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Blind adaptive identification of FIR channel in chaotic communication systems*

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In this paper we study the problem of blind channel identification in chaotic communications. An adaptive algorithm is proposed, which exploits the boundness property of chaotic signals. Compared with the EKF-based approach, the proposed algorithm achieves a great complexity gain but at the expense of a slight accuracy degradation. However, our approach enjoys the important advantage that it does not require the *a priori* information such as nonlinearity of chaotic dynamics and the variances of measurement noise and the coefficient model noise. In addition, our approach is applicable to the ARMA system.

Keywords: chaotic communication, blind identification, LMS algorithm, extended Kalman filtering
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1. Introduction

In communications, channel identification can be achieved by transmitting the training sequence. Inevitably, this will cost much bandwidth of the channel. Moreover, in some circumstances, no training sequence is available. Hence, blind channel identification has received considerable attention in recent years.[1]

Blind channel identification attempts to identify the channel by using the received signals. The *a priori* information about the statistics of input signals is assumed to be available and will also be used in the identification. However, the transmitted sequence is inaccessible.

In this paper we consider blind identification of a finite impulse response (FIR) system driven by the chaotic signal. This emerges in chaotic communications. In most chaotic communication schemes, it is assumed that the transmitter is connected to the receiver by an ideal channel.[2−10] However, in a realistic communication system, the distortion due to the propagation channel can corrupt the transmitted signal and make it difficult for the receiver to directly recover the transmitted signal. In such a situation, channel equalization is required to compensate the channel distortion.[11]

Since a chaotic sequence is a deterministic signal, its statistics cannot extract fully the information contained in the signal. It can be imagined that the statistics-based identification techniques would not achieve the optimum estimation accuracy.[12,13] In Ref.[14], Leung and Huang introduced a new complexity measure called the phase space volume (PSV) to describe the finite dimensional property of a chaotic signal. Later, they showed that the parameter estimation could be obtained by minimizing the PSV.[15] Although the minimum phase space volume (MPSV) technique can offer very high estimation accuracy, the computation spent for PSV is very heavy. Recently, in Ref.[12], Zhu and Leung proposed an identification approach called minimum nonlinear prediction error (MNPE). MNPE approach is derived by considering the short-term predictability of a chaotic signal. However, they performed the identification by expanding a moving-average (MA) system as an infinite-order autoregressive (AR) model. The estimation accuracy is limited by the truncation error in approximating the infinite order AR model by a finite-order AR system. Both the MPSV and MNPE approaches require random searching, which is very expensive in computation.

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In addition to the above batch algorithms, Zhu and Leung presented an extended Kalman filter-based adaptive equalization algorithm.[11] Compared with MPSV and MNPE, its computational complexity is very low. But the extended Kalman filtering (EKF) based method needs the *a priori* information such as the nonlinearity of chaotic dynamics and the variances of measurement noise and coefficient model noise.

In Ref.[14], we proposed to apply the variable-gain least mean square (LMS) algorithm to the identification of the AR system driven by a chaotic sequence. The main idea is to employ the boundness property of chaotic signals. The parameter estimates are found so that the resultant estimate of the driving signal is bounded by the known bound. For an AR system, the driving signal can be directly expressed by the observation signal and the estimated parameters. However, it is not straightforward to extend the idea to the identification of a MA system as considered in this paper.

In this paper we propose to exploit the variable-gain LMS algorithm[16] for blind channel identification in chaotic communications. This is derived from the fact that the input chaotic signal is bounded. Compared with the EKF-based approach, a great complexity gain is achieved at the expense of some accuracy degradation. Our approach does not require any *a priori* information about the nonlinearity of the chaotic dynamics or variances of the coefficient model noise.

2. Description of the problem

Consider a chaotic communication system, in which the information-bearing chaotic signal is transmitted through a non-ideal channel before it reaches the receiver. Suppose that the chaotic transmitter system is described by

$$x_n = f(x_{n-1}, \cdots, x_{n-\delta}), \quad x_n \in R^m. \tag{1}$$

The signal of transmission s_t is assumed to be stored in the bifurcation parameter of the chaotic system (1). By keeping the bifurcation parameter in the chaotic regime, the output of the chaotic system is chaotic and hence occupies a wide bandwidth.

In practice, the ideal channel does not exist. The transmitted signal passes through a propagation channel before arriving at the receiver. It was found that a typical fading and multipath channel could be modelled by an FIR filter.[17] Thus, the received signal would be

$$y_n = \sum_{l=0}^{L} a_l x_{n-l} + v_n, \tag{2}$$

where $a = [1 \ a_1 \ \cdots \ a_L]^{\mathrm{T}}$ is the channel coefficient vector, and v_n is a white Gaussian noise with variance σ_v^2. The objective of blind identification/equalization is to estimate the channel parameters a_k and recover the transmitted signal x_n from the received signal y_n.

In most blind algorithms that deal with non-chaotic signals, the *a priori* information about the statistics of the input signal is exploited.[1] However, instead of the statistics of chaotic signals, we will exploit the knowledge about the bound of the chaotic signal, which is assumed to be known, i.e. $-M \le x_n \le M$.

3. Blind adaptive algorithm for channel identification in chaotic communications

As a parameter estimation problem, channel identification will be developed under a certain criterion. In this paper, we require that the channel estimate should ensure the corresponding input signal estimate to be bounded by the known bound M. In other words, an effective estimate of channel should be in the following set:

$$\{\hat{a} = [\hat{a}_1 \ \cdots \ \hat{a}_L] \mid -M \le \hat{x}_t \le M\}. \tag{3}$$

If we can find an estimate of x_t as

$$\hat{x}_t = \mathrm{BLF}(y_t \ \cdots \ y_{t-P} \ \hat{a}_1 \cdots \ \hat{a}_L), \tag{4}$$

where BLF (.) is a bilinear function, then Eq.(3) can be directly used to derive an identification algorithm. For an AR system, Eq.(4) is straightforward. But this is not the case for a FIR system.

Since we assume that the variance of measurement noise is unknown, the measurement noise is ignored during the derivation of the new approach. From Eq.(2), it follows that

$$x_t = y_t - \sum_{l=1}^{L} a_l x_{t-l}. \tag{5}$$

In the above equation, only y_t is known.

From Eq.(5), it seems that we cannot obtain an expression such as Eq.(4) for the input signal x_t. This problem can be solved naturally by exploiting the technique used in the recursive nonlinear least-square algorithm.[18–20]

Since the true channel parameters are unknown, in contrast to Eq.(5), we have

$$\hat{x}_t = y_t - \sum_{l=1}^{L} \hat{a}_l \hat{x}_{t-l} + \varepsilon_t. \tag{6}$$

By minimizing the weighted sum of squares of the sequence $\{\hat{x}_t\}$, it is readily obtained that

$$\hat{x}_t = y_t - \sum_{l=1}^{L} \hat{a}_l \hat{x}_{t-l}, \tag{7}$$

$$u_t = \hat{x}_t - \sum_{l=1}^{L} \hat{a}_l u_{t-l}. \tag{8}$$

The gradient vector would be $\hat{x}_t u_t^{t-L+1}$. Considering the requirement (3), we should assign a large step size if $|\hat{x}_t| > M$; otherwise, we should take a very small step size. The proposed algorithm is summarized in Table 1.

Table 1. Variable-gain LMS algorithm.

Initialization: $\hat{a}^{(0)} = 0_{L\times 1}$; $\hat{x}_{-L} = \cdots = \hat{x}_{-1} = 0$; $u_{-L} = \cdots = u_{-1} = 0$.

For t=0, 1, $\cdots$ do

$$\hat{x}_{t+1} = y_{t+1} - (\hat{x}_t^{t-L+1})^{\mathrm{T}} \hat{a}^{(t)}, \ \hat{x}_t^{t-L+1} = [\hat{x}_t, \ \cdots, \ \hat{x}_{t-L+1}]^{\mathrm{T}}$$

$$u_{t+1} = \hat{x}_{t+1} - (u_t^{t-L+1})^{\mathrm{T}} \hat{a}^{(t)}, \ u_t^{t-L+1} = [u_t, \ \cdots, \ u_{t-L+1}]^{\mathrm{T}}$$

$$\hat{a}^{(t+1)} = \hat{a}^{(t)} + \lambda_t \hat{x}_{t+1} u_t^{t-L+1}$$

where

$$\lambda_t = \begin{cases} \max\left\{ \dfrac{1}{\|u_t^{t-L+1}\|^2}\left(1 - \dfrac{M}{|\hat{x}_t|}\right), \lambda^u \right\}, & \text{if } |\hat{x}_t| > M \\ \lambda^l, & \text{otherwise} \end{cases} \qquad 0 < \lambda^l \le \lambda^u$$

This algorithm requires about $4L$ multiplication operations at each iteration, i.e. its computational complexity is $O(L)$. The EKF-based algorithm has a complexity of $O(L^2)$.[11] In Table 2, we make an overall comparison of the EFK-based algorithm and our algorithm.

Table 2. Overall comparison of EKF-based algorithm and variable gain LMS based algorithm.

	EKF-based algorithm	Variable-gain LMS based algorithm
a priori knowledge	1. Nonlinearity of chaotic system; 2. Variance of the measurement noise; 3. Variance of the coefficient model noise.	Bound of the transmitted chaotic signal.
Applicability	Identification of MA system	1. Identification of MA system; 2. Identification of ARMA system.
Complexity	$O(N^2)$	$O(N)$
Stability	Potential non-stability problem	Always stable

For logistic or Chebysev sequences $\{u_t\}_{t=1}^{\infty}$, there exist some constants $0 < T^l < T^u < \infty$ such that $T^l \le \|u_t^{t-L+1}\| \le T^u$ for all t.

Assume that the bifurcation parameter is always in the chaotic regime, and $L \ge 2$. The right inequality is very straightforward since we suppose that $\|u_t\| \le M$. Hence only the left inequality needs to be proved. At instant t, if $|u_t|$ is very small, assumed to be less than 0.1, then $|u_{t+1}| > 0.1$ and $\|u_{t+1}^t\| > 0.1$. It leads to $\|u_t^{t-L+1}\| > 0.1$.

Intuitively, if $\|\hat{x}_t^{t-L+1}\|$ is very small, the channel parameters cannot be estimated reliably. Fortunately, the above discussion indicates that $\|\hat{x}_t^{t-L+1}\|$ is not too small and supplies enough information for extracting a reliable estimate of channel parameter.

Remark 1 It can be observed that the proposed algorithm acts like the conventional LMS with a fixed small step size when $|\hat{x}_t| \le M$. On the other hand,

the algorithm acts more like the normalized LMS algorithm when $|\hat{x}_t| \gg M$.

In fact, the proposed algorithm can be extended to cope with ARMA model. The extended algorithm is presented in Table 3.

Table 3. Variable-gain LMS algorithm for ARMA model.

ARMA Model:

$$y_t = \sum_{i=1}^{p} a_i y_{t-i} + \sum_{i=0}^{q} b_i x_{t-i};\ b_0 = 1;\ \theta = [a_1\ \cdots\ a_p\ b_1\ \cdots\ b_q]^{\mathrm{T}}$$

Initialization:

$$\hat{a}^{(0)} = 0_{p\times 1};\ \hat{b} = 0_{q\times 1};\ u_{-q} = \cdots = u_{-1} = 0;\ w_{-q} = \cdots = w_{-1} = 0;\ \hat{x}_{-q} = \cdots = \hat{x}_{-1} = 0$$

For $t = 0, 1, \cdots$ do

$$\hat{x}_{t+1} = y_{t+1} - [(y_t^{t-p+1})^{\mathrm{T}}\ (\hat{x}_t^{t-q+1})^{\mathrm{T}}]\hat{\theta}^{(t)},\ \hat{x}_t^{t-q+1} = [\hat{x}_t,\ \cdots, \hat{x}_{t-q+1}]^{\mathrm{T}}$$

$$u_{t+1} = y_{t+1} - (u_t^{t-q+1})^{\mathrm{T}}\hat{b}^{(t)},\ u_t^{t-L+1} = [u_t,\ \cdots,\ u_{t-q+1}]^{\mathrm{T}}$$

$$w_{t+1} = \hat{x}_{t+1} - (w_t^{t-q+1})^{\mathrm{T}}\hat{b}^{(t)},\ w_t^{t-L+1} = [w_t,\ \cdots,\ w_{t-q+1}]^{\mathrm{T}}$$

$$\phi_t = [u_t\ \cdots\ u_{t-p+1}\ w_t\ \cdots w_{t-q+1}]^{\mathrm{T}}$$

$$\hat{\theta}^{(t+1)} = \hat{\theta}^{(t)} + \lambda_t \hat{x}_{t+1}\phi_t$$

where

$$\lambda_t = \begin{cases} \max\left\{\dfrac{1}{\|\phi_t\|^2}\left(1 - \dfrac{M}{|\hat{x}_t|}\right), \lambda^u\right\}, & \text{if } |\hat{x}_t| > M \\ \lambda^l, & \text{otherwise} \end{cases} \qquad 0 < \lambda^l \le \lambda^u$$

4. Simulation results

In this section, simulation results are presented to evaluate the proposed algorithm.

Example 1 Consider a chaotic communication system, in which the chaotic transmitter is the logistic map

$$x_n = \lambda x_{n-1}(1 - x_{n-1}),$$

where the bifurcation parameter λ is controlled in the chaotic regime [3.7 4.0]. The signal of transmission x_n is assumed to pass through a third-order FIR channel $a = [1,\ 0.45,\ -0.22]$. The mean square error (MSE) between the true channel coefficients and the estimated channel parameters is exploited to measure the identification performance. Figure 1 shows the steady-state MSE of the proposed approach and EKF-based approach under various SNR. About 3dB performance degradation is paid for the complexity reduction. Our algorithm enjoys another advantage that it does not require any *a priori* information about the nonlinearity and variance of the coefficient model noise. Figure 2(a) shows the convergent process of the proposed algorithm. In Fig.2(b) we present the results obtained by using the normalized LMS algorithm (NLMS). Since no chaotic characteristic is exploited, NLMS algorithm is inferior to the proposed algorithm. Figure 3 shows the error in the equalized data.

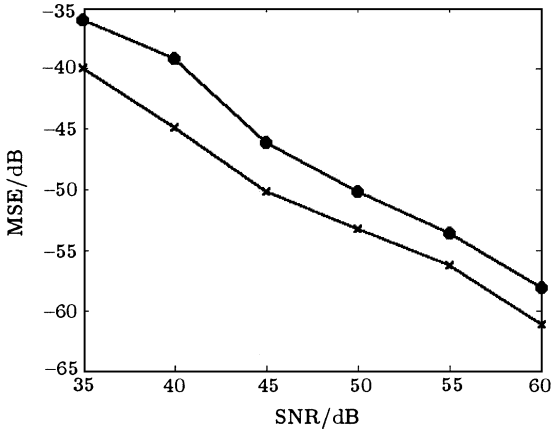


Fig.1. MSE of EKF-based algorithm and the proposed approach at six different SNR levels. Circle—variable gain LMS algorithm, "×"—EKF-based approach.

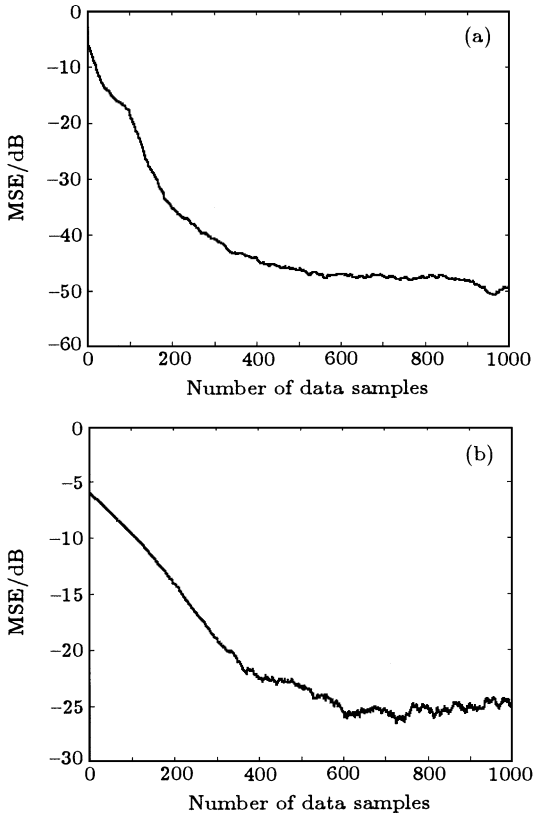


Fig.2. The convergent process of the proposed algorithm and NLMS algorithm, SNR=50dB. (a) the proposed algorithm, (b) normalized LMS algorithm.

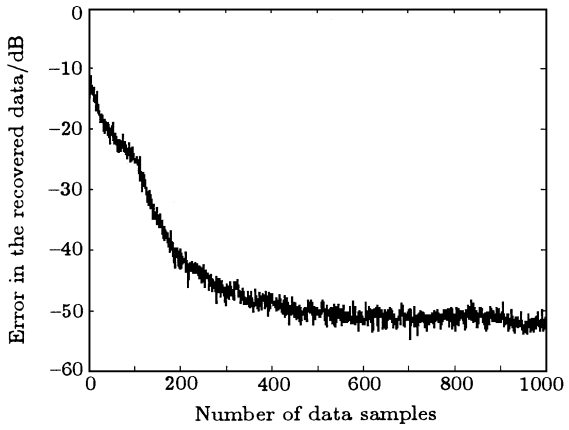


Fig.3. Error in the equalized data, SNR=50dB.

Quite interestingly, it is noted that the proposed algorithm is very robust to the overestimation of channel order. In Fig.4, we plot the MSE corresponding to three different estimated channel orders. No obvious degradation takes place when the channel order is overestimated to be $L+2$. For an AR system, the performance degradation was observed.[13]

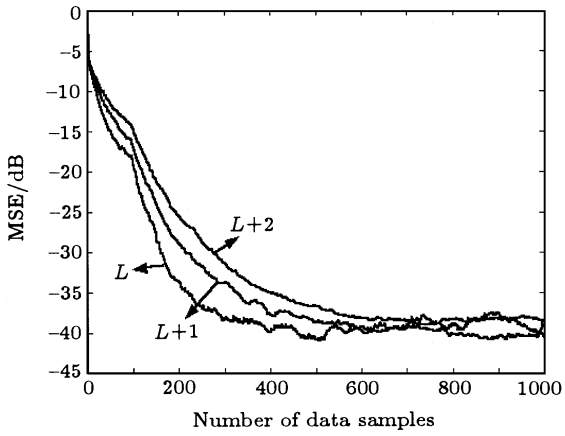


Fig.4. Performance of the proposed algorithm with the estimated channel order, SNR=40dB.

Example 2 In this example, we consider the blind identification of an ARMA model with a Henon chaotic sequence as input signal,

$$\begin{aligned} z_t =& 0.195z_{t-1} - 0.95z_{t-2} + x_t \\ &+ 0.45x_{t-1} - 0.22x_{t-2}, \\ y_t =& z_t + n_t. \end{aligned}$$

The algorithm shown in Table 3 is applied to the observations y_t. The MSE is plotted in Fig.5 as a solid curve. Also included in the figure are the MSE due to AR parameters and that due to MA parameters. It is observed that the proposed algorithm can effectively identify the ARMA model. The estimates of AR parameters are more accurate than the estimate of MA parameters.

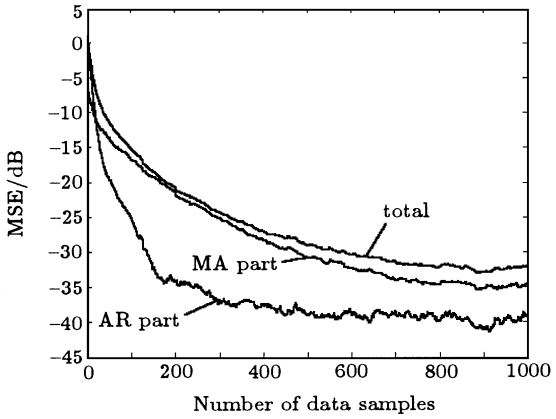


Fig.5. Performance of the proposed algorithm in identifying ARMA model, SNR=45dB.

5. Conclusion

In this paper a blind adaptive algorithm has been proposed to blindly identify FIR channel in chaotic communications. The bounded property of chaotic

signals is exploited. Compared with the EKF-based approach, the proposed algorithm has achieved a large complexity gain but at the expense of the accuracy degradation. However, our approach requires less *a priori* information. Knowledge about the nonlinearity of chaotic dynamics and the variance of coefficient model noise is not required. The simplicity and high accuracy make our approach applicable in the receiver design of chaotic communication systems.

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