Vol 13 No 2, February 2004
1009-1963/2004/13(02)/0201-04

Chinese Physics

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Numerical study of the scaling of the maximum kinetic energy per unit length for imploding Z-pinch liner *

Zeng Zheng-Zhong(曾正中)† and Qiu Ai-Ci(邱爱慈)

Northwest Institute of Nuclear Technology, Xi'an 710024, *China*

(Received 7 March 2003; revised manuscript received 31 July 2003)

Numerical computation based on a zero-dimensional thin-plasma-shell model has been carried out to study the scaling of the maximum kinetic energy per unit length, the current amplitude and the compression ratio for the imploding Z-pinch liner driven by peaked current pulses. A dimensionless scaling constant of 0.9 with an error less than 10% is extracted at the optimal choice of the current and liner parameters. Deviation of the chosen experimental parameter from the optimal exerts a minor influence on the kinetic energy for wider-shaped and slower-decaying pulses, but the influence becomes significant for narrower-shaped and faster-decaying pulses. The computation is in reasonable agreement with experimental data from the Z, Saturn, Blackjack 5 and Qiangguang-I liners.

Keywords: Z-pinch, plasma x-ray sources
PACC: 5255E, 5250D

1. Introduction

Imploding plasma liners in the Z-pinch schemes [1−3] have proved to be capable of producing high power radiation in the soft x-ray waveband for applications in radiation and fusion physics research.[1−4] The x-ray power has been demonstrated to be dependent largely on the liner kinetic energy that is related in general to the liner parameters and the current flowing through the liner. It is therefore a general pursuit to obtain the largest possible kinetic energy under certain practical conditions, which demands a reasonable general evaluation of the kinetic energy scaling. Although the kinetic energy per unit liner length for a certain compression ratio can be derived theoretically to be proportional to the square of the current amplitude, a largely varying constant (from about 0.6 up to 1.0) is used in the scaling equation to account for the current pulse shape.[5−7] An argument can thus be raised that not only the current pulse shape, but also the current and liner parameter choice, dominate the dynamics of the plasma implosion. So the complex influence of current pulse shape and parameter choice on the scaling should be considered. In this work, detailed numerical computation is performed to gain an improved evaluation of the kinetic energy scaling.

2. Theoretical treatment

It is assumed that a uniform thin cylindrical plasma shell is created when the driving current begins to flow through the liner. This consideration can be quite reasonable for annular gas-puff liners and appropriately arranged annular wire-arrays that satisfy

$$d \cong 2kv_{\mathrm{p}}t_{\mathrm{m}}, \tag{1}$$

where d is the distance between two neighbouring wires, v_{p} the thermodynamic expansion speed of a single wire plasma generated by the current passing through the wire, t_{m} the time corresponding to the current maximum, and k=0.15 an empirical number. Equation (1) means that during the early phase of the current rise, the plasma columns from individual wires merge to produce a nearly uniform cylindrical plasma shell.[8]

Under this consideration, the kinetic equation for the zero-dimensional thin-plasma-shell model [7−9] can be written as

$$\mathrm{d}^2R/\mathrm{d}\tau^2 = -Ai^2(\tau)/R \tag{2}$$

with initial conditions $R(0) = 1$ and $\dot{R}(0) = 0$, where

*Project supported by the National Natural Science Foundation of China (Grant No 10035020).
†E-mail: zzzeng@nint.ac.cn

(in MKS units)

$$A = \mu_0 I_{\rm m}^2 t_{\rm m}^2/(4\pi m r_0^2) \tag{3}$$

is a factor depending on current and liner parameter choice, $m = M/l$ the linear specific mass of the shell with total mass M and shell length l, μ_0 the permeability in vacuum, $I_{\rm m}$ the current amplitude, r_0 the initial liner radius, $R = r/r_0$, $\tau = t/t_{\rm m}$, $i(\tau) = I(t)/I_{\rm m}$ with the instantaneous radius $r(t)$ of the shell, t the time since the current begins to rise, and $I(t)$ the instantaneous driving current. In order to be faithful to practical situations in which the inductance of the imploding shell changes with time, experiment-related $I(t)$ is used which is either taken directly from experimental waveforms or fitted from published experimental curves.

Integration of Eq.(2) with the initial conditions gives

$$({\rm d}R/{\rm d}\tau)^2 = 2A \int_{\tau}^{0} i^2(\tau')/R(\tau'){\rm d}R(\tau'), \tag{4}$$

resulting in the imploding kinetic energy per unit length as

$$\begin{aligned} E_{\rm ku} =& m({\rm d}r/{\rm d}t)^2/2 = m r_0^2({\rm d}R/{\rm d}\tau)^2/t_{\rm m}^2/2 \\ =& \mu_0 I_{\rm m}^2({\rm d}R/{\rm d}\tau)^2/(8\pi A) \\ =& \mu_0 I_{\rm m}^2 \int_{\tau}^{0} [i^2(\tau')/R(\tau')]{\rm d}R(\tau')/(4\pi) \\ =& \mu_0 I_{\rm m}^2 \int_{\tau}^{0} [i^2(\tau')/R(\tau')][{\rm d}R(\tau')/{\rm d}\tau']{\rm d}\tau'/(4\pi). \end{aligned} \tag{5}$$

Equation (5) can be rearranged into a scaling equation

$$E_{\rm ku}/I_{\rm m}^2 = \mu_0/(4\pi)\alpha \ln(r_0/r_\tau), \tag{6}$$

where r_τ is the liner radius at τ, and

$$\alpha = \left\{ \int_{\tau}^{0} [i^2(\tau')/R(\tau')][{\rm d}R(\tau')/{\rm d}\tau']{\rm d}\tau' \right\} / \ln(r_0/r_\tau) \tag{7}$$

is a dimensionless scaling constant. For $i\ (\tau')$=1, α=1; and for $0 < i\ (\tau') < 1$, $0 < \alpha < 1$ there is no general analytical solution.

Using units kJ/cm for $E_{\rm ku}$ and MA for $I_{\rm m}$, Eq.(6) can be rewritten as

$$E_{\rm ku}/I_{\rm m}^2 = \alpha \ln(r_0/r_\tau). \tag{8}$$

Equations (6) and (7) indicate that for a certain compression ratio r_o/r_τ, $E_{\rm ku}/I_{\rm m}^2$, and equally α, is not only the function of $i\ (\tau') \neq 0$ or 1, but also the function of A that acts to determine $R(\tau')$, ${\rm d}R(\tau')/{\rm d}\tau'$ and τ in Eq.(7) according to Eq.(2). For a peaked $i(\tau')$, a large A will lead to a small τ for r_0/r_τ, so that α can be small; a small A will give rise to a small ${\rm d}R(\tau')/{\rm d}\tau'$ so that α can be small again. Is there then an optimal A at which $E_{\rm ku}/I_m^2$ and α reach their maxima? Furthermore, what will be the difference between the values of the maximal $E_{\rm ku}/I_{\rm m}^2$ and α for different $i(\tau')$?

These questions are difficult to answer quantitatively in analytical form but can be solved numerically in fair detail. For a chosen waveform of $i(\tau')$, Eq.(2) is numerically solved for a certain A to obtain $R(\tau')$ and ${\rm d}R(\tau')/{\rm d}\tau'$. Then $E_{\rm ku}/I_{\rm m}^2$ and α are calculated from Eqs.(5) and (6) for a specific r_0/r_τ. The same procedure can be carried out with A changing in an appropriate range at a properly fine step. The maximal $E_{\rm ku}/I_{\rm m}^2$ and α will thus be determined.

3. Computational results

Numerical computations are carried out using the current waveforms from the experiments on the PBFA-*Z* (*Z*),[1] Saturn (ST),[2,10] Blackjack 5 (BJ) [11] and Qiangguang-I (QG) [12] pulsed power apparatus.

The calculated curves of α versus A under different conditions are plotted in Fig.1, and the experimental waveforms of $i(\tau)$ used for calculating these curves are shown in Fig.2.

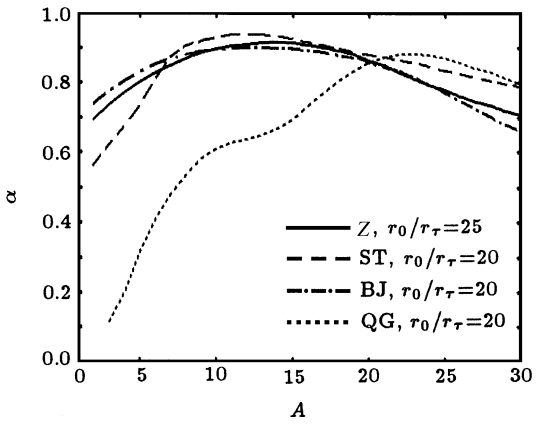


Fig.1. Calculated curves of α versus A for different compression ratios and driving current waveforms.

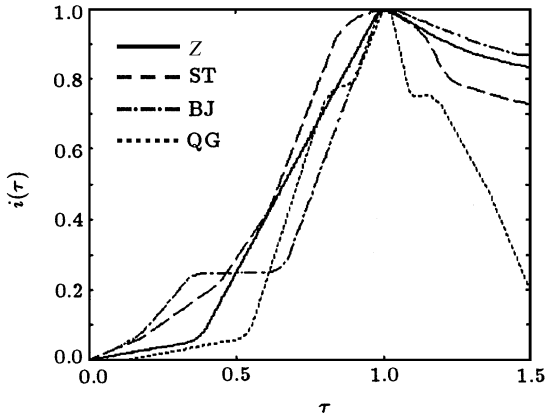


Fig.2. Experimental waveforms of $i(\tau)$ used to calculate curves shown in Fig.1.

From Fig.1 one can find that the maximal values of α for different r_0/r_τ and machines (current shapes) indeed exist and are fairly close to 0.9. Detailed computation shows that the maximal α can be rounded to 0.9 within 5% for the used waveforms in Fig.2 and for r_0/r_τ=10–25; in even larger range of r_0/r_τ and for more different waveforms (taking $i(\tau)$=sin($\pi\tau/2$); (1–cos($\pi\tau$))/2; τ ($\tau\leq 1$) and 1–(τ–1) ($\tau > 1$); and $\exp\left(-\frac{(\tau-1)^2}{2\times 0.33^2}\right)$ in addition to the experimental current pulses), the maximal α will not deviate in excess of 10% beyond 0.9 although the α curves themselves and the optimal values of A regarding the maximal α are quite different from each other.

The scaling of the maximum kinetic energy per unit length, $E_{\rm kum}$, the current amplitude, $I_{\rm m}$, and the compression ratio, $r_0/r_{\rm f}$, can then be found to be

$$\frac{E_{\rm kum}}{I_{\rm m}^2\ \ln(r_0/r_{\rm f})} = 0.9 \qquad (9)$$

for general evaluation purposes. This revelation of scaling indicates that a proper choice of A for a chosen compression ratio will make a peaked current nearly as effective as a constant current in driving thin Z-pinch liners.

4. Discussion and conclusion

Listed in Table 1 are the values of the predicted kinetic energy of the whole liner ($E_{\rm kp}$) from Eq.(8) and Fig.1 based on the experimental data in columns 2–6 and column 8, in comparison with the experimentally measured total kinetic energy ($E_{\rm ke}$) based on the measured imploding velocity ($v_{\rm fe}$, column 7) and the experimental data in the columns 4 and 6, and with the predicted maximum kinetic energy ($E_{\rm km}$) from Eq.(9) based on the data in columns 2, 6 and 8 for several experiments concerning peaked current pulses.[1,10–12] The predicted kinetic energy $E_{\rm kp}$ and $E_{\rm km}$ are close to each other and in reasonable agreement with the experimentally measured energy $E_{\rm ke}$ for Z, Saturn and Blackjack 5, indicating that these experiments are nearly optimal. For Qiangguang-I the situation is slightly complicated: $E_{\rm kp}$ could be in agreement with $E_{\rm ke}$ considering the measurement error of $v_{\rm fe}$ (taken as measured average value over 14 ns near the late phase of implosion, evaluated to be half as large as the instant velocity at the end of implosion),[12] while $E_{\rm ke}$ is far from $E_{\rm km}$ due to the mismatch between the relatively heavy liner (limited by the gas-puff system) and the relatively small driving current (limited by the pulsed power source). If $I_{\rm m}$ for Qiangguang-I can be raised up to 3 MA while keeping the other parameters unchanged, the value of A will then be much closer to its optimum, giving reasonable agreement between $E_{\rm ke}$ and $E_{\rm km}$.[12]

Table 1. Comparison of calculated total kinetic energy from Fig.1, Eq.(8) and Eq.(9) with experiment-related data. Columns 2–10 are experimental data, and columns 11–14 are calculated values.

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Experiment	$I_{\rm m}$ /MA	$t_{\rm m}$ /ns	m /mg·cm^{-1}	r_0 /cm	l /cm	$v_{\rm fe}$ /cm·μs^{-1}	$r_0/r_{\rm fe}$	A (used)	$E_{\rm ke}$ /kJ	$E_{\rm kp}$ /kJ	A (optimal)	$E_{\rm km}$ /kJ	τ for $r_0/r_{\rm fe}$ at optimal A
Z (W, wire-array)	17	140	2	2	2	92	25	7.1	1693	1587	13.9	1674	1.06
ST (W, wire-array)	6.5	57	0.45	0.88	2	65	20	3.9	190	170	12.1	228	1.03
BJ (Ar, gas-puff)	3.5	170	0.33	1.25	3	53	20	6.9	139	95	12.7	99	1.11
QG (Kr, gas-puff)	1.5	70	0.025	1.00	3	16*	20	4.4	1.0	5.3	21.9	18	1.02

* Measured average value over 14 ns near the late phase of implosion, which was evaluated to be half as large as the instantaneous velocity at the end of implosion because of the fast accelerating nature of imploding liner.[12]

From Table 1 and Fig.1, one can see that for a certain compression ratio the implosion time corresponding to the optimal value of A and thus to the maximal value of α is very close to and just a little later than the current peak. Bearing in mind Eq.(2) that determines the imploding dynamics, this indicates that a larger A than the optimal is related to the implosion taking place during the current wave front and a smaller A to that during the wave tail. The sharp drop of the α curves for ST and QG in the small A region can therefore be interpreted as being due to the sharp drop of the current at the wave tail. In the meantime, the deviation of the experimental A beyond the optimal exerts a minor influence on α for wider-shaped and slower-decaying pulses while the energy is more sensitive to the choice of A for narrower-shaped and faster-decaying pulses. Therefore, in the case of building a fast Z-pinch driving machine, the current pulse shape, in addition to the amplitude, should be considered with attention.

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