Vol 13 No 3, March 2004
1009-1963/2004/13(03)/0341-03

Chinese Physics

© 2004 Chin. Phys. Soc.
and IOP Publishing Ltd

Two-pion interferometry at small relative momentum for pion sources with transverse and longitudinal expansions in relativistic heavy ion collisions

Chen Xiao-Fan(陈小凡)†, Yang Xue-Dong(杨学栋), and Han Ling(韩 玲)

Department of Physics, Harbin Institute of Technology, Harbin 150001, *China*

(Received 16 May 2003; revised manuscript received 26 August 2003)

A two-pion correlation function at small relative momentum for pion sources with transverse and longitudinal expansions in relativistic heavy ion collisions is obtained using two-pion interferometry at small relative momentum, and the relations between the real and apparent parameters of the pion source are given. The relations can be used to extract both the temperature and the transverse and longitudinal expansion velocities of pion sources and to verify the correctness of relativistic transformation $T' = T\sqrt{1-v^2}$ of temperature in relativistic statistical mechanics and thermodynamics.

Keywords: two-pion interferometry, relativistic heavy ion collisions, relativistic
PACC: 2570N, 2000, 0760L, 0788

Relativistic heavy ion collisions provide the study on quantum chromodynamics (QCD) with the earth-based experiments when the momentum transfer is large (the case in which pQCD is applicable) or small. Intensity interferometry has been widely used to probe the spacetime structure and properties (e.g., transition of hadronic matter to quark–gluon plasma) of the matter at high temperature and density produced in relativistic heavy ion collisions.[1−5] Our papers show that intensity interferometry at small relative momentum for the static sources in relativistic and ultra-relativistic heavy ion collisions can (a) give the relations among the pion source parameters when different means of pion interferometry are applied,[6−11] (b)express explicitly the effects of collective flow on two-pion interferometry,[12,13] (c) explain quantitatively the insensitivity of pion interferometry analyses to the life parameters of pion sources,[14] and (d) determine the scattering length for the strong interaction between nucleons with the same isospin.[15]

In this paper, we present two-pion interferometry at small relative momentum for pion sources with transverse and longitudinal expansions in relativistic heavy ion collisions.

In relativistic heavy ion collisions, the two-pion correlation function is defined as

$$C_2(\boldsymbol{p}_1,\boldsymbol{p}_2) = \frac{P_2(\boldsymbol{p}_1,\boldsymbol{p}_2)}{P_1(\boldsymbol{p}_1)P_1(\boldsymbol{p}_2)}, \tag{1}$$

where $P_2(\boldsymbol{p}_1,\boldsymbol{p}_2)$ is the probability of pion pairs appearing in relativistic heavy ion collisions, and $P_1(\boldsymbol{p})$ is the probability of a single pion meson created in relativistic heavy ion collisions. In the Pratt formulation of two-pion interferometry,[5] the two-pion correlation function at small relative momentum is expressed as

$$\begin{aligned} &C_{2p}(\boldsymbol{p}_1,\boldsymbol{p}_2) \\ &=1+\lambda\frac{\left|\iint \mathrm{d}x\cdot\exp(\mathrm{i}x\cdot q)g(x,p)\right|^2}{\left|\iint \mathrm{d}x\cdot g(x,p)\right|^2}, \end{aligned} \tag{2}$$

where $g(x,p)$ is the emission function for pion sources, x the four-dimensional coordinate of a pion meson, p the four-dimensional momentum of the pion meson, $q = p_2 - p_1$ the four-dimensional momentum difference of a pion pair, and λ the two-pion correlation factor. For our model of pion sources with transverse and longitudinal expansions in relativistic heavy ion collisions, the emission function $g(x,p)$ is

†E-mail: xfchenxf@yahoo.com.cn

$$g(x,p) = B\cdot\exp\left(-\frac{\rho^2}{R_t^2}-\frac{z^2}{R_l^2}\right)\cdot\exp\left(-\frac{t^2}{\tau^2}\right)\cdot\exp\left(-\frac{E_p - v_t p_t\cos(\phi_\rho-\phi_{p_t}) - v_l p_l \mathrm{sgn}(z)}{T\sqrt{1-v^2}}\right), \quad (3)$$

where v_t and v_l are the constant transverse and longitudinal expansion velocities of a pion source respectively, T is the temperature of the source, R_t and R_l are the real transverse and longitudinal spatial parameters of the source respectively, τ is the temporal parameter, $E_p=\sqrt{p^2+m^2}$ is the energy of the pion meson, and B is the normalization factor. At small relative momentum, substituting Eq.(3) into Eq.(2) gives the two-pion correlation function for such a pion source,

$$\begin{aligned} C_{2e}(\boldsymbol{p}_1,\boldsymbol{p}_2) =& 1+\lambda_e-\frac{1}{2}\lambda_e q_t^2\left\{R_t^2\left[(|J_0(-\mathrm{i}a_t)|^2\right.\right.\\ &\left.\left.-\frac{\pi}{4}|J_1(-\mathrm{i}a_t)|^2\right]+b\tau^2\right\}\\ &-\frac{1}{2}\lambda_e q_l^2\left[R_l^2\left(1-\frac{2}{\pi}\mathrm{th}^2 a_l\right)+b\tau^2\right], \end{aligned} \quad (4)$$

where

$$a_t = \frac{v_t p_t}{T\sqrt{1-v^2}}, \quad (5)$$

$$a_l = \frac{v_l p_l}{T\sqrt{1-v^2}}, \quad (6)$$

$$b = \frac{mT}{3mT+m^2}. \quad (7)$$

For experiments, the two-pion correlation function at small relative momentum is [7–15]

$$\begin{aligned} C_{2a}(\boldsymbol{p}_1,\boldsymbol{p}_2) =& 1+\lambda_a-\frac{1}{2}\lambda_a q_t^2(R_{at}^2+b\tau^2)\\ &-\frac{1}{2}\lambda_a q_l^2(R_{al}^2+b\tau^2), \end{aligned} \quad (8)$$

where R_{at} and R_{al} are called, respectively, the apparent spatial transverse and longitudinal parameters of a pion source, which can be measured directly from experiments. For the same pion source, setting $C_{2e}(\boldsymbol{p}_1,\boldsymbol{p}_2)$ in Eq.(4) equal to $C_{2a}(\boldsymbol{p}_1,\boldsymbol{p}_2)$ in Eq.(8) gives

$$\lambda_a = \lambda_e, \quad (9)$$

$$R_{at} = R_t\left(|J_0(-\mathrm{i}a_t)|^2-\frac{\pi}{4}|J_1(-\mathrm{i}a_t)|^2\right)^{1/2}, \quad (10)$$

$$R_{al} = R_l\left(1-\frac{2}{\pi}\mathrm{th}^2 a_l\right)^{1/2}. \quad (11)$$

Figure 1 shows R_{at} as a function of transverse momentum of the pion meson, and Fig.2 shows R_{al} as a function of longitudinal momentum of the pion meson, when T=130MeV, v_t=0.55c, v_l=0.45c, R_t=4.1fm, and R_l=3.1fm. The figures indicate that the bigger the momenta, the smaller the apparent spatial parameters. Figure 2 shows that the apparent spatial longitudinal parameter R_l of the pion source approaches a constant when the longitudinal momentum p_l is large.

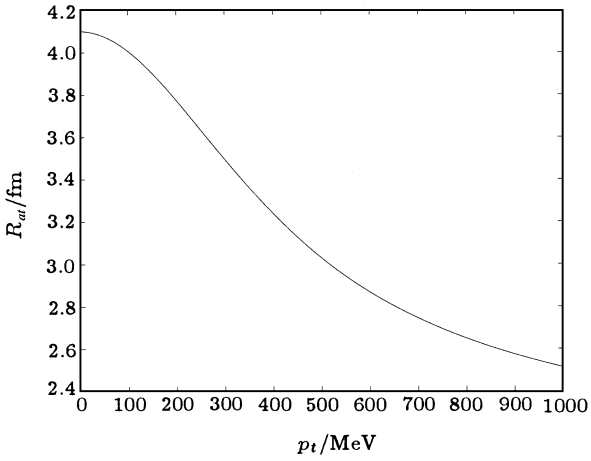


Fig.1. R_{at} as a function of transverse momentum of the pion meson, when T=130MeV, v_t=0.55c, v_l=0.45c, R_t=4.1fm, and R_l=3.1fm.

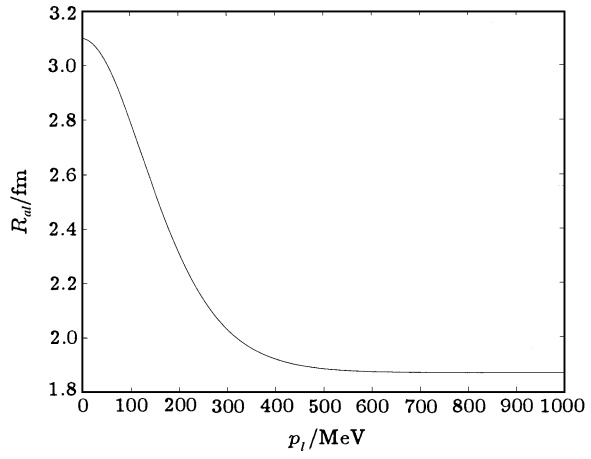


Fig.2. R_{al} as a function of longitudinal momentum of the pion meson, when T=130MeV, v_t=0.55c, v_l=0.45c, R_t=4.1fm, and R_l=3.1fm.

By integrating the emission function $g(x,p)$ in Eq.(3) over spacetime coordinates, the pion momentum spectrum $P_1(\boldsymbol{p})$ can be obtained, which is

$$\begin{aligned} P_1(\boldsymbol{p}) =& A\exp\left(-\frac{E_p}{T_{\mathrm{eff}}}\right)\left[\exp\left(\frac{v_l p_l}{T_{\mathrm{eff}}}\right)\right.\\ &\left.+\exp\left(-\frac{v_l p_l}{T_{\mathrm{eff}}}\right)\right]D\left(\frac{v_t p_t}{T_{\mathrm{eff}}}\right), \end{aligned} \quad (12)$$

where A is a normalization factor,

$$T_{\rm eff} = T\sqrt{1-v^2}, \tag{13}$$

and

$$D\left(\frac{v_t p_t}{T_{\rm eff}}\right) = \int_0^{2\pi} \exp\left(-\frac{v_t p_t \cos\phi}{T_{\rm eff}}\right) {\rm d}\phi. \tag{14}$$

$T_{\rm eff}$, the transverse expansion velocity v_t and the longitudinal expansion velocity v_l for a pion source can be obtained by fitting the experimental pion momentum spectrum to Eq.(12). And the temperature T of the source is determined by

$$T = \frac{T_{\rm eff}}{\sqrt{1-(v_t^2+v_l^2)}}. \tag{15}$$

There is another method to determine the temperature T, transverse expansion velocity v_t and longitudinal expansion velocity v_l for a pion source. Experimentally, the relations between apparent spatial parameters and momenta are obtained by performing the interferometry analyses with cuts in the magnitude of the pion momenta. By fitting the data, the real transverse and longitudinal spatial parameters R_t and R_l, d_t and d_l are determined, where

$$d_t = \frac{v_t}{T_{\rm eff}}, \tag{16}$$

$$d_l = \frac{v_l}{T_{\rm eff}}, \tag{17}$$

therefore,

$$v_t = d_t T_{\rm eff}, \tag{18}$$

$$v_l = d_l T_{\rm eff}, \tag{19}$$

$$T = \frac{T_{\rm eff}}{\sqrt{1-T_{\rm eff}^2(d_t^2+d_l^2)}}. \tag{20}$$

To verify the correctness of the relativistic transformation $T' = T\sqrt{1-v^2}$ ($T_{\rm eff} = T\sqrt{1-v^2}$ in our case) of temperature in relativistic statistical mechanics and thermodynamics, one just needs to know how $T_{\rm eff}$ varies with v when T is a constant. For the different particles (e.g., protons, neutrons) from the nuclear fireball produced in the same collision, T is the same, and $T_{\rm eff}$ and v can be obtained with the above methods, which can then be used to obtain the relation between $T_{\rm eff}$ and v.

In conclusion, the expression for the two-pion correlation function at small relative momentum for pion sources with transverse and longitudinal expansions in relativistic heavy ion collisions is Eq.(4), and the relations between the real and apparent parameters of the pion sources are Eqs.(9)–(11). These relations can be used to extract both the temperature and transverse and longitudinal expansion velocities of the pion sources and to verify the correctness of the relativistic transformation $T' = T\sqrt{1-v^2}$ of temperature in relativistic statistical mechanics and thermodynamics.

References

[1] Weiner R M 2000 *Phys. Rep.* **327** 249

[2] Wiedemann U A and Heinz U 1999 *Phys. Rep.* **319** 145

[3] Boal D H, Gelbke C K and Jennings B K 1990 *Rev. Mod. Phys.* **62** 553

[4] Gyulassy M, Kauffman S K and Wilson L W 1979 *Phys. Rev.* C **20** 2267

[5] Pratt S 1984 *Phys. Rev. Lett.* **53** 1219

[6] Chen X F *et al* 2002 *High Energy Nucl. Phys.* **26** 955 (in Chinese)

[7] Chen X F *et al* 2001 *High Energy Nucl. Phys.* **25** 1002 (in Chinese)

[8] Chen X F *et al* 2001 *High Energy Nucl. Phys.* **25** 149 (in Chinese)

[9] Chen X F and Chen Z L 1999 *High Energy Nucl. Phys.* **23** 1097 (in Chinese)

[10] Chen X F 1998 *High Energy Nucl. Phys.* **22** 903 (in Chinese)

[11] Chen X F 1998 *High Energy Nucl. Phys.* **22** 424 (in Chinese)

[12] Chen X F *et al* 2002 *High Energy Nucl. Phys.* **26** 383 (in Chinese)

[13] Chen X F *et al* 2000 *High Energy Nucl. Phys.* **24** 524 (in Chinese)

[14] Chen X F *et al* 2002 *HIT Journal* **2** 258

[15] Chen X F *et al* 2001 *Chin. Phys. Lett.* **18** 1452