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Scheme for directly measuring the Wigner characteristic function via the driven Jaynes–Cummings model *

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A scheme is proposed for measuring the Wigner characteristic function of a cavity field. In the scheme an atom is sent through a slightly detuned cavity and driven by a strong resonant classical field. Then the population of the atom in the ground state directly yields the Wigner characteristic function of the cavity field.

Keywords: Wigner characteristic function, driven Jaynes–Cummings model, cavity field
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In recent years, there has been considerable interest in the preparation and reconstruction of quantum states. In cavity quantum electrodynamics (QED), schemes have been proposed for the generation of nonclassical states.[1−4] Lutterbach and Davidovich [5] have presented a scheme for direct measurement of the Wigner function in both cavity QED and ion traps, using the dispersive interaction. Following the scheme of Ref.[5], Bertet *et al*[6] have reported the measurement of the Wigner function of a cavity field.

On the other hand, a scheme for direct observation of the Wigner characteristic function of a cavity field was proposed by Wilkens and Meystre.[7] In the scheme the atom interacts with both the cavity field and a reference driving field. After the interaction the inversion of the atom is directly related to the Wigner characteristic function of the initial state of the cavity field. In a recent paper, Kim *et al*[8] have made a similar proposal for measuring the Wigner characteristic function of a cavity field, in which the cavity mode needed to be displaced by a larger magnitude before the atom–field interaction.

The scheme of Ref.[7] requires that the cavity field and the classical field be at exact resonance and thus the classical field may couple to the cavity mode, which may cause the scheme invalid. In this paper we propose an alternative scheme for the measurement of the Wigner characteristic function of a cavity field. In our scheme the atom interacts with a slightly off-resonant cavity field and a strong resonant classical field. As the cavity field and the classical field are not at exact resonance the drawback of Ref.[7] is overcome. In contrast with the scheme of Ref.[8], our scheme does not require the displacement of the cavity field before the atom–field interaction.

We consider a two-level atom interacting with a single-mode cavity field and being driven by a classical field simultaneously. In the rotating-wave approximation, the Hamiltonian is (assuming $\hbar = 1$) [9,10]

$$H = \omega_0 S_z + \omega_a a^+ a + \frac{g}{2}(a^+ S^- + a S^+) + \frac{\Omega}{2}(S^+ \mathrm{e}^{-\mathrm{i}\omega t - \mathrm{i}\phi} + S^- \mathrm{e}^{\mathrm{i}\omega t + \mathrm{i}\phi}), \quad (1)$$

where $S^+ = |e\rangle\langle g|$, $S^- = |g\rangle\langle e|$, $S_z = \frac{1}{2}(|e\rangle\langle e| - |g\rangle\langle g|)$, with $|e\rangle$ and $|g\rangle$ being the excited and ground states of the atom, a^+ and a the creation and annihilation operators for the cavity mode, g the atom–cavity coupling strength, ω_0 the atomic transition frequency, ω_a the cavity frequency, and ω, Ω, and ϕ the frequency, Rabi frequency, and phase of the classical field, respectively. Assume that $\omega_0 = \omega$. Then the interaction Hamiltonian, in the interaction picture, is

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written as

$$H_{\rm i} = \frac{g}{2}(\mathrm{e}^{-\mathrm{i}\delta t}a^{+}S^{-} + \mathrm{e}^{\mathrm{i}\delta t}aS^{+}) + \frac{\Omega}{2}(S^{+}\mathrm{e}^{-\mathrm{i}\phi} + S^{-}\mathrm{e}^{\mathrm{i}\phi}), \quad (2)$$

where $\delta = \omega_0 - \omega_a$. The free Hamiltonian $\omega_0 S_z + \omega_a a^{+}a$ is used for the transformation to the interaction picture. Define a new atomic basis

$$|+\rangle = \frac{1}{\sqrt{2}}(|g\rangle + \mathrm{e}^{-\mathrm{i}\phi}|e\rangle),$$
$$|-\rangle = \frac{1}{\sqrt{2}}(|g\rangle - \mathrm{e}^{-\mathrm{i}\phi}|e\rangle). \quad (3)$$

Then we can rewrite $H_{\rm i}$ as

$$H_{\rm i} = \left\{\frac{g}{2}\left[\mathrm{e}^{-\mathrm{i}\delta t}a^{+}\left(\sigma_z + \frac{1}{2}\sigma^{+} - \frac{1}{2}\sigma^{-}\right)\mathrm{e}^{-\mathrm{i}\phi} + \mathrm{e}^{\mathrm{i}\delta t}a\left(\sigma_z + \frac{1}{2}\sigma^{-} - \frac{1}{2}\sigma^{+}\right)\mathrm{e}^{\mathrm{i}\phi}\right] + \Omega\sigma_z\right\}, \quad (4)$$

where $\sigma_z = \frac{1}{2}(|+\rangle\langle+| - |-\rangle\langle-|)$, $\sigma^{+} = |+\rangle\langle-|$, and $\sigma^{-} = |-\rangle\langle+|$.

The time evolution of this system is decided by the Schrödinger equation

$$\mathrm{i}\frac{\mathrm{d}|\psi(t)\rangle}{\mathrm{d}t} = H_{\rm i}|\psi(t)\rangle. \quad (5)$$

Perform the unitary transformation

$$|\psi(t)\rangle = \mathrm{e}^{-\mathrm{i}H_0 t}|\psi'(t)\rangle, \quad (6)$$

with

$$H_0 = \Omega\sigma_z. \quad (7)$$

Then we obtain

$$\mathrm{i}\frac{\mathrm{d}|\psi'(t)\rangle}{\mathrm{d}t} = H'_{\rm i}|\psi'(t)\rangle, \quad (8)$$

where

$$H'_{\rm i} = \frac{g}{2}\left[\mathrm{e}^{-\mathrm{i}\delta t}a^{+}\left(\sigma_z + \frac{1}{2}\sigma^{+}\mathrm{e}^{\mathrm{i}\Omega t} - \frac{1}{2}\sigma^{-}\mathrm{e}^{-\mathrm{i}\Omega t}\right)\mathrm{e}^{-\mathrm{i}\phi} + \mathrm{e}^{\mathrm{i}\delta t}a\left(\sigma_z + \frac{1}{2}\sigma^{-}\mathrm{e}^{-\mathrm{i}\Omega t} - \frac{1}{2}\sigma^{+}\mathrm{e}^{\mathrm{i}\Omega t}\right)\mathrm{e}^{\mathrm{i}\phi}\right]. \quad (9)$$

Assuming that $\Omega \gg \delta, g$, we can neglect the terms oscillating fast. Then $H'_{\rm i}$ reduces to

$$H'_{\rm i} = \frac{g}{2}(\mathrm{e}^{-\mathrm{i}\delta t - \mathrm{i}\phi}a^{+} + \mathrm{e}^{\mathrm{i}\delta t + \mathrm{i}\phi}a)\sigma_z = \frac{g}{2}(\mathrm{e}^{-\mathrm{i}\delta t - \mathrm{i}\phi}a^{+} + \mathrm{e}^{\mathrm{i}\delta t + \mathrm{i}\phi}a)S_x, \quad (10)$$

where

$$S_x = \frac{1}{2}(S^{+}\mathrm{e}^{-\mathrm{i}\phi} + S^{-}\mathrm{e}^{\mathrm{i}\phi}). \quad (11)$$

The evolution operator for Hamiltonian $H'_{\rm i}$ can be written in the form [11]

$$U'(t) = \mathrm{e}^{-\mathrm{i}A(t)S_x^2}\mathrm{e}^{-\mathrm{i}B(t)S_x a}\mathrm{e}^{-\mathrm{i}C(t)S_x a^{+}}. \quad (12)$$

Using the Schrödinger equation

$$\mathrm{i}\frac{\mathrm{d}U'(t)}{\mathrm{d}t} = H_{\rm i}U'(t), \quad (13)$$

we obtain

$$B(t) = \int_0^t \frac{g}{2}\mathrm{e}^{\mathrm{i}\delta t'}\mathrm{e}^{\mathrm{i}\phi}\mathrm{d}t' = \frac{g}{2\mathrm{i}\delta}\mathrm{e}^{\mathrm{i}\phi}(\mathrm{e}^{\mathrm{i}\delta t} - 1), \quad (14)$$

$$C(t) = \int_0^t \frac{g}{2}\mathrm{e}^{-\mathrm{i}\delta t'}\mathrm{e}^{-\mathrm{i}\phi}\mathrm{d}t' = \frac{g}{-2\mathrm{i}\delta}\mathrm{e}^{-\mathrm{i}\phi}(\mathrm{e}^{-\mathrm{i}\delta t} - 1), \quad (15)$$

$$A(t) = \mathrm{i}\int_0^t B(t')\frac{g}{2}\mathrm{e}^{-\mathrm{i}\delta t'}\mathrm{e}^{-\mathrm{i}\phi}\mathrm{d}t' = \frac{g^2}{4\delta}\left[t + \frac{1}{\mathrm{i}\delta}(\mathrm{e}^{-\mathrm{i}\delta t} - 1)\right]. \quad (16)$$

The evolution operator of the system is given by

$$U(t) = \mathrm{e}^{-\mathrm{i}H_0 t}U'(t) = \mathrm{e}^{-\mathrm{i}\Omega t S_x}\mathrm{e}^{-\mathrm{i}A(t)S_x^2}\mathrm{e}^{-\mathrm{i}B(t)S_x a}\mathrm{e}^{-\mathrm{i}C(t)S_x a^{+}} = \mathrm{e}^{-\mathrm{i}A(t)S_x^2}\mathrm{e}^{-2|\alpha(t)|^2 S_x^2}\mathrm{e}^{-\mathrm{i}\Omega t S_x}\mathrm{e}^{2(\alpha a^{+} - \alpha^{*}a)S_x}, \quad (17)$$

where

$$\alpha = -\mathrm{i}\frac{g}{2\delta}\mathrm{e}^{-\mathrm{i}\phi}\mathrm{e}^{-\mathrm{i}\delta t/2}\sin(\delta t/2). \quad (18)$$

Assume that the initial density operator of the whole system is

$$\hat{\rho}(0) = |g\rangle\langle g|\,\hat{\rho}_f, \quad (19)$$

where $\hat{\rho}_f$ is the unknown density operator for the cavity field. Then after an interaction time τ, the density operator for the whole system is

$$\hat{\rho}(\tau) = \frac{1}{4}[|g\rangle(\mathrm{e}^{-\mathrm{i}\Omega t/2}D(\alpha) + \mathrm{e}^{\mathrm{i}\Omega t/2}D(-\alpha)) + \mathrm{e}^{-\mathrm{i}\phi}|e\rangle(\mathrm{e}^{-\mathrm{i}\Omega t/2}D(\alpha) - \mathrm{e}^{\mathrm{i}\Omega t/2}D(-\alpha)) \times \hat{\rho}_f[(\mathrm{e}^{\mathrm{i}\Omega t/2}D^{+}(\alpha) + \mathrm{e}^{-\mathrm{i}\Omega t/2}D^{+}(-\alpha))\langle g| + (\mathrm{e}^{\mathrm{i}\Omega t/2}D^{+}(\alpha) - \mathrm{e}^{-\mathrm{i}\Omega t/2}D^{+}(-\alpha))\mathrm{e}^{-\mathrm{i}\phi}\langle e|], \quad (20)$$

where $D(\alpha)$ is the displacement operator,

$$D(\alpha) = \mathrm{e}^{(-\alpha^{*}a + \alpha a^{+})}. \quad (21)$$

We now detect the atomic state. The probability of measuring the atom in the ground state $|g\rangle$ is

$$P_g(\alpha, \varphi) = \frac{1}{2} + \frac{1}{2}\mathrm{Tr}\{\mathrm{e}^{-\mathrm{i}\varphi}D(\beta)\hat{\rho}_f\}, \quad (22)$$

where $\beta = 2\alpha$ and $\varphi = \Omega t/2$. The modulus of β is controlled by t, and the phase by ϕ. The parameter φ is controllable by the Rabi frequency Ω. We can rewrite Eq.(22) as

$$P_g(\alpha, \varphi) = \frac{1}{2} + \frac{1}{2}\mathrm{Re}[\mathrm{e}^{-\mathrm{i}\varphi}\chi(\beta)], \tag{23}$$

where $\chi(\beta)$ is the Wigner characteristic function,

$$\chi(\beta) = \mathrm{Tr}\{\hat{\rho}_f \hat{D}(\beta)\}. \tag{24}$$

From Eq.(23) we obtain

$$\chi(\beta) = 2P_g(\beta, 2n\pi) - 1 + \mathrm{i}\left[2P_g\left(\beta, 2n\pi + \frac{\pi}{2}\right) - 1\right]. \tag{25}$$

Thus a measurement of $P_g(\beta, \varphi)$ for two phases $\varphi = 2n\pi, 2n\pi + \dfrac{\pi}{2}$ directly yields the Wigner characteristic function of the initial field state at point β.

In conclusion, we have proposed a scheme for directly measuring the Wigner characteristic function of a cavity field via the driven Jaynes–Cummings model. In our scheme the atom is resonantly driven by a strong classical field, which is slightly detuned from the cavity. Therefore, the coupling between the classical field and the cavity mode can be avoided and the drawback of Ref.[7] is overcome. In contrast with the scheme of Ref.[8], our scheme does not require the displacement of the cavity field before the atom–field interaction and thus our scheme might be more feasible.

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