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The asymmetry of the Autler–Townes doublet in a three-level system

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The effect of the Bloch–Siegert shift on a strongly driven transition is studied in a three-level double-resonance configuration and the result is presented in this paper. We show that when a resonantly driven transition is probed to a third level, the Bloch–Siegert shift leads to an asymmetric Autler–Townes doublet. An important conclusion is that the asymmetry depends only on the driving field intensity, in contrast to a previous study where it is reported that the asymmetry depends not only on the driving field intensity but also on the characteristics of the three-level system. Our result implies an alternative way of measuring the Bloch–Siegert shift.

Keywords: Autler–Townes doublet, Bloch–Siegert shift, three-level system
PACC: 3270, 3260, 3335, 4265

1. Introduction

The study of a strongly driven two-level atom is important for the understanding of light+atom interactions, and the nonlinear processes of a two-level atom in a resonant electromagnetic field are associated with many interesting concepts and observations in the fields of laser physics, nonlinear optics and quantum optics.[1−5,16] The rotating wave approximation (RWA) is one of the most widely used approximations in studies of the interaction between a two-level atom and a coherent electromagnetic field. Considering a two-level atom as a magnetic spin of $s = 1/2$ in a static Zeeman field, and decomposing the driving field into two components, one resonant and the other far off resonant at twice the two-level atom transition frequency, the essence of the RWA is to discard the effect of the far off-resonant counter-rotating component. It is well known that the counter-rotating component may introduce a shift in the transition frequency of the driven two-level atom, and this frequency shift is the Bloch–Siegert shift.[6] It is also well known that there is a doublet structure if such a driven transition is probed to a third level (see Fig.1) in a three-level double-resonance configuration. This splitting of the level induced by a strong driving field, known as the Autler–Townes doublet,[7] has been studied extensively and is well understood in terms of dynamic Stark effect.[8] Under RWA, a transition subjected to a resonant driving field displays the well-known symmetric Autler–Townes doublet with two components having equal intensity.

Recently, there have been studies that examine the effect of the Bloch–Siegert shift on the Autler–Townes doublet using a three-level double-resonance configuration.[9−11] Mandal and Ghosh [9] showed that the Bloch–Siegert shift results in an asymmetric Autler–Townes doublet even for a resonantly driven transition. However, the asymmetric Autler–Townes doublet is a result of an artificially introduced driving field detuning, there lacks of evidence that links the asymmetry to the Bloch–Siegert shift in the theoretical approach adopted in Ref.[9]. Wei *et al*[10] showed that the asymmetric Autler–Townes doublet is a natural consequence upon taking into account the counter-rotating component and hence established a direct link between the Bloch–Siegert shift and the asymmetry in the Autler–Townes doublet. They also showed, rather unexpectedly, that there is a complicated correlation between the Bloch–Siegert shift and the asymmetry in the Autler–Townes doublet, which depends not only on the driving field intensity but also on the charac-

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teristics of the three-level system including relaxation rates and equilibrium population distributions. Furthermore, the origin of such an unexpected correlation was not investigated due to its complexity. It has been pointed out recently that the measurement of the Bloch–Siegert shift through the Autler–Townes doublet is an order of magnitude better resolved than monitoring shifts of power broadened transition for equivalent signal to noise levels.[11]

In a driven two-level transition, it is well established that the Bloch–Siegert shift depends only on the driving field intensity, and to lowest order is given by $\chi^2/4\omega_{21}$ (χ is the driving field Rabi frequency and ω_{21} is the two-level transition frequency).[6,12–15] In a three-level double-resonance configuration, a second field is used to probe the effect of a strong driving field on a two-level transition via a separated transition. If this second field is a weak field, it will not perturb the strongly driven transition. Thus, it is expected that the effect of the counter-rotating component observed in a three-level configuration should have the same characteristics as observed in a two-level configuration. It is therefore of interest to investigate closely the unexpected correlation reported in Ref.[10] and the result is presented in this paper. Our results show that the asymmetry in the Autler–Townes doublet depends only on the driving field intensity. This is in contrast to the previous study and the reason for the unexpected correlation reported in Ref.[10] is found to be due to an inappropriate approximation used in numerical calculations. We show that, as expected, there is a simple one-to-one correlation between the asymmetry and the Bloch–Siegert shift. Our result implies an alternative experimental method for measuring the Bloch–Siegert effect.

2. Theory

Figure 1 shows a three-level double-resonance configuration considered in this paper. A strong linearly polarized driving field (frequency ω_1, Rabi intensity χ_1) is applied resonantly on the $|1\rangle \leftrightarrow |2\rangle$ transition. A second field (frequency ω_2, Rabi intensity χ_2) is used to examine the effect of the counter-rotating component of the driving field by probing a transition between one of the driven two levels $|1\rangle$ to a third level $|3\rangle$. Such a three-level double-resonance configuration gives rise to the well-known Autler–Townes doublet. To study the effect of the Bloch–Siegert shift on the Autler–Townes doublet profile, it is adequate to consider the case of a weak probe field. In this situation, the counter-rotating component associated with the weak probe can be neglected whereas that associated with the strong driving field is taken into consideration in the calculation. Therefore, the non-zero elements of the interaction Hamiltonian are given by

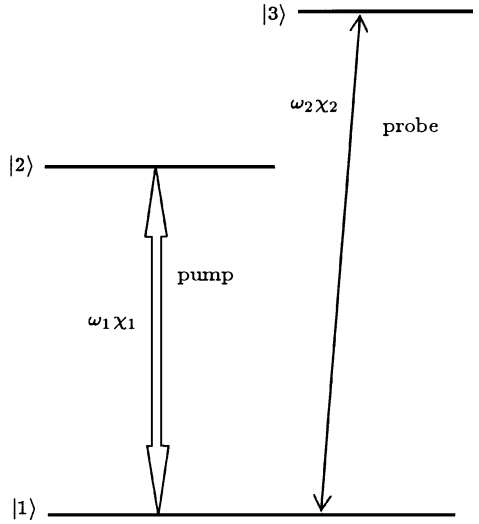


Fig.1. Schematic of the energy levels for studying the Bloch–Siegert shift in a three-level double-resonance configuration. A strong linearly polarized field (frequency ω_1, Rabi intensity χ_1) drives the $|1\rangle \leftrightarrow |2\rangle$ transition and a weak probe field (frequency ω_2, Rabi intensity χ_2) scans the $|1\rangle \leftrightarrow |3\rangle$ transition.

$$V_{21} = V_{12}^* = -\hbar\chi_1 \cos\omega_1 t = -\frac{\hbar\chi_1}{2}[\exp(-\mathrm{i}\omega_1 t) + \exp(\mathrm{i}\omega_1 t)], \quad (1a)$$

$$V_{31} = V_{13}^* = -\frac{\mu_{31}B_2}{2}\exp(-\mathrm{i}\omega_2 t) = -\frac{\hbar\chi_2}{2}\exp(-\mathrm{i}\omega_2 t), \quad (1b)$$

where χ_1 and χ_2 are the Rabi frequencies of the driving and probing fields, respectively.

The equations of motion of the density matrix are given by [2,3]

$$\dot\rho_{11} = \mathrm{i}\chi_1\cos\omega_1 t(\rho_{21}-\rho_{12}) + \mathrm{i}\frac{\chi_2}{2}(\rho_{31}\mathrm{e}^{\mathrm{i}\omega_2 t} - \rho_{13}\mathrm{e}^{-\mathrm{i}\omega_2 t}) - \gamma(\rho_{11}-\rho_{11}^0), \quad (2a)$$

$$\dot\rho_{22} = -\mathrm{i}\chi_1\cos\omega_1 t(\rho_{21}-\rho_{12}) - \gamma(\rho_{22}-\rho_{22}^0), \quad (2b)$$

$$\dot\rho_{33} = -\mathrm{i}\frac{\chi_2}{2}(\rho_{31}\mathrm{e}^{\mathrm{i}\omega_2 t} - \rho_{13}\mathrm{e}^{-\mathrm{i}\omega_2 t}) - \gamma(\rho_{33}-\rho_{33}^0), \quad (2c)$$

$$\dot\rho_{31} = -(\mathrm{i}\omega_{31}+\gamma_{31})\rho_{31} - \mathrm{i}\chi_1\cos\omega_1 t\rho_{32} - \mathrm{i}\frac{\chi_2}{2}\mathrm{e}^{-\mathrm{i}\omega_2 t}(\rho_{33}-\rho_{11}), \quad (2d)$$

$$\dot\rho_{32} = -(\mathrm{i}\omega_{32}+\gamma_{32})\rho_{32} - \mathrm{i}\chi_1\cos\omega_1 t\rho_{31} + \mathrm{i}\frac{\chi_2}{2}\mathrm{e}^{-\mathrm{i}\omega_2 t}\rho_{12}, \quad (2e)$$

$$\dot\rho_{21} = -(\mathrm{i}\omega_{21}+\gamma_{21})\rho_{21} - \mathrm{i}\chi_1\cos\omega_1 t(\rho_{22}-\rho_{11}) - \mathrm{i}\frac{\chi_2}{2}\mathrm{e}^{-\mathrm{i}\omega_2 t}\rho_{23}, \quad (2f)$$

where γ is the longitudinal relaxation rate and assumed, for simplicity, to be the same for all levels, γ_{ij} is the transverse relaxation rate between the $|i\rangle \Leftrightarrow |j\rangle$ transition, ρ^0_{ii} is the equilibrium population of level $|i\rangle$ in the absence of applied fields, and ω_{ij} is the transition frequency between the $|i\rangle \Leftrightarrow |j\rangle$ transition. In the following calculations, the above equations are solved to the first order of the weak field Rabi intensity χ_2 to obtain the Autler–Townes spectrum. By letting $\chi_2 = 0$ in Eq.(2), we obtain a set of equations to the zero order of weak probe field intensity χ_2, which describes a driven two-level transition. Adopting notation used in Ref.[10], a solution to zero order of the weak probe field intensity χ_2 is given by [10]

$$\rho^{(0)}_{12} = \sum_n a_n \exp(\mathrm{i}n\omega_1 t), \tag{3a}$$

$$w^{(0)} = \rho^{(0)}_{22} - \rho^{(0)}_{11} = \sum_n a_n \exp(\mathrm{i}n\omega_1 t), \tag{3b}$$

where

$$\begin{aligned}
a_n &= \frac{\chi_1}{2}\frac{(d_{n-1}+d_{n+1})}{(n\omega_1-\omega_{21}-\mathrm{i}\gamma_{21})},\\
d_0 &= \frac{\gamma(\rho^0_{22}-\rho^0_{11})}{P_0 - 2\chi_1^2 \mathrm{Re}(R_0 Z_2)},\\
d_n &= Z_n d_{n-2}, \quad n \neq 0,\\
Z_n &= \frac{\chi_1^2 Q_n}{P_n - \chi_1^2 R_n Z_{n+2}},\\
Q_n &= \frac{\mathrm{i}(n-1)\omega_1+\gamma_{21}}{[(n-1)\omega_1-\mathrm{i}\gamma_{21}]^2-\omega_{21}^2},\\
R_n &= \frac{\mathrm{i}(n+1)\omega_1+\gamma_{21}}{[(n+1)\omega_1-\mathrm{i}\gamma_{21}]^2-\omega_{21}^2},\\
P_n &= \mathrm{i}n\omega_1+\gamma-\chi_1^2(Q_n+R_n).
\end{aligned}$$

Substituting the above zero order solution into Eqs.(2), we obtain a set of equations to the first order of the weak probe field intensity χ_2. To calculate the response of the probe field, the first order equation for ρ_{31} is derived as

$$\rho^{(1)}_{31} = \sum_n b_n \exp(\mathrm{i}n\omega_1 t)\exp(-\mathrm{i}\omega_2 t), \tag{4a}$$

$$\rho^{(1)}_{32} = \sum_n c_n \exp(\mathrm{i}n\omega_1 t)\exp[-\mathrm{i}(\omega_2-\omega_1)t], \tag{4b}$$

with b_n given by

$$\begin{aligned}
&\left[h_n - \frac{\chi_1^2}{4}\left(\frac{1}{g_n}+\frac{1}{g_{n-2}}\right)\right]b_n\\
=&\frac{\chi_1^2}{4g_n}b_{n+2} + \frac{\chi_1^2}{4g_{n-2}}b_{n-2} - \frac{\chi_1\chi_2}{4}\left(\frac{a_{n+1}}{g_n}+\frac{a_{n-1}}{g_{n-2}}\right)\\
&-\frac{\chi_2}{4}d_n + \frac{\chi_2}{2}\left(\frac{1}{2}-\frac{3}{2}\rho^0_{33}\right)\delta_{n0},
\end{aligned} \tag{5}$$

$$\begin{aligned}
h_n =& n\omega_1 - \omega_2 + \omega_{31} - \mathrm{i}\gamma_{31}\\
=& n\omega_1 + \delta_2 - \mathrm{i}\gamma_{31},
\end{aligned} \tag{6a}$$

$$\begin{aligned}
g_n =& n\omega_1 - \omega_2 + \omega_{32} + \omega_1 - \mathrm{i}\gamma_{32}\\
=& n\omega_1 - \delta_1 + \delta_2 - \mathrm{i}\gamma_{32},
\end{aligned} \tag{6b}$$

where δ_1 (δ_2) is the driving (probing) field detuning. In particular, the weak probe field response is described by the term including frequency ω_2 in the expanded series of ρ_{31} (see, Eq.(4)), which involves solving its coefficient b_0. By letting n=0 in Eq.(5), we obtain the following equation for b_0:

$$\begin{aligned}
&\left[h_0 - \frac{\chi_1^2}{4}\left(\frac{1}{g_0}+\frac{1}{g_{-2}}\right)\right]b_0\\
=&\frac{\chi_1^2}{4g_0}b_2 + \frac{\chi_1^2}{4g_{-2}}b_{-2} - \frac{\chi_1\chi_2}{4}\left(\frac{a_1}{g_0}+\frac{a_{-1}}{g_{-2}}\right)\\
&-\frac{\chi_2}{4}d_0 + \frac{\chi_2}{2}\left(\frac{1}{2}-\frac{3}{2}\rho^0_{33}\right).
\end{aligned} \tag{7}$$

This is a recursive solution and proper truncation is required in order to numerically calculate b_0. The truncation needs to be carried out in such a way that the difference of including an extra term should be negligible. In Ref.[10] the solution for b_0 is obtained by truncating terms with the order higher than 2. By closely inspecting Eq.(7), it is seen that the coefficient of b_2: $\dfrac{\chi_1^2}{4g_0} = \dfrac{\chi_1^2}{\delta_2-\delta_1-\mathrm{i}\gamma_{32}}$ is significant for strong driving field, and therefore such a truncation is inappropriate. Here, the effect of terms of different order is examined, and it is found that the most compact form for the solution of b_0 is given as

$$b_0 = \frac{\chi_2}{4}\cdot\frac{\left[(-\chi_1^2)\cdot\left(\dfrac{\chi_1\left(\dfrac{a_1}{g_0}+\dfrac{a_3}{g_2}\right)+d_2}{g_0\left[4h_2-\chi_1^2\left(\dfrac{1}{g_0}+\dfrac{1}{g_2}\right)\right]}\right)\right] - \chi_1\left(\dfrac{a_1}{g_0}+\dfrac{a_{-1}}{g_{-2}}\right) - d_0 + (1-3\rho^0_{33})}{h_0 - \dfrac{\chi_1^2}{4}\left[\left(\dfrac{1}{g_0}+\dfrac{1}{g_{-2}}\right)+\dfrac{\chi_1^2}{g_0^2\left[4h_2-\chi_1^2\left(\dfrac{1}{g_0}+\dfrac{1}{g_2}\right)\right]}\right]}. \tag{8}$$

For a given driving field frequency ω_1 and Rabi intensity χ_1, the absorptive Autler–Townes profile can be obtained by plotting the imaginary part of Eq.(8) as a function of the probe field frequency ω_2 and the results are presented in the following section.

3. Numerical results and discussion

Figure 2 shows the absorption spectra exhibiting Autler–Townes doublet. It is obtained by plotting the imaginary part of b_0 [see Eq.(8)] as a function of probe field detuning for a resonant driving field $\omega_1 = \omega_{21}$. It is well known that, if the counter-rotating component is neglected, a resonant driving field gives rise to a symmetric Autler–Townes doublet, and the asymmetry originates from a detuning in the driving field. From Fig.2 it can be seen that, when the counter-rotating component is taken into consideration, the intensity of Autler–Townes doublet becomes unequal and this asymmetry increases as the driving Rabi intensity increases. Also shown in Fig.2 are results for which higher order terms are included in the solution of b_0, and it can be seen that there is no noticeable difference. The origin behind such an asymmetry is that a driving field resonant with a two-level transition ($\omega_1 = \omega_{21}$) is no longer resonant due to the Bloch–Siegert effect, and such an off resonant field results in an asymmetric Autler–Townes doublet.

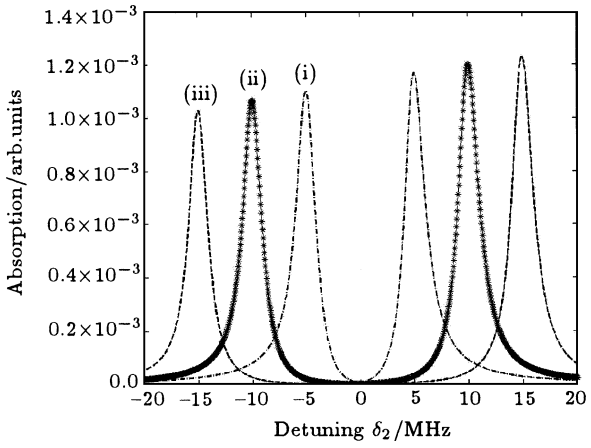


Fig.2. Absorptive Autler–Townes doublet in the presence of a resonant driving field. The Rabi intensities used are 10, 20 and 30MHz for traces (i), (ii) and (iii) respectively. The results are obtained using b_0 given in Eq.(8). For comparison, the results (curve with *) using solution for which higher order terms are included are also shown superimposed.

The driving field frequency is then tuned around the two-level transition frequency to recover a symmetric Autler–Townes doublet, indicating that the driving field is resonant again with the shifted two-level transition. The frequency shift of the driving field required to obtain a symmetric Autler–Townes doublet gives a measure of the frequency shift of the two-level transition and is plotted as a function of driving field intensity as shown in Fig.3 for a range of combinations of parameters of the three-level system.

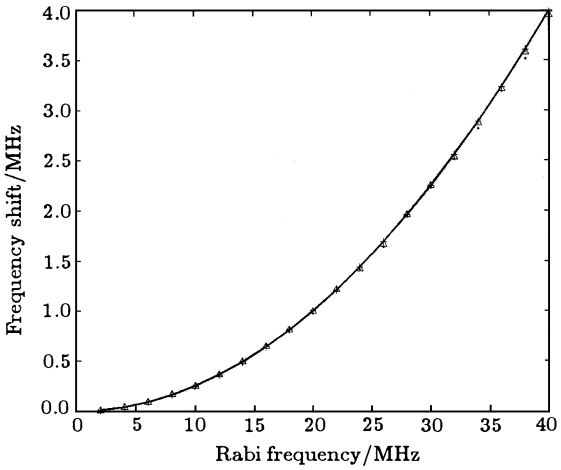


Fig.3. Bloch–Siegert shift in a three-level double-resonance configuration for various combination of parameters. The curve in solid line is a plot of Eq.(9). The frequency shift of the driving field required to minimize the level of asymmetry of the Autler–Townes doublet is plotted as a function of driving field Rabi frequency for the following combinations of the three-level system parameters: γ_{32}=0.2, $w^{(0)}$=–1.0 in curve (ii) (triangles); γ_{32}=0.2, $w^{(0)} = -0.4$ in curve (iii) (dots); γ_{32}=2.0, $w^{(0)}$=–1.0 in curve (iv) (crosses).

As mentioned above, in a driven two-level transition, the Bloch–Siegert shift depends only on the driving field intensity, and to lowest order is given by $\chi^2/4\omega_{21}$.[6,12–15] In a three-level configuration, a strong field drives the two-level transition while probed using a second weak field which does not perturb the strongly driven transition via a separated transition. Thus, it is expected that the effect of the counter-rotating term observed in a three-level configuration will be the same as that observed in a two-level configuration. In this paper, the expression of the driving field frequency shift (Δ) required to recover a symmetric Autler–Townes doublet can be obtained from Fig.3 as

$$\Delta = \frac{\chi_1^2}{4\omega_{21}}. \qquad (9)$$

The above equation shows the correlation between the driving field frequency shift and the driving field intensity. We can know from it, for a three-level

system (ω_{21} is a constant), that the driving field frequency shift is only a function of the driving field intensity. It also shows that the asymmetry property of the Autler–Townes doublet only depends on the driving field intensity, since the asymmetry of the Autler–Townes doublet could be described by the pump frequency shift. Moreover, this asymmetry degree is as a direct ratio to the square of the driving field intensity. So, as expected, the asymmetry depends on the driving field intensity only. There is no dependence of the asymmetry on the characteristics of the three-level system as reported previously.[10] More significantly, this shift is in good agreement with the lowest order of the Bloch–Siegert shift.[6,12–15]

Our result indicates that the frequency shift due to the counter-rotating component is the same for both two-level and three-level configurations. It is interesting to note that up to now the Bloch–Siegert shift is a widely accepted theoretical prediction; there has been no experimental observation of this shift. There are two main factors that hinder a direct measurement of the Bloch–Siegert shift. The first factor is saturation, which diminishes the signal strength, and the second one is power broadening, which increases the transition linewidth and makes a precise determination of line centre difficult. In the three-level double-resonance configuration studied in this paper, the Bloch–Siegert shift is measured through a separation transition. By using a weak field there is no saturation and power broadening. Therefore, our result implies an alternative method for measuring the Bloch–Siegert shift.

4. Conclusion

In this paper, the effect of the Bloch–Siegert shift on the Autler–Townes doublet profile is reconsidered, and a new first-order solution is derived. An important observation is that the level of asymmetry of the Autler–Townes doublet depends only on the driving field intensity, and the frequency shift of the driving field required to yield an Autler–Townes doublet of equal intensity is the same as the Bloch–Siegert shift in two-level configuration. This work rectifies the conclusion of Ref.[10] and suggests an alternative experimental method for measuring the Bloch–Siegert shift.

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