbol{v}_i)^2]\}\mathrm{e}^{-\mathrm{i}k\cdot x_i}. \tag{A2}$$

The second one describes the organized behaviour of the system. From Eqs.(10), (12), (16) and (17) we obtain

$$\begin{aligned} \ddot{\eta}_k =& \ddot{\rho}_k - a_k\ddot{q}_k \\ =& \sum_i\{-(\boldsymbol{k}\cdot\boldsymbol{v}_i)^2 - \omega_p^2 \\ &+ a_k\omega^2/[\omega^2 - (\boldsymbol{k}\cdot\boldsymbol{v}_i)^2]\}\mathrm{e}^{-\mathrm{i}k\cdot x_i}. \end{aligned} \tag{A3}$$

According to Eq.(A2), the above equation may be changed into

$$\ddot{\eta}_k = \ddot{\eta}_k^{(1)} + \sum_i [a_k - \omega_p^2]\mathrm{e}^{-\mathrm{i}k\cdot x_i}. \tag{A4}$$

So we choose $a_k = \omega_p^2$ to ensure that η_k has no collective component. Consequently, from Eq.(A1) we

obtain

$$\eta_k = \sum_i \frac{\omega^2 - \omega_p^2 - (\boldsymbol{k}\cdot\boldsymbol{v}_i)^2}{\omega^2 - (\boldsymbol{k}\cdot\boldsymbol{v}_i)^2} \mathrm{e}^{-\mathrm{i}k\cdot x_i}. \tag{A5}$$

If we include the coefficient ω_p^2 in the basic definition of q_k, we have

$$q_k = \sum_i \{\omega_p^2/[\omega^2 - (\boldsymbol{k}\cdot\boldsymbol{v}_i)^2]\}\mathrm{e}^{-\mathrm{i}k\cdot x_i}. \tag{A6}$$

Appendix B

We find that it is more convenient to introduce the quantities $\xi_{k,\omega}$ as follows:

$$\xi_{k,\omega} = \sum_i [\omega_p^2/(\omega - \boldsymbol{k}\cdot\boldsymbol{v}_i)]\mathrm{e}^{-\mathrm{i}k\cdot x_i}. \tag{B1}$$

q_k may be re-expressed as

$$q_k = (1/2\omega)\cdot(\xi_{k,\omega} - \xi_{k,-\omega}). \tag{B2}$$

If one can prove

$$\dot{\xi}_{k,\omega} + \mathrm{i}\omega\xi_{k,\omega} = 0, \tag{B3}$$

one may easily obtain the harmonic oscillator equation for q_k,

$$\ddot{q}_k + \omega^2 q_k = 0. \tag{B4}$$

By differentiating Eq.(B3) and using the equations of motion (5) for $\dot{\boldsymbol{v}}_i$, and with the random phase approximation, we may obtain the following equation:

$$\begin{aligned}&\dot{\xi}_{k,\omega} + \mathrm{i}\omega\xi_{k,\omega}\\ &= \mathrm{i}\omega_p^2 \sum_i \left\{1 - \sum_j \frac{4\pi e^2}{m_\mathrm{e}\gamma(1+\beta^2\gamma^2)(\omega - \boldsymbol{k}\cdot\boldsymbol{v}_j)^2}\right\}\mathrm{e}^{-\mathrm{i}k\cdot x_i}.\end{aligned} \tag{B5}$$

It is obvious that the term in the right-hand side of Eq.(B5) must vanish for arbitrary $\boldsymbol{x}_i$ in order to satisfy Eq.(B3). And hence, we obtain:

$$[4\pi e^2/m_\mathrm{e}\gamma(1+\beta^2\gamma^2)]\sum_j 1/(\omega - \boldsymbol{k}\cdot\boldsymbol{v}_j)^2 = 1. \tag{B6}$$

References

[1] Mourou G and Umstadter D 1992 *Phys. Fluids* B **4** 2315
Perry M D and Mourou G 1994 *Science* **264** 917

[2] Umstadter D 2001 *Phys. Plasmas* **8** 1774
Du C G, Zeng G H and Xu Z Z 1999 *Acta Phys. Sin.* **48** 86 (in Chinese)
Najmudin Z, Krushelnick K, Tatarakis M, Clark E L, Danson C N, Malka V, Neely D, Santale M I K and Dangor A E 2003 *Phys. Plasmas* **10** 438

[3] Macchi A, Cornolti F Pegoraro F, Liseikina T V, Ruhl H and Vshivkov V A 2001 *Phys. Rev. Lett.* **87** 205004

[4] Anatoly Spitkovsky and Pisin Chen 2000 *AIP Conf. Proc.* **569** 183

[5] Barov N, Rosenzweig J B and Thompson M C 2002 *Preprint* physics / 0205007

[6] Malka G and Miquel J L 1996 *Phys. Rev. Lett.* **77** 75

[7] Stardubtsev S V and Romanov A M 1962 *Prokhozhdeniye Zaryazhennykh Chastits cherez Veshchestvo* (Penetration of Charged Particles into Matter) Tashkent: Akad. Nauk USSR

[8] Deutsch C, Furukawa H, Mima K, Murakami M and Nishihara K 1996 *Phys. Rev. Lett.* **77** 2483

[9] *Stopping Power of Electrons and Positrons* 1987 Doklad 37 MKRE (Reports of 37 MKRE), Moscow: Energoizdat

[10] Val'chuck V V, Volkov N B, and Yalovets A P 1995 *Plasma Phys. Rep.* **21** 159

[11] Pines D and Bohm D 1952 *Phys. Rev.* **85** 338