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Moments of the intensity of a single-mode laser driven by coloured pump noises with a cross-correlation between the real and imaginary parts of the quantum noise*

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By means of the unified coloured noise approximation and phase lock, we study in this paper the stationary intensity distribution of the single-mode laser cubic model driven by coloured pump noise with cross-correlation between the real and imaginary parts of the quantum noise. We present a thorough discussion of how the cross-correlation λ_q between the real and imaginary parts of the quantum noise and the self-correlation time τ of the pump noise determine the behaviours of the mean intensity $\langle I\rangle$ and variance $\lambda_2(0)$ for both below and above the threshold operation, and many new phenomena are discovered. When the laser is operated above the threshold, increasing the cross-correlation intensity λ_q makes the $\langle I\rangle$–τ curves exhibit a "re-entrant phase transition". Whether the laser is above or below the threshold, the whole $\langle I\rangle$–τ curve moves down as λ_q increases; however, when $\lambda_q = 1$ (perfect cross-correlation), the curve abruptly runs up. A nonzero value of cross-correlation time τ ($\tau \neq 0$) leads to the entire $\lambda_2(0)$–p' curve being suppressed (here p' is the pump noise intensity). This indicates the increasing precision of the laser beam due to the existence of the self-correlation time τ. The behaviour of the $\lambda_2(0)$–p' curve as a function of λ_q is similar to that of the $\langle I\rangle$–τ curve against λ_q: that is, whether the laser is above or below the threshold, the $\lambda_2(0)$–p' curve moves up as λ_q increases; however, when $\lambda_q = 1$, the curve suddenly moves down. Through the study in this paper, we can find a way to optimize for output laser intensity.

Keywords: moments of the laser intensity, coloured pump noises, cross-correlation
PACC: 4250, 0540, 4260K

1. Introduction

Scientists usually take into consideration the cross-correlation between the quantum and pump noises when studying the statistical fluctuation properties in a single-mode laser.[1–3] With the appearance of new phenomena in the laser field, they begin to explore the new statistical properties by taking into account the correlation between the quantum noise and the pump noise.[4–9] There are a few previous studies in which the effect of the cross-correlation between the real and imaginary parts of quantum noise is considered. However, the fluctuations of a complex field can be coherent, and the real and imaginary parts of the quantum noise generally originate from a common bath, thus they may also be coherent. In 1996 Zhou *et al*[10] first took into account the cross-correlation between the real and imaginary parts of the quantum noise. In their research, the amplitude was not decoupled from the phase equation, and they dealt with it in the two-dimensional case. Soon afterwards Ke *et al*[11] also considered the cross-correlation between the real and imaginary parts of the quantum noise, but they adopted the locked phase method to make the amplitude equation decoupled from the phase equation, and this decoupled amplitude equation contained the results obtained in Ref.[12]. It was also revealed that the locked phase of a laser is caused by the cross-correlation between the real and imaginary parts of the quantum noise. Subsequently, Zhang *et al*[13,14]

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further studied the effect of the cross-correlation between the real and imaginary parts of the quantum noise in terms of the white-gain model: the corresponding stationary and transient properties (the first passage time, variance, and skewness) were calculated and compared with the experimental data. In all the aforementioned papers, the pump noise and the quantum noise are both approximated as Gaussian white noise. To approach the actual circumstance, the system driven by coloured noise should be fully taken into account.

In this paper the steady-state fluctuations of a single-mode laser driven by coloured pump noises with the cross-correlation between the real and imaginary parts of quantum noise are investigated theoretically. Due to the coloured nature of the pump fluctuations, and the system driven by coloured pump noises undergoing a non-Markovian process, we cannot obtain the Fokker–Planck equation (FPE). Thus several approximate theoretical treatments are used to obtain an approximate FPE. [15−17] In Section 2, we use the unified coloured noise approximation (UCNA) [18,19] and the method of locked phase to derive the decoupled amplitude and phase equations. The reason for this approximation is that it is a good approximation for both small and large correlation time τ, and for the intermediate value of τ it still gives a useful result. In Section 3 an analytic expression of the stationary intensity distribution (SID) is derived and the mean, variance, and skewness of the laser intensity are calculated. In Section 4, we analyse the numerical results and identify the effects of pump noise "colour" and the effect of the cross-correlation of the complex quantum noise on the first three moments of single-mode laser below and above the threshold. Many new phenomena are discovered.

2. The Langevin equation

We consider the coloured cubic model. It can be described by the following Langevin equation:

$$\frac{\mathrm{d}E}{\mathrm{d}t} = a_0 E - A|E|^2 E + Ep(t) + q(t), \qquad (1)$$

where a_0 and A stand for the net gain and self-saturation coefficients respectively, E is the complex laser field. The random variable $q(t)$ represents the complex quantum noises, and the noise term is assumed to be the Gaussian white noise with zero mean and correlation function

$$\langle q_i(t)q_j(s)\rangle = [\delta_{ij} + \lambda_q(1-\delta_{ij})]p\delta(t-s) \quad (i,j = I,R), \qquad (2)$$

where p is the strength of the quantum noise. I and R represent the imaginary and real parts of the noise respectively. λ_q, the cross-correlation coefficient, measures the degree of cross-correlation between the real and imaginary parts of the quantum noise, and $|\lambda_q| \leq 1$. The random variable $p(t)$ represents the complex pump noises. The noise term is assumed to be O–U noise with the correlation function

$$\langle p_i(t)\rangle = 0,$$
$$\langle p_i(t)p_j(s)\rangle = \delta_{ij}\frac{p'}{2\tau}\mathrm{e}^{\frac{-|t-s|}{\tau}} \quad (i,j = I,R). \qquad (3)$$

Here p' is the pump noise intensity, and τ is the pump noise self-correlation time. a_0, A, p, p' and τ are real parameters, and the quantum noise $q(t)$ is independent of the pump noise $p(t)$.

Changing the variables of Eq.(1) into the polar coordinates according to $E = r\mathrm{e}^{\mathrm{i}\phi}$, Eq.(1) can be written as follows:

$$\frac{\mathrm{d}r}{\mathrm{d}t} = a_0 r - Ar^3 + rp_R(t) + \varepsilon_r(t), \qquad (4)$$
$$\frac{\mathrm{d}\phi}{\mathrm{d}t} = p_I(t) + \frac{\varepsilon_{\phi(t)}}{r}, \qquad (5)$$

where r and ϕ represent the amplitude and phase of the laser field respectively, and

$$\varepsilon_r(t) = q_R(t)\cos\phi + q_I(t)\sin\phi, \qquad (6)$$
$$\varepsilon_\phi(t) = -q_R(t)\sin\phi + q_I(t)\cos\phi. \qquad (7)$$

Since $\langle\varepsilon_r(t)\rangle \neq 0$, $\langle\varepsilon_\phi(t)\rangle \neq 0$, applying the techniques developed in Ref.[20], $\varepsilon_r(t)$ and $\varepsilon_\phi(t)$ can be changed into

$$\varepsilon_r(t) = \frac{p}{2r}(1 - \lambda_q\sin 2\phi) + \tilde{\varepsilon}_r(t), \qquad (8)$$
$$\varepsilon_\phi(t) = -\frac{p}{2r}\lambda_q\cos 2\phi + \tilde{\varepsilon}_\phi(t), \qquad (9)$$

in which pure noises $\tilde{\varepsilon}_r(t)$ and $\tilde{\varepsilon}_\phi(t)$ are zero mean with correlations

$$\langle\tilde{\varepsilon}_r(t)\tilde{\varepsilon}_r(s)\rangle = p(1 + \lambda_q\sin 2\phi)\delta(t-s),$$
$$\langle\tilde{\varepsilon}_\phi(t)\tilde{\varepsilon}_\phi(s)\rangle = p(1 - \lambda_q\sin 2\phi)\delta(t-s),$$
$$\langle\tilde{\varepsilon}_r(t)\tilde{\varepsilon}_\phi(s)\rangle = p\lambda_q\cos 2\phi\delta(t-s). \qquad (10)$$

Inserting Eqs.(8) and (9) into Eqs.(4) and (5), respectively, we have

$$\frac{\mathrm{d}r}{\mathrm{d}t} = a_0 r - Ar^3 + \frac{p}{2r}(1 - \lambda_q\sin 2\phi) + rp_R(t) + \tilde{\varepsilon}_r(t), \qquad (11)$$

$$\frac{\mathrm{d}\phi}{\mathrm{d}t} = -\frac{p}{2r^2}\lambda_q\cos 2\phi + p_I(t) + \frac{\tilde{\varepsilon}_{\phi(t)}}{r}. \qquad (12)$$

It can be seen that Eqs.(11) and (12) couple each other, and $p_i(t)(i = I, R)$ is the coloured noise, so it is a non-Markovian process. In order to obtain the FPE of the amplitude, we proceed to the following approximation. First, we use the UCNA in the phase equation (12); it is found the phase also can be locked by the method of phase lock.[11,13,14] So we make the amplitude equation decoupled from the phase equation, and the decoupled amplitude equation can be obtained. By means of the UCNA again, we deal with the decoupled amplitude equation. Finally, the corresponding approximate FPE can be written.

In Eqs.(11) and (12) $p_i(t)(i = I, R)$ is an O–U noise, which is equivalent to the following process:

$$\dot{p}_i = -\frac{1}{\tau}p_i + \frac{1}{\tau}\eta_i(t), \tag{13}$$

$$\langle \eta_i(t)\rangle = 0,$$

$$\langle \eta_i(t)\eta_j(s)\rangle = \delta_{i,j}p'\delta(t-s), \qquad (i,j = I, R).$$

Thus the two-dimensional differential equations (11) and (12) become the four-dimensional ones (11), (12), and (13); obviously, this is a Markovian process.

Using the UCNA in the phase equation (12), we obtain the approximate phase Langevin equation

$$\frac{\mathrm{d}\phi}{\mathrm{d}t} = \frac{1}{A(r,\tau)}\left[-\frac{p}{2r^2}\lambda_q \cos 2\phi + \eta_I(t) + \frac{\tilde{\varepsilon}_{\phi(t)}}{r}\right]. \tag{14}$$

Here

$$\begin{aligned} A(r,\tau) =& 1 - \tau\frac{\mathrm{d}}{\mathrm{d}\phi}\left[-\frac{p}{2r^2}\lambda_q\cos 2\phi\right] \\ =& 1 - \frac{\tau p}{r^2}\lambda_q \sin 2\phi. \end{aligned}$$

Equation (14) also can be transformed into the following form:

$$\frac{\mathrm{d}\phi}{\mathrm{d}t} = \frac{-\frac{p}{2r^2}\lambda_q\cos 2\phi}{A(r,\tau)} + \Gamma(t), \tag{15}$$

with

$$\Gamma(t) = \frac{\eta_I}{A(r,\tau)} + \frac{\tilde{\varepsilon}_\phi(t)}{A(r,\tau)r}.$$

The statistical properties of $\Gamma(t)$ are

$$\begin{aligned} &\langle \Gamma(t)\rangle = 0, \\ &\langle \Gamma(t)\Gamma(s)\rangle = \frac{1}{A^2(r,\tau)}\left[p' + \frac{p}{r^2}(1 - \lambda_q \sin 2\phi)\right]\delta(t-s). \end{aligned} \tag{16}$$

From Eq.(15) we can see clearly that the presence of a new drift term, $-\dfrac{\frac{p}{2r^2}\lambda_q\cos 2\phi}{A(r,\tau)}$, arising from the nonzero cross-correlation coefficient λ_q, leads the laser phase to be locked at a stable value ϕ_s. Based on the conditions of phase lock $\langle \mathrm{d}\phi/\mathrm{d}t\rangle = 0$, and the most stable lock being subject to

$$\frac{\partial(\mathrm{d}\phi/\mathrm{d}t)}{\partial\phi} - \frac{1}{2}\frac{\partial^2\langle\Gamma(t))^2\rangle}{\partial\phi^2} < 0, \tag{17}$$

we can deduce that ϕ_s satisfies the following equation:

$$\lambda_q \sin 2\phi_s = |\lambda_q|. \tag{18}$$

From Eq.(18), we have

$$\phi_s = (2n+1)\pi/4, \quad n = 0, \pm 1, \pm 2, \cdots.$$

Inserting Eq.(18) into Eq.(11), we can obtain the decoupled amplitude equation

$$\frac{\mathrm{d}r}{\mathrm{d}t} = a_0 r - Ar^3 + \frac{p}{2r}(1 - |\lambda_q|) + rp_R(t) + \tilde{\varepsilon}_r(t), \tag{19}$$

assuming that

$$\tilde{f}(r) = a_0 r - Ar^3 + \frac{p}{2r}(1 - |\lambda_q|), \quad \tilde{g}(r) = r.$$

So a Markovian process can be written by means of the UCNA again:

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{1}{c(r,\tau)}[\tilde{f}(r) + \tilde{g}(r)\eta_R(t) + \tilde{\varepsilon}_r(t)], \tag{20}$$

where

$$\begin{aligned} c(r,\tau) =& 1 - \tau\left[\tilde{f}'(r) - \frac{\tilde{g}'(r)}{\tilde{g}(r)}\tilde{f}(r)\right] \\ =& 1 + 2Ar^2\tau + \frac{\tau p(1-|\lambda_q|)}{r^2}. \end{aligned} \tag{21}$$

Equation (21) shows that $c(r,\tau) > 0$, and the UCNA is valid in the whole region of τ.

3. The SID and three moments of the laser intensity

The FPE corresponding to Eq.(20) is obtained as

$$\frac{\partial Q(r,t)}{\partial t} = -\frac{\partial}{\partial x}F(r)Q(r,t) + \frac{\partial^2}{\partial x^2}D(r)Q(r,t), \tag{22}$$

where

$$\begin{aligned} F(r) =& \frac{a_0 r - Ar^3 + \frac{p}{2r}(1+|\lambda_q|)}{c(r,\tau)} \\ &+ \frac{p'}{2}\left(\frac{\partial}{\partial x}\left(\frac{r}{c(r,\tau)}\right)\right)\frac{r}{c(r,\tau)} \\ &+ \frac{p(1+|\lambda_q|)}{2}\left(\frac{\partial}{\partial x}\left(\frac{1}{c(r,\tau)}\right)\right)\frac{1}{c(r,\tau)}, \\ D(r) =& \frac{p'}{2}\frac{r^2}{c^2(r,\tau)} + \frac{p(1+|\lambda_q|)}{2}\frac{1}{c^2(r,\tau)}. \end{aligned}$$

The stationary probability distribution of laser intensity is derived from Eq.(22) as

$$Q_{\rm st}(I) = N_0 \mathrm{e}^{f(I)} I^{-\alpha} [p'I + P(1+|\lambda_q|)]^{\beta} [2A\tau I^2 + I + p\tau(1-|\lambda_q|)], \tag{23}$$

where

$$f(I) = -\frac{p\tau(1-|\lambda_q|)^2}{2(1+|\lambda_q|)I} - \frac{A(1-2a_0\tau)I}{p'} + \frac{A^2\tau I[2p(1+|\lambda_q|)-p'I]}{p'^2},$$

$$\alpha = \frac{2(1-a_0\tau)+P'\tau+2|\lambda_q|(3-P'\tau)+|\lambda_q|^2(4+2a_0\tau+P'\tau)}{2(1+|\lambda_q|)^2},$$

$$\beta = \frac{1}{2(1+|\lambda_q|)^2}\left[-2+p'\tau(1+|\lambda_q|)^2 - \frac{4A^2P^2\tau(1+|\lambda_q|)^4}{P'^3} - \frac{2AP(1+|\lambda_q|)^3(2a_0\tau-1)}{P'^2} - 2|\lambda_q|(1+P'\tau) + \frac{2a_0(1+|\lambda_q|)((1+|\lambda_q|-P'\tau(1-|\lambda_q|))}{P'}\right].$$

The normalization constant N_0 is determined by

$$\int_0^\infty Q_{\rm st}(I)\mathrm{d}I = 1.$$

The nth moment of the laser intensity I is given by the following integral:

$$\langle I^n\rangle = \int_0^\infty I^n Q_{\rm st}(I)\,\mathrm{d}I. \tag{24}$$

The mean, the variance, and the skewness of the laser intensity can be obtained by numerical integration of Eq.(24). The mean of the laser intensity reads as

$$\langle I\rangle = \int_0^\infty I\,Q_{\rm st}(I)\,\mathrm{d}I. \tag{25}$$

The variance and the skewness of the laser intensity are defined as

$$\lambda_2(0) = \frac{\langle I^2\rangle}{\langle I\rangle^2} - 1,$$

$$\lambda_3(0) = \frac{\langle I^3\rangle}{\langle I\rangle^3} - 3\lambda_2(0) - 1. \tag{26}$$

4. Conclusion and discussion

In the following, we will discuss the effects of the cross-correlation λ_q between the real and imaginary parts of the quantum noise and the self-correlation time τ of the pump noise on the three moments of the single-mode laser for below and above the threshold operation respectively.

4.1. The effect of the cross-correlation λ_q on the mean intensity $\langle I\rangle$

We plot in Fig.1(a) the curves of mean laser intensity $\langle I\rangle$ versus the self-correlation time τ of the pump noise for operation below the threshold. Figure 1(a) shows an interesting phenomenon, the $\langle I\rangle$–τ curves decrease monotonically, and the whole $\langle I\rangle$–τ curve moves down as λ_q increases, however when $\lambda_q = 1$ (perfect correlated), the curve does not move down, but moves up abruptly. Moreover, the magnitude of

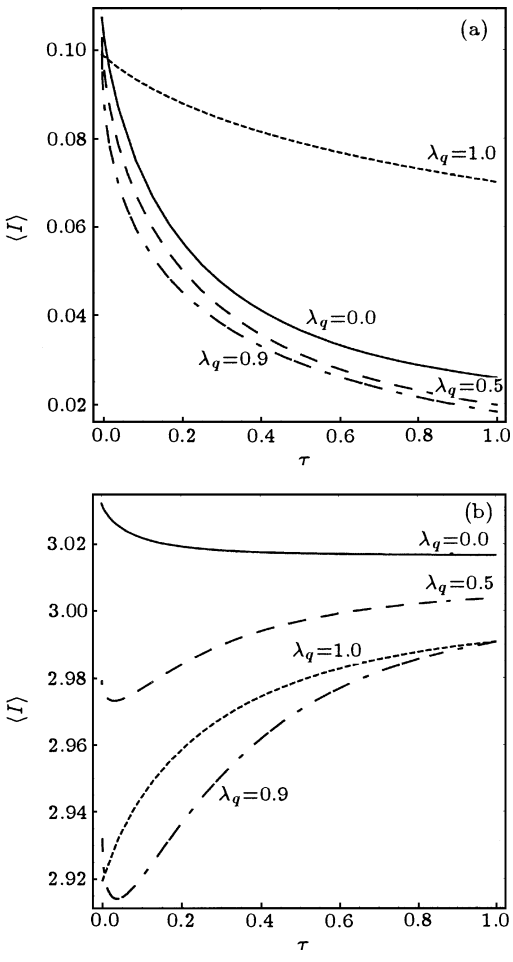


Fig.1. The curves of mean laser intensity $\langle I\rangle$ versus self-correlation time τ of the pump noise. The values of parameters: $\lambda_q = 0$ (solid), 0.5 (dashed), 0.9 (dot-dashed), 1 (dotted), with p=0.1, p'=3, A=1, (a) $a_0 = -3$; (b) $a_0 = 3$.

the curve with $\lambda_q = 1$ is larger than that of the curve with $\lambda_q = 0$: i.e. when $\lambda_q = 1$, the output of the laser arrives at the maximum.

The $\langle I\rangle$–τ curves for above the threshold operation are plotted in Fig.1(b). The curves show that, when $\lambda_q = 0$, the curve decreases monotonically; when $\lambda_q \neq 0$ ($\lambda_q = 0.5$), the curve has a minimum, the location of the minimum is lowered as λ_q is increased (λ_q=0.9); however, when $\lambda_q = 1$, the curve increases with a_0 monotonically. That is, when the value of λ_q is increased, the $\langle I\rangle$–τ curves undergo the transition from being monotonic $\rightarrow$ possessing single extremum $\rightarrow$ being monotonous. Here we have seen a "re-entrant phase transition", which is in parallel to that in the laser intensity. Moreover, when $\lambda_q = 1$, the curve of $\langle I\rangle$–τ also moves up abnormally, which is similar to the interesting phenomenon of Fig.1(a), but the magnitude of moving up is not so acute as that in Fig.1(a).

4.2. The effect of the self-correlation time τ on the variance $\lambda_2(0)$

The $\lambda_2(0)$–p' curves for below and above the threshold are plotted in Figs.2(a) and (b) respectively (p' is the strength of pump noise), τ is a parameter, λ_q=0.5. Figure 2(a) shows that, when $\tau = 0$, the $\lambda_2(0)$–p' curves have a maximum and a minimum for below the threshold operation; when $\tau \neq 0$, the $\lambda_2(0)$–p' curves only have a maximum, which is similar to the existence of multiple "phase" in the stationary intensity distribution. The magnitude of $\lambda_2(0)$ decreases as the self-correlation time τ is increased. Figure 2(b) shows that the whole $\lambda_2(0)$–p' curve moves down as τ is increased. The behaviour of $\lambda_3(0)$ is similar to that of $\lambda_2(0)$; and for brevity, the $\lambda_3(0)$–p' curves are not given in this paper. This phenomenon indicates that whether below or above threshold operation, $\tau \neq 0$ will induce the increase of the output laser precision.

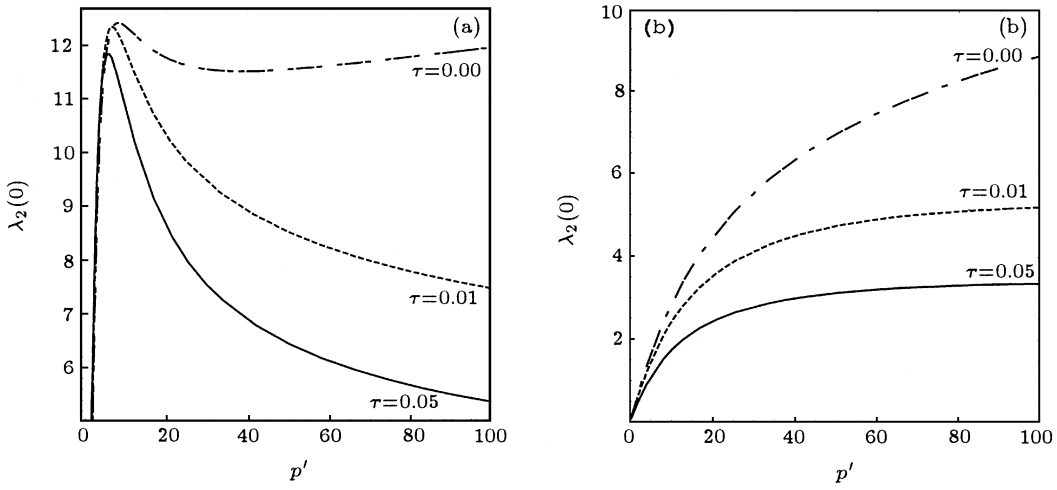


Fig.2. The curves of normalized variance $\lambda_2(0)$ of laser intensity versus the pump noise intensity p'. The values of parameters: τ=0 (dot-dashed), 0.01 (dotted), 0.05 (solid), with p=0.1, λ_q=0.5, A=1, (a) $a_0 = -3$; (b) a_0=3.

4.3. The effect of λ_q on the variance $\lambda_2(0)$ for $\tau \neq 0$

In Figs.3(a) and (b), we plot the $\lambda_2(0)$–p' curves for below and above the threshold operation respectively, in which λ_q is a parameter, τ=0.01. Figure 3 shows that the $\lambda_2(0)$–p' curves have a maximum for below the threshold operation, and the curves increase as p' is increased for above the threshold operation, but whether the laser is operated below or above the threshold, the whole $\lambda_2(0)$–p' curves moves up as λ_q increases; however, when λ_q=1 (perfect cross-correlation), the $\lambda_2(0)$–p' curve suddenly moves down, even crossing the $\lambda_2(0)$–p' curve of λ_q=0: that is, the value of the variance abruptly decreases for perfect cross-correlation. The behaviour of $\lambda_3(0)$ is similar to that of $\lambda_2(0)$, and for brevity, $\lambda_3(0)$–p' curves are not given in this paper. If we assume τ=0, the $\lambda_2(0)$–p' curves do not have this interesting phenomenon, but the whole $\lambda_2(0)$–p' curve moves up as λ_q increases, and when λ_q=1, the magnitude of $\lambda_2(0)$ is the largest. So we think this abnormal interesting phenomenon is induced by the joint effect of the cross-correlation between the real and imaginary parts of the quantum noise and the colouredness of the pump noises.

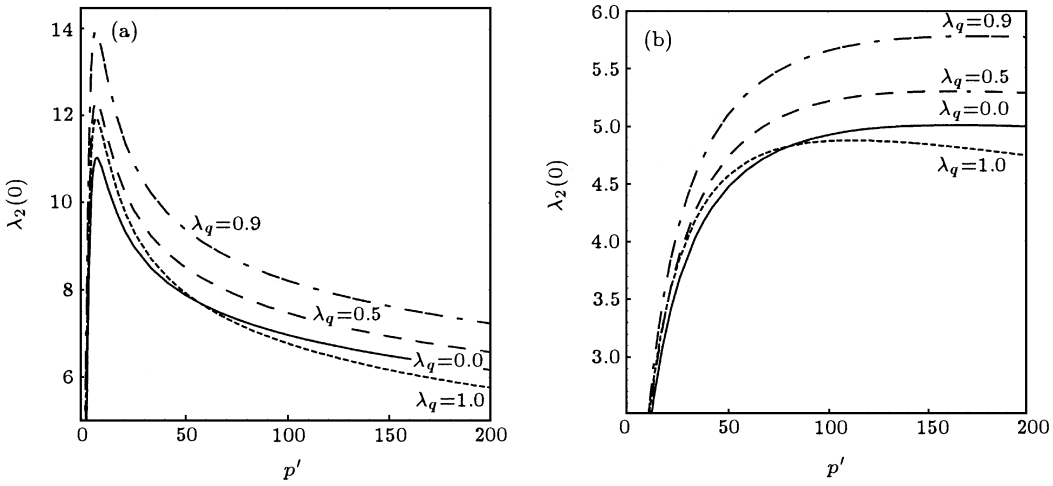


Fig.3. The curves of normalized variance $\lambda_2(0)$ of the laser intensity versus the pump noise intensity p'. The values of parameters: $\lambda_q = 0$ (solid), 0.5 (dashed), 0.9 (dot-dashed), 1 (dotted), with p=0.1, τ=0.01, A=1, (a) $a_0 = -3$; (b) a_0=3.

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