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The persistent current in an Aharonov–Bohm ring with a side-coupled quantum dot

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We have investigated the persistent current in a mesoscopic ring with a side-coupled quantum dot. The problems are probed by using the one-impurity Anderson Hamiltonian and are treated with the slave boson mean field theory. It is shown that the persistent current in this system has the spin fluctuations, and the charge transfers between the two subsystems are suppressed in the limit of $\Delta/T_{\rm K}^0 \ll 1$. The minimum value of the persistent current for $\xi_{\rm K}/L = 5$ of the odd parity system provides an opportunity to detect the Kondo screening cloud.

Keywords: quantum dot, Kondo effect, persistent current, Kondo screening cloud
PACC: 7000, 7335, 7335C, 7127

1. Introduction

The Kondo effect is a well-known phenomenon in solid state physics.[1] Initially the problem was studied to detect the enhancement of resistivity of magnetic impurities imbedded in a metallic host. With the progress in the nanofabrication technique of electronic devices, we can investigate the Kondo effect by means of quantum dots (QDs) [1] in a controllable way. QDs are small droplets of electrons confined in a small region in a semiconductor material with a size of about 100 nm.[1] The persistent current (PC) in an A–B ring [1−4] induced by the magnetic flux is one of the most exciting and intensively studied subjects. The QD coupled to a ring can mimic an impurity in the metallic host, which is studied extensively by many groups of scientists all over the world.[3−14] For a ring with an inserted QD, the Kondo effect has the signature of an enhancement of the persistent current because the Kondo resonance at the Fermi level provides a new channel for electric current flowing through the QD. For a ring with a side-coupled QD, the PC is very sensitive to the ratio between the length of the ring and the size of the Kondo screening cloud. The renormalization-group scaling approach in all orders of the perturbation was used in Ref.[4] to calculate the current for various values of $\xi_{\rm K}/L$ ($\xi_{\rm K}$ is the Kondo correlation length). By using the slave boson mean field approximation (MFA),[15] Ref.[5] drew the conclusion that I equals I_0 approximately in the limit of $\Delta/T_{\rm K}^0 \to 0$, and examined the effect of spin fluctuations on the PCs and their phase-coherent properties. The PC gave a nontrivial algebraic suppression to the charge fluctuations in the continuum limit of ring energy levels in Ref.[6]. The conductance of a quantum wire with a side-coupled quantum dot [16,17] was discussed also in Refs.[7,8].

In this paper, we study theoretically the equilibrium properties of a QD side-attached to a mesoscopic ring threaded by an A–B flux. The PC is calculated as a function of the level spacing Δ and the total numbers of electrons $N_{\rm e}$, and as a universal scaling function of $\xi_{\rm K}/L$. The results are presented in Section 3 in the limit of $U \to \infty$, T=0. The fluctuations affect the PC and the phase-coherent properties. Whether it is paramagnetic or not depends on the parity of the system.

2. Model and solution

A mesoscopic ring threaded by an A–B flux in our study can be described by a ($N_\sigma = 2$)-fold degenerate one impurity Anderson Hamiltonian.[8,16] The Hamiltonian can be written in terms of auxiliary slave boson operators [9,10,15] plus constraints in the limit $U \to \infty$

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(U is the Coulomb repulsion in the QD):

$$H = \sum_{m\sigma} \varepsilon_m c^+_{m\sigma} c_{m\sigma} + \sum_{\sigma} \varepsilon_d f^+_\sigma f_\sigma + \sum_{m\sigma} (t_m c^+_{m\sigma} b^+ f_\sigma + \text{H.C.}) + \lambda\left(b^+ b + \sum_\sigma f^+_\sigma f_\sigma - 1\right). \quad (1)$$

The ring is described by a one-dimensional tight-binding model with N_r lattice sites, $c^+_{m\sigma}(c_{m\sigma})$ is the creation (annihilation) operator for the electrons in the ring, and λ is Lagrange multiplier, ε_d represents the single particle energy in the QD. $\varepsilon_m = -2t\cos\left[\frac{2\pi}{N_\text{r}}(m+\phi)\right]$, where m is an integer within the interval $[-(N_\text{r}-1)/2, N_\text{r}/2]$, the phase factor ϕ is defined by $\phi = \Phi/\Phi_0$ with Φ and Φ_0 being the magnetic flux and the flux quantum ($\Phi_0 = hc/e$), and t is the hopping integral between the nearest-neighbouring sites, where $t_m = -t'/\sqrt{N_\text{r}}$, t' is a tuned-matrix element, describing the hopping between the dot and the site "0" of the ring.

In the slave boson representation, the dot level is represented as $d^+ = f^+_\sigma b$, where the pseudo-fermion operator f^+_σ and the slave boson operator b^+ describe the singly occupied and empty dot states, respectively. Since the dot is either empty or singly occupied, the constraint $b^+b + \sum_\sigma f^+_\sigma f_\sigma = 1$ has to be fulfilled.

In the MFA, the slave boson operator b is replaced by a constant number $b_0 =< b(t) >$, neglecting the fluctuation around the average $< b(t) >$ of the slave boson.[9,10] At T=0, this MFA is correct only for describing the spin fluctuations (Kondo regime). In the chosen gauge in Eq.(1), the ring electron contribution to the free energy does not depend on the phase factor and can be omitted. So we start from the impurity part of the free energy, which is given by

$$F = \frac{N_\sigma}{\pi}\int_{-\infty}^{\infty} f(\omega)\text{Im}\ln G_f(\omega^+)\text{d}\omega + (\bar{\varepsilon}_d - \varepsilon_d)\left(\frac{\Delta}{\Delta_0} - 1\right), \quad (2)$$

where $f(\omega)$ is the Fermi–Dirac distribution function, $\bar{\varepsilon}_d = \varepsilon_d + \lambda$, $\Delta = b_0^2\Delta_0$, and $\Delta_0 = \pi\rho(\varepsilon_f)\,|t(\varepsilon_f)|^2$, where Δ_0 is the bare resonance level width characterizing the coupling strength between the QD and the ring. $\rho(\varepsilon_f)$ and $t(\varepsilon_f)$ are the density of states and the hopping amplitude at the Fermi energy level, respectively. For the half-filled case, ($\varepsilon_f = 0$), we have $N_\text{r} = N_\text{e}$, $\rho(\varepsilon_f) = 1/\delta = N_\text{r}/2\pi$. Here δ is the mean level spacing in the ring. $\rho(\varepsilon_f)$ is the density of states at the highest occupied level energy ε_f of the ring. $G_f(\omega^+)$ is the pseudo-fermion f_σ retarded propagator given by

$$G_f(\omega^+) = \left(\omega^+ - \bar{\varepsilon}_d - \frac{\Delta}{\Delta_0}\sum_m \frac{|t_m|^2}{(\omega^+ - \varepsilon_m)}\right)^{-1}. \quad (3)$$

The PC is defined by the relation

$$I(\phi, T) = -\frac{e}{h}\frac{\partial}{\partial\phi}F(\phi, T). \quad (4)$$

At zero temperature, by using the free energy in Eq.(2), and the retarded propagator in Eq.(3), the current in the ring is $I(\phi,T) = I_0(\phi,T) + I_\text{QD}(\phi,T)$. Here $I_0(\phi,T)$ denotes the current contributions of free electrons in the ring, expressed as

$$I_0(\phi, T) = \sum_{m\sigma} f(\varepsilon_m) I_m(\phi). \quad (5)$$

Contribution from each level is given by

$$I_m(\phi) = -\frac{2e}{h}\delta\sin\left[\frac{2\pi}{N_\text{r}}(m+\phi)\right]. \quad (6)$$

With the oscillation amplitude $\delta = 2\pi t/N_\text{r}$, we can obtain the following expression:

$$I_\text{QD}(\phi,T) = -N_\sigma\sum_m \frac{e}{h}\frac{4\pi t^2}{N_\text{r}^2} < b_0^2 > \times \sin\left[\frac{2\pi}{N_\text{r}}(m+\phi)\right] \times \frac{(\varepsilon_f + \lambda - \varepsilon_m)^2 - t'^4 < b_0^4 > /(4t^2)}{[(\varepsilon_f + \lambda - \varepsilon_m)^2 + t'^4 < b_0^4 > /(4t^2)]^2}. \quad (7)$$

The renormalized parameters $\bar{\varepsilon}_d$ and Δ give, respectively, the position and width of the resonance peaks in the QD, and the Kondo correlation energy is $T_\text{K} = \sqrt{\bar{\varepsilon}_d^2 + \Delta^2}$. At T=0, we obtain from minimizing F with respect to $\bar{\varepsilon}_d$ and Δ, the self-consistent set of the two equations with two unknowns ($\bar{\varepsilon}_d$ and Δ):

$$\frac{\partial F}{\partial\bar{\varepsilon}_d} = -\sum_k \frac{N < b_0^2 >< V_{dk}^2 >}{(\varepsilon_f + \lambda - \varepsilon_m)^2 + (< b_0^2 > \pi < V_{dk}^2 > \rho)^2} + < b_0^2 > -1,$$
$$\frac{\partial F}{\partial\Delta} = \sum_k \frac{(\varepsilon_f + \lambda - \varepsilon_m) * 2N < b_0 >< V_{dk}^2 >}{(\varepsilon_f + \lambda - \varepsilon_m)^2 + (< b_0^2 > \pi < V_{dk}^2 > \rho)^2} + 2\lambda < b_0 >, \quad (8)$$

where $\bar{\varepsilon}_d = \varepsilon_d + \lambda$, $\Delta/\Delta_0 = b_0^2$, $< V_{dk}^2 >= -t'/\sqrt{N_\text{r}}$.

3. Results and discussion

In the Kondo regime, the physical quantities in the system are universal scaling functions of ξ_K/L or

$\delta/T_{\rm K}^0$ ($\xi_{\rm K} = \hbar v_{\rm F}/T_{\rm K}^0$, where $v_{\rm F}$ is the Fermi velocity and $T_{\rm K}^0$ is the Kondo temperature,[5,13] L is the ring circumference),[18] and so we can solve numerically the set of Eq.(8) as functions of the phase factor for various values of $\xi_{\rm K}/L$.

From the conclusions of Ref.[19], we can obtain the current that depends on the number of electrons, with modulo 4. There are four classes i (i=0,1,2,3). For $I_0^{N_{\rm e}}$, $N_{\rm e}$ is defined as $N_{\rm e} = 4n - i$. The parameters are taken to be $\varepsilon_d = -0.7$, $\phi = 0.20$, $t' = \sqrt{0.3}$, $T_{\rm K}^0 = 6.033 \times 10^{-3}$, T=0, t=1.0 (all energies in units of t). We plot the $I/I_0 \sim \Delta/T_{\rm K}^0$ curves, as shown in Fig.1. The behaviour of I/I_0 depends strongly on which number class $N_{\rm e}$ belongs to. In the limit of $\Delta/T_{\rm K}^0 \ll 1$, for the four classes, we have $I = I_0$. When $\Delta/T_{\rm K}^0 > 0.5$, the difference is obvious.

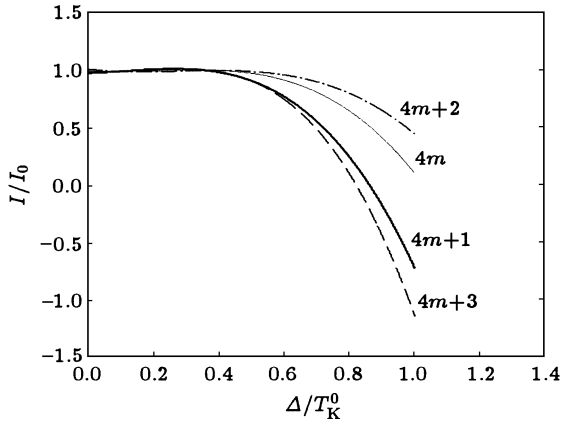


Fig.1. The persistent current as a function of the energy-level spacing of the ring.

The Kondo screening cloud theory [4,5,18,20] explains the properties. For $\Delta/T_{\rm K}^0 \ll 1$, in the Kondo regime, when the electrons of the ring drop to the QD, some electrons in the ring take part in forming the Kondo screening cloud, which compensates the trapped electrons on the dot. So the current I remains the same as I_0. If $\Delta/T_{\rm K}^0 \sim 1$, the current is positive paramagnetic for $N = 4m$ and $4m+2$, but is negative diamagnetic for $N = 4m + 1$ and $4m + 3$, which is the effect of the spin fluctuations on the ground state. But in Ref.[6], contrary to our opinions, no dependence on parity is presented. Of course, this is different from Ref.[5], where the direction of the current does not depend on parity, as we demonstrate here. For $\Delta/T_{\rm K}^0 > 1$, since the finite size effect suppresses the Kondo effect, the two systems decouple each other.

Figures 2 and 3 illustrate Ih/e–ϕ/π curves for different $\xi_{\rm K}/L$ values. We can conclude that the current is paramagnetic for even parity ($N_{\rm e} = 4m, 4m + 2$), and diamagnetic for odd parity ($N_{\rm e} = 4m+1, 4m+3$). There are phase translations by π between $N_{\rm e} = 4m$ and $N_{\rm e} = 4m + 2$, between $N_{\rm e} = 4m + 1$ and $N_{\rm e} = 4m + 3$, which correspond to the conclusions of Fig.1. Our conclusions are in agreement with those in Ref.[4], where the periodicity of the current is 2π, but it is 4π in our result.

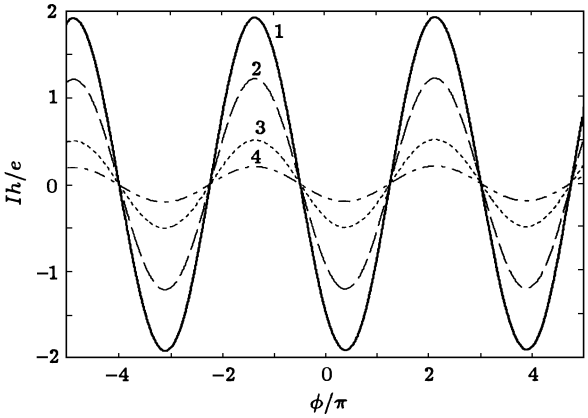


Fig.2. The current I_0 in an ideal mesoscopic ring and I in a ring with a side-coupled QD as functions of the phase factor Φ/π for N being an even number. Curve 1: $\xi_{\rm K}/L \ll 1$, curve 2: $\xi_{\rm K}/L = 0.15$, curve 3: $\xi_{\rm K}/L = 5$, curve 4: $\xi_{\rm K}/L = 15$.

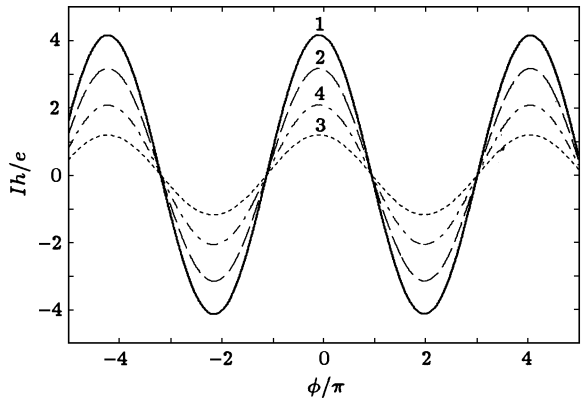


Fig.3. The current I_0 in an ideal mesoscopic ring and I in a mesoscopic ring with a side-coupled QD as functions of the phase factor Φ/π for N being an odd number. Curve 1: $\xi_{\rm K}/L \ll 1$, curve 2: $\xi_{\rm K}/L = 0.15$, curve 3: $\xi_{\rm K}/L = 5$, curve 4: $\xi_{\rm K}/L = 15$.

In Fig.2, the amplitude of the current drops from 2 to 0.3 for $N_{\rm e}$ even, when $\xi_{\rm K}/L$ increases from 0.15 to 15. But for $N_{\rm e}$ odd, as shown in Fig.3, as $\xi_{\rm K}/L$ increases from 0.15 to 5, the amplitude reduces from 4 to 1.3. With $\xi_{\rm K}/L$ increasing further, it begins to increase again. If $\xi_{\rm K}/L \ll 1$, due to the Kondo effect for both odd and even parity, the PC presents a sawtooth-like form perfect transmission, corresponding to an ideal ring without impurity. The amplitude

of the current for N_e odd is half of that for N_e even, as shown in Ref.[19]. When ξ_K/L increases, the current cuts down because of the interference between the ballistic channel and the resonant channel from the QD, implying the suppression of the Kondo-resonant transmission. When an electron drops to the QD, the N_e in the ring becomes $N_e - 1$. So the parity of the system becomes reversed, and the amplitude of the current for N_e even is half of that for N_e odd. The free electrons in the ring, which can compensate the loss of the charge partly, screen the hole originating from the trapped electrons. With increasing ξ_K/L, the Kondo effect becomes weaker due to the incomplete screening of the free electrons. For $N_e - 1$ odd, there is a single electron in the Fermi level, which forms a singlet with the QD. As a result, it screens these channels. So the current reduces with increasing ξ_K/L. For $N_e - 1$ even, no such singlet originates. The current has a minimum at $\xi_K/L = 5$; when ξ_K/L increases further, the current goes up again.

4. Conclusion

In conclusion, we have shown that the properties of the PC are determined by parameters ξ_K/L and δ/T_K^0 in a parity-dependent manner. For a ring with an odd number of electrons, the Kondo effect leads to a higher transmission than that of the even parity system. The minimum value of the PC for $\xi_K/L = 5$ of the odd parity system provides an opportunity to detect the Kondo screening cloud.

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