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Chaos synchronization based on intermittent state observer *

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This paper describes the method of synchronizing slave to the master trajectory using an intermittent state observer by constructing a synchronizer which drives the response system globally tracing the driving system asymptotically. It has been shown from the theory of synchronization error-analysis that a satisfactory result of chaos synchronization is expected under an appropriate intermittent period and state observer. Compared with continuous control method, the proposed intermittent method can target the desired orbit more efficiently. The application of the method is demonstrated on the hyperchaotic Rössler systems. Numerical simulations show that the length of the synchronization interval τ_s is of crucial importance for our scheme, and the method is robust with respect to parameter mismatch.

Keywords: chaos synchronization, state observer, intermittent method
PACC: 0545

1. Introduction

Chaos in nonlinear systems is characterized by their extreme sensitive dependence on initial conditions, which is known as the "butterfly effect". Accordingly, for a long time, synchronization between two chaotic systems was mistakenly considered to be impossible. However, since the pioneering work by Carroll and Pecora,[1,2] researchers have realized that chaotic systems can be synchronized, and hence there has been considerable work on chaos synchronization.[3−6] Particularly, its practical application in secure communication [5,6] has made it a distinct subfield of nonlinear dynamics.

In recent years different methods of chaos synchronization have been put forward. But the state observer for synchronization has received little attention.[7] In comparison with continuous control of chaos, a new synchronization scheme based on occasional coupling has been proposed.[8,9] This scheme, as well as others proposed in the literature, has a potential application for secure communications. In Ref.[9] we presented an occasional driving synchronization for a class of chaotic systems with the computation of conditional Lyapunov exponents (CLEs), and deduced that only few driving periods are used to synchronize two chaotic systems. However, it is very tedious and difficult to compute CLEs, especially for high dimensional systems. On the basis of Refs.[7–9], in this paper, we propose an intermittent chaos synchronization with the help of the designed state observer and present some simulation results concerning the hyperchaotic Rössler system. In this scheme, the synchronization signal is used in the response system at intervals of time τ_s during which the response system tracks the orbit of the driving system. It is noted that the length of the intervals is of crucial importance.

This paper is organized as follows. Section 2 introduces the intermittent state observer, which shows that under certain conditions synchronization between the driving and the driven systems is realized, and we also stress that the scheme is robust with respect to noise and parameter mismatch. In Section 3 the proposed method is applied to synchronize the well-known hyperchaotic Rössler system. Finally, we summarize with concluding remarks.

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2. System description and design of intermittent state observer

In this section, a brief description of intermittent state observer is given. Assume the nonlinear dynamical systems considered here to be, without loss of generality, described by the following state and output equations:

$$\dot{x} = f(x, \alpha), \tag{1}$$

and define

$$s = s(x) = h(x) + Kx. \tag{2}$$

Here the state vector $x \in R^n$, $\alpha \in R^p$ is a parameter vector, $K \in R^{m\times n}$ will be addressed shortly, and f is a nonlinear, but smooth function of state vector. Apparently, dynamical behaviour of the system or the solution of Eq.(1) varies with parameter vector. We assume that for some α, the solutions of Eq.(1) would exhibit chaotic behaviour. The state observer of Eq.(1) is given by [7]

$$\dot{y} = f(y, \alpha) + u(h(x), h(y)), \tag{3a}$$

where $u(h(x), h(y))$ is a controller to be designed.

Definition 1 To prove the synchronization property, let us define the synchronization error as $e(t) = x - y$. If

$$\lim_{t\to\infty} |e(t)| = \lim_{t\to\infty} |x - y| = 0, \tag{4}$$

then Eq.(3a) becomes the observer of system (1). The goal of control is to find a controller $u(h(x), h(y))$ that satisfies Eq.(4).

Assumption 1 We assume that the system given by Eq.(1) has the form

$$\dot{x} = f(x, \alpha) = Ax + Bh(x), \tag{5}$$

and the controller is

$$u(h(x), h(y)) = B(s(x) - s(y)). \tag{6}$$

Hence we have

$$\dot{y} = f(y, \alpha) + B(h(x) + Kx - h(y) - Ky), \tag{3b}$$

where $A \in R^{n\times n}$ is a constant matrix, $B \in R^{n\times m}$, $h : R^n \to R^m (m \le n)$ are nonlinear smooth maps.

Thus, the time derivative of synchronization error or the error system dynamics is given by

$$\dot{e} = \dot{x} - \dot{y} = (A - BK)e. \tag{7a}$$

Then the error system (7a) is linear and time-invariant. Therefore, in order to make system (3b) an observer for the state of system (5), it is necessary to stabilize system (7a) at the origin via a suitable K. If the pair (A, B) is observable then there exists a constant gain matrix K such that $(A-BK)$ is stable. The necessary and sufficient condition for the existence of a feedback gain vector K such that system (3b) becomes a global observer of system (5) is that all the eigenvalues of the error system (7a), if any, have negative real parts. Therefore, as time elapses, $e(t) = x-y$ decays exponentially to zero, that is, system (1) and system (3a) are synchronized perfectly.

The method of chaos synchronization mentioned above based on state observer belongs to the category of continuous drive method. First of all, in application, sometimes it is impossible to drive the driven system all along. Secondly, efficiency of intermittent driving is higher than that of continuous driving, especially in secure communication. In the following, we will construct an appropriate intermittent state observer that satisfies Eq.(4).

An intermittent state observer is based on imposing the controller periodically. In a general way, let $\tau = \tau_s + \tau_a$, where $\tau_s > 0$ and $\tau_a > 0$ denote the intervals for synchronization and autonomy phases, respectively. Then, for $j = 1, 2, \cdots$, our scheme is as follows.

Remark 1

(i) The jth synchronization phase

For $(j-1)(\tau_s + \tau_a) \le t \le j\tau_s + (j-1)\tau_a$, the slave system given by (3b) tracks the orbit of the driving system (5) or (1).

(ii) The jth autonomy phase

For $j\tau_s + (j-1)\tau_a \le t \le j(\tau_s + \tau_a)$, the slave system is a duplicate of the master system (1) which implies that the slave system becomes autonomous: that is, the controller takes no actions.

According to (i) and (ii), the controller (6) can be rewritten as

$$u = \begin{cases} B(s(x) - s(y)), & (j-1)(\tau_s + \tau_a) \le t \le j\tau_s + (j-1)\tau_a, \\ 0, & j\tau_s + (j-1)\tau_a \le t \le j(\tau_s + \tau_a). \end{cases} \tag{7b}$$

So, the synchronization error dynamical system is described by

$$\dot{e} = \begin{cases} J \cdot e, & j\tau_s + (j-1)\tau_a \le t \le j(\tau_s + \tau_a), \quad (8a) \\ (A - BK)e, & (j-1)(\tau_s + \tau_a) \le t \le j\tau_s + (j-1)\tau_a, \quad (8b) \end{cases}$$

where J represents the Jacobian matrix of the system, $J_{ij} = \partial f_i / \partial x_j$, $(i, j = 1, \cdots, n)$. Since in the synchronization phase the error decays to zero exponentially, at the end of this phase the error becomes extremely small, provided that τ_s is sufficiently large. Equations (8a), (8b) imply that $e = 0$ is an exponentially stable equilibrium point of the error system.

The result stated above belongs to the ideal case when there is no noise and when the parameter vectors are the same between the drive and the driven systems. Hereinafter we consider the nonideal case and show that the scheme proposed is robust with respect to noise and parameter mismatch. Note that, for the nonideal case, the response system (3b) should be replaced by

$$\dot{y} = f(y, \alpha') + m(t)B(h(x) + Kx - h(y) - Ky + n), \quad (3c)$$

where α' is the parameter vector for the driven system, and n is a random noise term, and

$$m(t) = \begin{cases} 1, & (j-1)(\tau_s + \tau_a) \le t \le j\tau_s + (j-1)\tau_a, \\ 0, & j\tau_s + (j-1)\tau_a \le t \le j(\tau_s + \tau_a). \end{cases}$$

Consider the drive and response systems given respectively by Eqs.(5) and (3c). Assume that f is Lipschitz in both variables, then there exists τ_s which is sufficiently large such that Eq.(4) holds.[8] From Ref.[8], it is obvious that in the present of noise and/or parameter mismatch, the scheme is robust in the nonideal case. Note that some other factors may also contribute to the error bound, hence τ_s may be larger. As a consequence, the length of the synchronization interval τ_s is of crucial importance for our scheme.

3. Example and simulation results

To illustrate the chaotic synchronization scheme proposed here, the hyperchaotic Rössler system, which is described by the set of ordinary differential equations,

$$\dot{x} = Ax + x_1 x_3 (0, 0, 1, 0)^{\mathrm{T}} + (0, 0, 3, 0)^{\mathrm{T}} \quad (9)$$

is considered and numerical simulations are performed, where

$$x = (x_1, x_2, x_3, x_4) \in R^4,$$
$$h(x) = [x_1 x_3],$$
$$s(x) = x_1 x_3 + \sum_{i=1}^{4} k_i x_i,$$
$$A = \begin{bmatrix} 0 & -1 & -1 & 0 \\ 1 & \alpha & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -\beta & \gamma \end{bmatrix},$$
$$B = (0, 0, 1, 0)^{\mathrm{T}},$$
$$C = (0, 0, 3, 0)^{\mathrm{T}}.$$

The parameters α, β and γ are taken in a range to ensure the hyperchaotic behaviour of system (9). Since the controllability matrix is full rank, there exists a gain vector K such that system (3b) becomes a global observer of system (9), i.e. $y \to x$ for any initial state. For example, the eigenvalues of $A - BK$ can be taken as $[-7, -6, -5 + 2i, -5 - 2i]$ for $K = [659.7 \quad -541.3 \quad 23.0 \quad -1718.2]$. The initial values are arbitrarily chosen as follows: $x(0) = [5.01, 0.012, 5.2, 20.5]$ and $y(0) = [0.012, 0.01, 0.015, 15.5]$, respectively, for the drive system and the response system. The period $\tau = \tau_s + \tau_a$=30 s. The fourth-order Runge–Kutta method is used to solve the systems of differential Eqs.(9) and (3c) with time step size equal to 0.001 in all numerical simulations.

Firstly, we considered the ideal case (i.e. no noise and no parameter mismatch) with $\alpha = 0.25$, $\beta = 0.5$ and $\gamma = 0.05$. Fig.1(a)–(c) shows that the synchronization can be eventually achieved at different values of interval τ_s. And the larger τ_s is, the faster and more accurately the synchronization will be achieved. When $\tau = \tau_s$=30 s, i.e. the control is continuously applied, the systems synchronize fast. This is because τ_a is in inverse proportion to control price. Thus, the transient time needed for the synchronization will last shorter for larger τ_s. On the other side, Fig.1(d) indicates there is hardly synchronization for two systems when τ_s is unreasonably small.

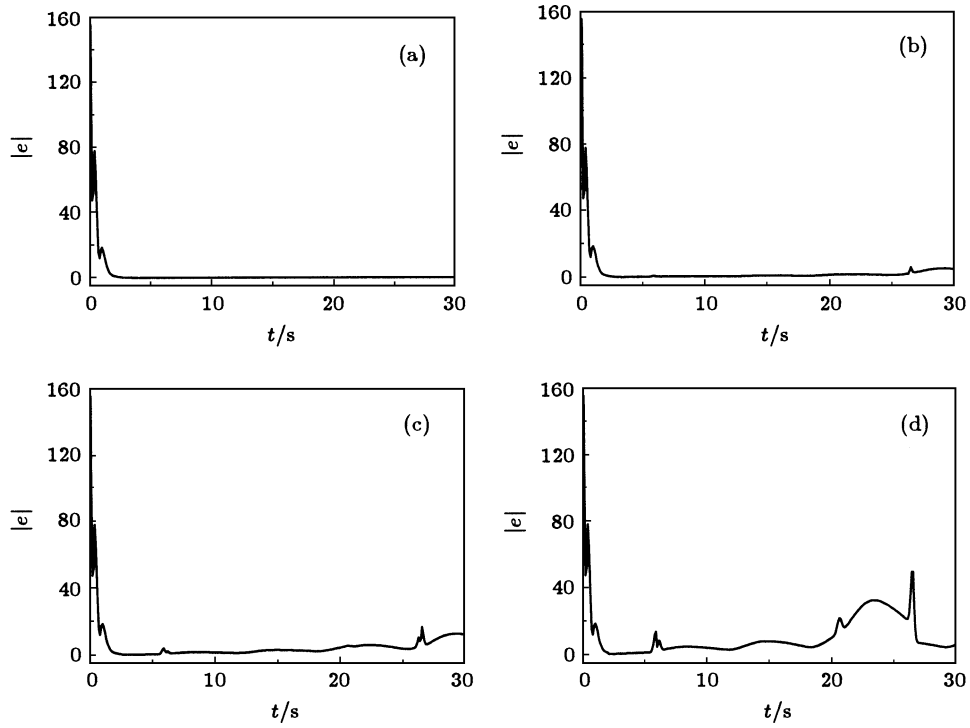


Fig.1. Synchronization error at different τ_s under ideal case. (a) τ_s=30 s, (b) τ_s=4 s, (c) τ_s=2.5 s, (d) τ_s=2 s.

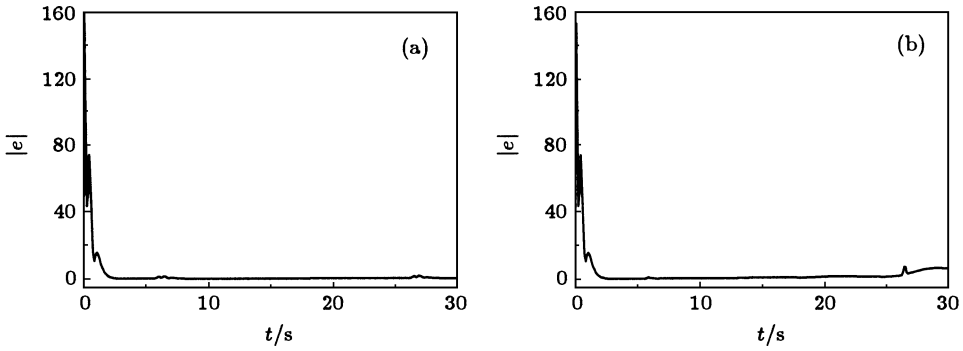


Fig.2. Synchronization error at different τ_s with parameter mismatch. (a) τ_s=30 s, (b) τ_s=4 s.

In the remaining simulations, the nonideal case is considered. All parameters are changed by 1% in the response system (i.e. multiplied by 1.01). The results of the synchronization of the two chaotic systems with different parameters are depicted in Fig.2, which implies the robustness of the synchronization. As can be seen from Fig.2, the synchronization of two Rössler systems is achieved. In addition, the method is also robust with respect to noise, but the simulation has not been shown for the limitation of space.

4. Conclusions

In this paper, we have provided a novel scheme for chaos synchronization using an intermittent state observer. The validity of the proposed method is verified by theoretical and numerical analysis. The results indicated that the length of the synchronization interval τ_s is of crucial importance for our scheme, emphasizing the fact that synchronization is achieved for synchronization interval τ_s greater than a certain threshold. From the above-mentioned analysis and simulations, we also found that the proposed method is robust with respect to parameter mismatch.

In contrast to continuous control method, the proposed discrete method can achieve target orbit more efficiently even under the circumstances that the control price is greatly reduced. As mentioned in Ref.[8], it is possible to send chaotically masked or coded mes-

sages for communication in the autonomous phases. In the scheme proposed in Ref.[9], the synchronization is achieved at discrete times (i.e. not in a time interval), whereas in this scheme the synchronization is achieved in an interval. Prior to Ref.[9], no computation of the conditional Lyapunov exponents is required.

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