Vol 13 No 2, February 2004
1009-1963/2004/13(02)/0234-04

Chinese Physics

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# Reduced quantum fluctuation in mesoscopic Josephson junction with nonclassical radiation field at finite temperature*

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(Received 6 June 2003; revised manuscript received 14 July 2003)

We have investigated the reduced fluctuation properties in a mesoscopic Josephson junction with the squeezed state at a finite temperature. It is shown that the fluctuations increase with increasing temperature and the mesoscopic Josephson junction subsystem can exhibit squeezing behaviour at an appropriately low temperature.

**Keywords:** mesoscopic Josephson junction, squeezed thermal state, quantum fluctuation
**PACC:** 7450, 4250

## 1. Introduction

In recent years, the mesoscopic Josephson junction(MJJ) has been widely studied.[1−3] There have been many new quantum phenomena observed in the systems, such as Coulomb blockade of Cooper pair tunnelling, Bloch oscillations and Zener tunnelling. Recently, the interaction of a quantized electromagnetic field (QEMF) with MJJ has been investigated [4−6] and some interesting results such as Shapiro steps,[4] supercurrent fluctuation,[5] the collapse and revival (CR) phenomenon and DC components of the supercurrent [6] have been obtained.

At the same time the nonclassical properties of the QEMF have been a centre of interest in quantum optics.[7] The squeezing of quantum fluctuations is one of the most fundamental manifestations of the Heisenberg uncertainty relation, which is one of the most important principles of quantum mechanics. It has been shown theoretically and experimentally that the squeezed field can be generated by various nonlinear systems such as Jaynes–Cummings model,[8−12] cooperative Dicke system,[13−15] etc. Recently, some authors [16−18] have studied the squeezing effect of external QEMFs acting on MJJs. However, most research is focused on the fluctuation of QEMF at zero temperature. In fact, any practical interaction of QEMF with MJJ is in a finite temperature environment, so the effect of temperature should be taken into account in calculating quantum fluctuations of the MJJ subsystem. In the present paper we explore the reduced quantum fluctuation in MJJ with squeezed states at a finite temperature, making use of thermal field dynamics (TFD).[19] Our results show that the fluctuations are related to the temperature.

The paper is organized as follows. In Section 2, the coupled field-junction system in MJJ is given. The reduced fluctuation properties in MJJ subsystem are investigated in Section 3. Section 4 contains a brief discussion and conclusion.

## 2. Coupled field-junction system

In this paper a circular superconducting ring with a MJJ is considered which is coupled to an external single-mode QEMF. The Hamiltonian describing the coupled field–junction system is (in units $\hbar = k_{\rm B} = c = 1$)

$$H = \frac{1}{2C}(q-Q)^2 + E_{\rm J}[1-\cos(2e\phi)] + \frac{1}{2L}(\phi-\Phi)^2, \quad (1)$$

where $E_{\rm J}$ is the Josephson coupling energy, $L$ is the inductance of the ring, $q$ is a charge tunnelling through the junction, $Q$ is the external charge which depends on the coupling between the junction and the external field, $\phi$ is the total magnetic flux through the ring, $\Phi$

*Project supported by the Natural Science Foundation of Education Bureau of Jiangsu Province, China(Grant No 01KJD140003).
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is the external flux and the charge $2e$ comes from the Cooper pairs.

The MJJ has a very small capacitance $C$ and operates at a very low temperature. As a sensitive quantum mechanical device, the flux $\phi$ and charge $q$ are a pair of quantum mechanical conjugate variables and satisfy the commutation relation $[\phi, q]$=i. The external charge $Q$ and the external magnetic flux $\Phi$ are also conjugated quantum variables satisfying the commutation relation $[\Phi, Q]$=i. We consider the case of small inductance, i.e. $e^2 E_{\rm J} << L^{-1}$, which means that the second term in Eq.(1) is small in comparison with the third term. From the Hamiltonian (1), we obtain the time evolution of $q, \phi, Q$ and $\Phi$ in the Heisenberg picture

$$\frac{{\rm d}q(t)}{{\rm d}t} = {\rm i}[H,q(t)] = -\frac{1}{L}[(4\lambda+1)\phi(t) - \Phi(t)], \quad (2)$$

$$\frac{{\rm d}\phi(t)}{{\rm d}t} = -\frac{{\rm d}\Phi(t)}{{\rm d}t} = \frac{1}{C}[q(t) - Q(t)], \quad (3)$$

$$\frac{{\rm d}Q}{{\rm d}t} = -\frac{1}{L}[\phi(t) - \Phi(t)], \quad (4)$$

where $\lambda = e^2 E_{\rm J} L$. The time evolution operators can be given by solving the above Heisenberg equations of motion. Using the initial conditions $q = q(0)$, $\phi = \phi(0)$, $Q = Q(0), \Phi = \Phi(0)$, we can express the variable $q, \phi, Q$ and $\Phi$ in terms of the field operators $a, a^+$ and the junction operators $b, b^+$ in the forms [14–16]

$$q = {\rm i}\sqrt{\frac{\omega' C}{2}}(b^+ - b), \quad \phi = \frac{1}{\sqrt{2\omega' C}}(b^+ + b), \quad (5)$$

$$Q = {\rm i}\sqrt{\frac{\omega C}{2}}(a^+ - a), \quad \Phi = \frac{1}{\sqrt{2\omega C}}(a^+ + a), \quad (6)$$

where $\omega' = \sqrt{1+4\lambda}/\sqrt{LC}$ is formally regarded as the angular frequency of the junction and $\omega = 1/\sqrt{LC}$ is the angular frequency of the single-mode radiation field. The field operators $a, a^+$ and the junction operators $b, b^+$ satisfy the usual Boson commutation relations

$$[a, a^+] = 1, \quad [b, b^+] = 1, \quad [a, b] = 0. \quad (7)$$

To investigate the nonclassical behaviours of the Josephson junction subsystem, we introduce a pair of quadrature operators of the junction in the forms

$$X_1 = \frac{b + b^+}{2}, \quad X_2 = \frac{b - b^+}{2{\rm i}}, \quad (8)$$

they satisfy the commutation relation $[X_1, X_2] = {\rm i}/2$. For small values of $\lambda$, we can neglect the higher order terms of $\lambda$ and obtain the time-dependent operator $X_{\rm J}(t)$ in the form [16–18]

$$X_{\rm J}(t) = A_{\rm J} a + A_{\rm J}^* a^+ + B_{\rm J} b + B_{\rm J}^* b^+, \quad (J = 1, 2), \quad (9)$$

where

$$A_1 = \frac{1}{4}\left[1 - \cos(\eta t) + {\rm i}\left(1 + \frac{1}{2}\lambda\right)\sin(\eta t)\right], \quad (10)$$

$$A_2 = \frac{1}{4}\left[\left(1 - \frac{1}{2}\lambda\right)\sin(\eta t) - \lambda\eta t - {\rm i} + {\rm i}\cos(\eta t)\right], \quad (11)$$

$$B_1 = \frac{1}{4}\Big[(1-\lambda) + (1+\lambda)\cos(\eta t) - {\rm i}\left(1 + \frac{3}{2}\lambda\right)\sin(\eta t)\Big], \quad (12)$$

$$B_2 = -\frac{1}{4}\Big[\left(1 + \frac{\lambda}{2}\right)\sin(\eta t) + \lambda\eta t + {\rm i}(1-\lambda) + {\rm i}(1+\lambda)\cos(\eta t)\Big], \quad (13)$$

and

$$\eta = (2 + \lambda)\omega. \quad (14)$$

## 3. Reduced quantum fluctuation in MJJ subsystem

In order to investigate the reduced quantum fluctuation of junction variables at a finite temperature, it is useful to consider a thermal state instead of a usual quantum state in MJJ. According to TFD, the thermal state (also called the "thermofield vacuum") is defined by [20,21]

$$|0\rangle_{\rm T} = T(\theta)|0\widetilde{0}\rangle_a, \quad (15)$$

where $|0\widetilde{0}\rangle_a$ is the usual vacuum state of the radiation field in a doubled Hilbert space, and the thermalizing operator is

$$T(\theta) = \exp[-\theta(a\widetilde{a} - a^+\widetilde{a}^+)]. \quad (16)$$

The additional mode (denoted by the tilde on the operators) is called the "fictitious" mode: it is introduced purely as an aid to calculation, and has no physical significance. The operator $T(\theta)$ defines a canonical Bogoliubov transformation,

$$\begin{aligned} T^+(\theta) a T(\theta) &= u a + v\widetilde{a}^+, \\ T^+(\theta) a^+ T(\theta) &= u a^+ + v\widetilde{a}, \end{aligned} \quad (17)$$

which mixes normal and tildian degrees of freedom, here

$$\begin{aligned} u &= \cosh\theta = [1 - \exp(-\omega/T)]^{-1/2}, \\ v &= \sinh\theta = [\exp(\omega/T) - 1]^{-1/2}. \end{aligned} \quad (18)$$

The parameter $\theta$ is related to $n_0$, the number of thermal photons, by

$$n_0 = {}_{\rm T}\langle 0|a^+a|0\rangle_{\rm T} = \sinh^2\theta. \tag{19}$$

We assume that the field is initially in a squeezed thermal state [21]

$$|\alpha, z\rangle_{\rm T} = D(\alpha)S(z)|0\rangle_{\rm T} = D(\alpha)S(z)T(\theta)|\widetilde{00}\rangle_a, \tag{20}$$

where $D(\alpha) = \exp(\alpha a^+ - \alpha^* a)$ is the displacement operator and $\alpha = |\alpha|\exp(\mathrm{i}\eta)$ is the displacement parameter, $S(z) = \exp[(za^{+2} - z^*a^2)/2]$ is the squeeze operator and $z = r\exp(\mathrm{i}\beta)$ is the squeeze parameter; the Josephson junction is initially in its lowest energy state (the vacuum state) $|0\rangle_b$. At $t = 0$, the field and junction operators are uncorrelated, and the interaction between field and MJJ takes place at $t > 0$. At an arbitrary time the state of the coupled system can be expressed as

$$|\varphi(t)\rangle = \mathrm{e}^{\mathrm{i}Ht}\ |\alpha, z\rangle_{\rm T} \otimes |0\rangle_b. \tag{21}$$

From Eqs.(8) (9), (20) and (21) we can obtain the variance

$$\begin{aligned}\langle(\Delta X_{\rm J})^2\rangle =& \langle X_{\rm J}^2\rangle - \langle X_{\rm J}\rangle^2 \\ =& 2|A_{\rm J}|^2\left(n_0\cosh 2r + \sinh^2 r\right. \\ &\left. + |\alpha|^2 + \frac{1}{2}\right) + |B_{\rm J}|^2 \\ &+ A_{\rm J}^2\left[\mathrm{e}^{\mathrm{i}\beta}\sinh 2r\left(n_0 + \frac{1}{2}\right) + \alpha^2\right] \\ &+ A_{\rm J}^{*2}\left[\mathrm{e}^{-\mathrm{i}\beta}\sinh 2r\left(n_0 + \frac{1}{2}\right) + \alpha^{*2}\right] \\ &- (A_{\rm J}\alpha + A_{\rm J}^*\alpha^*)^2. \end{aligned} \tag{22}$$

When the variance (22) is reduced below the level associated with the vacuum state, i.e. if $\langle(\Delta X_{\rm J})^2\rangle < 1/4$, there will be squeezing in the $X_{\rm J}$ quadrature. In order to see an apparent squeezing phenomenon, we consider a special time

$$\omega t = \frac{\pi}{2 + \lambda}, \tag{23}$$

then we obtain

$$A_1 = \frac{1}{2}, \quad A_2 = -\frac{1}{4}(\pi\lambda + 2\mathrm{i}), \tag{24}$$

$$B_1 = -\frac{\lambda}{2}, \quad B_2 = -\frac{\lambda}{4}(\pi - 2\mathrm{i}). \tag{25}$$

From Eqs.(22), (24) and (25), neglecting the second order terms of $\lambda$, we obtain

$$\langle(\Delta X_1)^2\rangle = \frac{1}{4}(2n_0+1)(\sinh 2r\cos\beta + \cosh 2r), \tag{26}$$

$$\begin{aligned}\langle(\Delta X_2)^2\rangle =& \frac{1}{4}(2n_0+1)[\cosh 2r - \sinh 2r \\ &\times(\pi\lambda\sin\beta + \cos\beta)]. \end{aligned} \tag{27}$$

From Eqs.(26) and (27), it can be seen that the quantum fluctuations of the junction subsystem depend on the temperature. Clearly, the fluctuations increase with increasing temperature. For Eq.(26), when $\beta = \pi$, we find that the condition of exhibiting squeezing in the $X_1$ quadrature, for fixed $r$, is

$$n_0 < \frac{\mathrm{e}^{2r} - 1}{2}. \tag{28}$$

It is easily found that no squeezing in $X_2$ when $\beta = \pi$. Similarly, for Eq.(27), we find that the condition of exhibiting squeezing in $X_2$ has been given in Eq.(28) when $\beta = 0$.

# 4. Conclusion

In this paper we have studied the reduced quantum fluctuation properties in MJJ subsystem at a finite temperature. It is shown that, if the field initially prepared in a squeezed thermal state and the Josephson junction initially in its lowest energy level state, the fluctuations depend on the temperature and increase as the temperature increases. It is also found that the MJJ subsystem can exhibit squeezing behaviour under an appropriately low average thermal photon number.

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