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The bending of light ray and unphysical solutions in general relativity

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In general relativity, according to Einstein, a gauge is related to the time dilation and the space contractions, and thus a physically realizable gauge should be unique for a given frame of reference. Since more than one metric solution for the same frame can produce the same deflection angle, this means that an invalid space–time metric can produce the correct deflection angle for a light ray. To demonstrate this with an unambiguous example, we consider a new extreme case that there is no space contraction in the radius direction while the conditions of asymptotic flatness and the requirement for gravitational red shifts are satisfied. This solution has a distinct characteristic of "space expansion" in the other directions. Nevertheless, it turns out that, in spite of requiring far more subtle calculations, the resulting deflection angle of a light ray is the same. An interesting property of this new solution is that its event horizon corresponds to an arbitrary integral constant. Thus, this calculation demonstrates beyond doubt that an unphysical solution can produce the correct first-order approximation of light bending. This makes it clear that there is a main difference between local effects such as the gravitational red shifts and the local light speeds, which are not gauge invariant, and integrated effects such as the bending of light, which can be (restricted) gauge invariant.

Keywords: Einstein's equivalence principle, Euclidean-like structure, bending of light, black hole, space expansion

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1. Introduction

The 1919 observational confirmation of Einstein's 1915 prediction of light bending is an historical event that marked the triumph of general relativity.[1] However, this triumph actually was somewhat over-evaluated since problems such as the appropriate gauge have not yet been settled.[2,3]

Einstein obtained the same bending of a light ray with two different methods: namely, the first-order approximations of the Schwarzschild solution in 1916 [4] and the harmonic solution in 1920.[5] A problem exposed by these facts is how to identify the solution that is more realistic.[2] However, Einstein failed to see this. Instead, he declared that the calculation and thus his theory is gauge independent.[5] Unfortunately, this "gauge invariance" is limited to the first-order approximation since the gravitational red shifts are distinct in the second-order [7] although this is beyond the accuracy of current experiments.

According to Einstein's equivalence principle, the local light speeds are measurable.[4,5] Since these two solutions have very different space contractions,[4,5] their local light speeds are different in the first-order. Moreover, since these two different solutions have the same frame of reference,[2,3] they cannot be both valid. In other words, the gauge invariance is not only theoretically invalid but also experimentally testable.[2] Currently, due to the influence of Pauli's version [1)] of the equivalence principle,[6] instead of questioning the gauge notion, it is incorrectly believed that coordinate light speeds have no physical meaning, and, to support this belief, the calculation of local light speeds is circumvented.[6−10]

While Einstein's equivalence principle requires the local space of a free falling observer to be Minkowski, Pauli's version requires only a proper metric signature. Nevertheless, in spite of Einstein's objection, Pauli's version is currently prevailing because the physical meaning of space–time coordinates was ambiguous in Einstein's theory.[11,12]

[1)] Einstein's objection to Pauli's version made clear that Einstein's version of infinitesimal equivalence principle is distinct from Pauli's. However, due to inadequate understanding, some theorists regarded that Einstein's equivalence principle as due to misunderstanding the basic physical and mathematical concepts.[11]

However, since the physical meaning of space–time coordinates has recently been clarified within the theoretical framework of general relativity,[2,3] the inadequacy of Pauli's version actually has already been demonstrated by the Michelson–Morley experiment.[3] Now, we are forced to recognize that an unphysical metric can lead to a correct angle of deflection.

It is useful to identify the reasons that a correct light deflection angle can be calculated from an unphysical solution. First, this is an integrated effect, and thus may not be subjected to the physical requirements for a local effect such as the local light speeds and gravitational red shifts.[3] Second, these metrics have a frame of reference such that a deflection angle can be defined.[3] Moreover, these metrics are not arbitrary because they satisfy the condition of an asymptotically flat metric, and have the same first-order for gravitational red shifts.[7] In current textbooks, these solutions are regarded as equally valid. Thus, to be convincing on the existence of such an unphysical solution, for some theorists, it would be necessary to show a clearly unphysical solution that satisfies the above conditions.

Experimentally, the Schwarzschild solution is in an unfavourable position, because there are experiments against it. Both the accurate Einstein–Infeld–Hoffman equation for tracking planets spacecraft [9] and the very accurate experiments on binary pulsars are based on the Maxwell–Newton approximation,[13−15] which is incompatible with the Schwarzschild solution.[16−18] Accordingly, it has been predicted [18] that the Gravity Probe-B gyroscopes [19] on the precessions would further confirm this approximation and Einstein's equivalence principle. Therefore, it is expected also that the experiments on horizontal local light speeds [2] will directly support the Maxwell–Newton approximation, but reject the Schwarzschild solution. However, theorists used to take the presumed validity of the Schwarzschild solution for granted.[9,10] Historically, there are great resistances to the local light speed experiment of Zhou [20] who conjectured the invalidity of the Schwarzschild solution.[2]

In this paper, to investigate this restricted gauge independence, the extreme case of no radius spatial contraction will be calculated. It will be shown that this solution has a distinct characteristic of "space expansion" in other directions. For this unusual feature, most theorists would agree that this metric is not a physical solution since gravity is expected to cause space contraction.[4,5] Nevertheless, it turns out that the resulting light deflection angle is the same (Section 4). Thus, the first-order approximation of light bending is (restricted) gauge invariant as Einstein claimed even though some solutions are unphysical.

Moreover, this new solution has an unusual property that an event horizon would correspond to an arbitrary integral constant although the correspondences of event horizons among other solutions are definite.[7] Then, if the advocates of black holes insisted that the theory is gauge invariant, they have to admit that the notion of event horizon has no physical meaning. For the convenience of the readers, first we shall clarify some conceptual problems.

2. The notion of gauge in general relativity

In general relativity, the notion of gauge was established for physically equivalent space–time coordinate systems.[4,5] For sometimes, the current notion of gauge seems to help resolve the difficulty due to the ambiguity of the coordinates. However, after the meaning of coordinates has been identified,[2,3] such a notion has been proven to be inadequate in physics. Here, for the convenience of the readers, a brief review and analysis is given instead of just the related references.

The nonlinear Einstein's field equation of 1915 version [4,5] for a space–time metric g_{ab} is

$$G_{ab} \equiv \boldsymbol{R}_{ab} - \frac{1}{2}\boldsymbol{R}g_{ab} = -K\boldsymbol{T}(m)_{ab}, \qquad (1)$$

where $\boldsymbol{R}_{ab}$ is the Ricci curvature tensor, $\boldsymbol{R} = \boldsymbol{R}_{cd}\ g^{cd}$, its source $\boldsymbol{T}(m)_{ab}$ is the energy-stress tensor for massive matter and can depend on g_{ab}. However, among these 10 equations of tensor components, only six of them are independent, since

$$\nabla^a G_{ab} \equiv 0. \qquad (2)$$

Note also that for G_{at} (a=x, y, z, t), there is no second-order time derivative.[7,17] It follows that, for Eq.(1), the initial condition of a Cauchy problem is restricted by four constraints. Thus, to solve the Einstein equation (1), four more conditions are needed. These four additional conditions are attributed as due to a certain freedom of choice of coordinates for a physical Riemannian space,[3,4] and are called the gauge conditions. An often-used gauge is the harmonic gauge condition.[7]

There are two extreme views on the question of gauge: (i) Fock,[11] who accepted Pauli's [6] "equivalence principle", argued that the harmonic gauge is the preferred gauge; (ii) another view, though rejected by Eddington [8] and Einstein,[5] is that the gauge condition is entirely arbitrary. However, it has been pointed out that this assumption of arbitrary gauge can lead to the acceptance of unphysical solutions.[21–23]

Certainly, there is some arbitrariness in the choice of gauge since a frame of reference is arbitrary according to the principle of general relativity. However, once a frame of reference is chosen, the gauge is also fixed since both time dilation and space contractions are measurable. Therefore, it is conceivable that, for some cases, the harmonic gauge may not be applicable. For instance, the space–time metric for Einstein's rotating disc [24] does not satisfy the harmonic gauge and is unique because, as Einstein stated,[4,5] it is necessary "to define time in such a way that the rate of a clock depends upon where the clock may be". Moreover, the validity of Einstein's equivalence principle can be proven for this case.[2,3]

However, it should be noted that the validity of Einstein's equivalence principle for other cases was assumed [2–5] although with plausible justifications. For example, the notion of black holes used a presumed validity of the Schwarzschild solution.[9,10,25,26] Thus, the advocates of black holes had to believe [1] Pauli's version, which requires only a proper metric signature, although Einstein objected to this as a misinterpretation.[27]

On the other hand, the theoretical framework of general relativity actually has included very clear physical meaning of the space–time coordinates although the realistic gauge is not yet known.[2,3] Thus, the criticism of Whitehead [12] toward general relativity as coordinates being confusing is actually only a problem of Einstein's shortcoming. Moreover, since space contraction and time dilation are measurable, the gauge is unique for a given frame for reference. Note that the gauge subsequently discussed is a mathematical notion for a given frame of reference. Moreover, it is only among specific diffeomorphic metrics, which satisfy some known physical requirements. To this end, let us clarify the physical meaning of space–time coordinates.

3. A frame of reference and the Euclidean-like structure

Einstein [28] once remarked, "If you want to find out anything from the theoretical physicists about the methods they use, I advise you stick closely to one principle: don't listen to their words, fix your attention on their deeds." Following his advice, it is found that, from his metric solutions (no matter whether exact or approximate) the space coordinates are characterized by (i) the Euclidean-like distance function $\mathrm{d}(P_1,P_2)=[(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2]^{1/2}$, (ii) the frame of reference is chosen before a solution is obtained, and (iii) the time coordinate is orthogonal to the space coordinates. The coordinates are independent of the gravity, and Einstein's predictions are related to the Euclidean-like distance instead of just the local metric distance.[3] Thus, Einstein's coordinate system has very specific physical meanings. The next question is whether these characteristics are coincident or intrinsic properties of general relativity. First, the Euclidean-like structure is related to the measuring rod attaching to the frame of reference (as in special relativity [29]). This measurement is different from Einstein's with the measuring rod at rest but in a free fall state. Second, a frame of reference is an integral part of Einstein's equivalence principle and Einstein's principle of general relativity. Third, as shown in his calculations, the orthogonality of the time coordinate [2] to the space coordinates is required by special relativity and Einstein's equivalence principle. Thus, the above three characteristics are inextricably included in the theoretical framework of general relativity.

[1] The advocates of black holes favour interpreting $\mathrm{d}t/\mathrm{d}r$ (instead of $\mathrm{d}r/\mathrm{d}t$) as velocity inside the "event horizon" of a Schwarzschild solution.[26] This disagrees with Einstein [5] who believes in a fixed signature for any space–time coordinate. However, such an exchange of signatures does not happen in an isotropic solution, which implies that the radius of the event of horizon would be $M\kappa/2$ instead of $2M\kappa$ in metric (9). Moreover, for the new solution in this paper, the radius of an event of horizon can even be arbitrary (see Section 4). Thus, the advocates had chosen a gauge because of its physical meaning. On the other hand, such a choice is incompatible with accepting Pauli's version as adequate in physics.

[2] In applying Einstein's equivalence principle, the time coordinate is transformed to be orthogonal to space coordinates. Landau and Lifshitz [31] showed that it is always possible, without changing the frame of reference, to transform an arbitrary metric to an orthogonal metric.

Nevertheless, Einstein was unable to see that this Euclidean-like structure is physically intrinsic to his theory. To measure space contraction, Einstein's measuring instruments are at rest but in a free fall state.[4,5] Based on such considerations, Einstein [4] believed, "In the general theory of relativity, space and time cannot be defined in such a way that differences of the spatial coordinates can be directly measured by the unit measuring-rod, or differences in the time coordinates by a standard clock." The fact is, however, that even the meaning of such measurements are unclear unless the physical meaning of the coordinates are clarified. Understandably, this confusion had led to the belief that coordinates have no meaning in general relativity.

Besides, the necessary existence of the Euclidean-like structure in his curved space–time, though required by physics, is not an integral part of the usual Riemannian geometry. Moreover, the Riemannian space is often illustrated with a curved surface embedded in a Euclidean space. Thus, many physicists including Dirac [30] invalidly believed that the coordinates of Einstein's physical space must be curves, and therefore also unclear.

In general relativity, for a frame of reference, there are two complementary distances: the local distance is decided by

$$\mathrm{d}s^2 = g_{ab}\mathrm{d}x^a\mathrm{d}x^b, \tag{3}$$

where g_{ab} is a space–time metric with a signature as that of the Minkowski metric. However, unlike the pseudo-Euclidean metric in special relativity, the meaning of the coordinates is not clear from metric (3) alone. This is a reason that the local distance formula of Landau and Lifshitz [31] does not always produce the "actual distances" as they claimed. Mathematically, a supplementary relation is necessary to clarify the meaning of coordinates in a curved space. However, Einstein [4,5,27] overlooked this; instead, he tried to have the physical meaning of coordinates from metric (3) alone.

Consequently, the meaning of coordinates was ambiguous, and Einstein's principles, which require a physical meaning of the coordinates,[11] were replaced with Pauli's version [6] of the equivalence principle and mathematical covariance since they do not require a physical meaning of coordinates. This ambiguity of coordinates was the main reason that Whitehead [12] objected that general relativity was a physical theory of confusion. However, relativists did not really have a better choice.

The supplementary spatial distance of two points $P_1(x_1,\ y_1,\ z_1)$ and $P_2(x_2,\ y_2,\ z_2)$ in the frame of reference is

$$\mathrm{d}(P_1, P_2) = [(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2]^{1/2}, \tag{4}$$

the Euclidean-like distance, which is independent of gravity. However, the space is non-Euclidean because it has a non-Euclidean metric (3). The confusion in current theory probably has a root in the fact that the Euclidean-like structure (4), though used, has not been explained adequately. In special relativity, these two complementary distances coincide because

$$\mathrm{d}s^2 = c^2\mathrm{d}t^2 - \mathrm{d}x^2 - \mathrm{d}y^2 - \mathrm{d}z^2, \tag{5}$$

where the units are centimetre and second, and c is the speed of light, 3×10^{10} cm/s.

Let us consider a solution of metric of $S(x,y,z,t)$ in the isotropic form,[7]

$$\begin{aligned}\mathrm{d}s^2 =&[(1-M\kappa/2r)^2/(1+M\kappa/2r)^2]c^2\mathrm{d}t^2\\ &-(1+M\kappa/2r)^4(\mathrm{d}x^2+\mathrm{d}y^2+\mathrm{d}z^2),\end{aligned} \tag{6}$$

where $\kappa = G/c^2$ (G=6.67 $\times$ $10^{-11}\mathrm{m}^3\cdot\mathrm{kg}^{-1}\mathrm{s}^{-2}$), M is the total mass of a spherical mass distribution with the centre at the origin of the frame of reference $K(x,y,z)$, and $r=[x^2+y^2+z^2]^{1/2}$. Since the metric is a function of r, which is defined in terms of the Euclidean-like structure $E(x,y,z)$, the Euclidean-like structure is necessarily included in such a Riemannian space–time of Einstein. In fact, Einstein [4] had made clear that the light velocity is "defined in the sense of Euclidean geometry".

However, since a Euclidean-like structure is physically meaningful only in a physical space, whose validity requires a Euclidean-like structure, this seems to have a problem of logical circulation. In practice, one may first assume a priori that a Euclidean-like structure exists in the Riemannian "space–time" under consideration. Accordingly, physical requirements are verified to confirm the existence of a Euclidean-like structure in a physical space. For instance, Einstein derived physical results by assuming that his equivalence principle is satisfied. However, if for a certain metric some results fail to fulfil a physical requirement, such a manifold would not be a physical space. Thus, the apparent problem of circular logic does not really exist in general relativity.

If Einstein's equivalence principle is valid, at a point resting at K, accordingly [4,5] there is a local

Minkowski space [1] with

$$ds^2 = c^2dT^2 - dX^2 - dY^2 - dZ^2, \quad (7)$$

such that

$$c^2dT^2 = [(1 - M\kappa/2r)^2/(1 + M\kappa/2r)^2]c^2dt^2, \quad (8a)$$

and

$$\begin{aligned} dl^2 =& dX^2 + dY^2 + dZ^2 \\ =& (1 + M\kappa/2r)^2(dx^2 + dy^2 + dz^2). \end{aligned} \quad (8b)$$

Then, time dilation and spatial contraction would be obtained as measured.[4,5] However, the physical meanings of formula (8) and metric (6) are clear only if the meaning of parameter r has been determined. According to Eqs.(8), the local light speed is

$$\begin{aligned} \frac{(dx^2 + dy^2 + dz^2)^{1/2}}{dt} =& c\frac{1 - M\kappa/2r}{(1 + M\kappa/2r)^3} \\ \approx& c\left[1 - \frac{2M\kappa}{r}\right]. \end{aligned} \quad (8c)$$

Equation (8c) further confirms the bending of light to be $KM/2\pi\Delta$, where Δ is the Euclidean-like distance from the centre of the sun.

Now, consider the Schwarzschild solution [4,5] of $S'(x', y', z', t')$

$$\begin{aligned} ds^2 =& (1 - 2M\kappa/\rho)c^2dt'^2 - (1 - 2M\kappa/\rho)^{-1}d\rho^2 \\ & - \rho^2 d\theta^2 - \rho^2 \sin^2\theta d\varphi^2, \end{aligned} \quad (9)$$

where

$$\begin{aligned} x' &= \rho \sin\theta \cos\varphi, \\ y' &= \rho \sin\theta \sin\varphi, \\ z' &= \rho \cos\theta \\ \rho &= [x'^2 + y'^2 + z'^2]^{1/2}. \end{aligned}$$

M is the total mass of a spherical mass distribution with the centre at the origin of the frame of reference $K'(x', y', z')$. Based on Einstein's equivalence principle,[4,5] at a point resting at K', there is a local Minkowski space (dX, dY, dZ, cdT) such that

$$c^2dT^2 = (1 - 2M\kappa/\rho)c^2dt'^2 \quad (10a)$$

and

$$\begin{aligned} dl^2 =& dX^2 + dY^2 + dZ^2 \\ =& (1 - 2M\kappa/\rho)^{-1}d\rho^2 + \rho^2 d\theta^2 + \rho^2 \sin^2\theta d\varphi^2, \\ =& \sum_{i=1}^{3}(dx'^i)^2 + \left[\left(1 - \frac{2M\kappa}{\rho}\right)^{-1} - 1\right] \\ & \times \frac{1}{\rho^2}\left[\sum_{i=1}^{3} x'^i dx'^i\right]^2 \\ \approx& \sum_{i=1}^{3}(dx'^i)^2 + \frac{2M\kappa}{\rho}\frac{1}{\rho^2}\left[\sum_{i=1}^{3} x'^i dx'^i\right]^2. \end{aligned} \quad (10b)$$

Then, time dilation and spatial contraction would be obtained.[4,5] Note that the space contraction is directional and no longer isotropic. Again, the physical meanings of Eqs.(9) and (10) are based on the Euclidean-like structure parameter ρ.

Moreover, both metrics have the same first-order gravitational red shifts. If ρ and r are related by [7]

$$\rho = r(1 + M\kappa/2r)^2, \quad \text{for } \rho > 2M\kappa, \quad (11)$$

then metrics (6) and (9) are mathematically equivalent. However, Eq.(11) also cannot be compatible with the fact that both r and ρ are distances. The frames of reference $K(x, y, z)$ and $K'(x', y', z')$ are related to the Euclidean coordinate system when M=0. The clock rates and the spatial contractions of metrics (6) and (9), though mathematically related by Eq.(11), are physically different.

Thus, the coordinate system $K(x, y, z)$, if S is related to reality, does have the meaning of a Euclidean-like structure in terms of physical measurements. The same arguments would also be applicable to S'. However, metric (6) and metric (9) have different types of spatial contraction. So only S or S' can be a physical space if they have the same frame of reference. In other words, should r and ρ have the same physical interpretation, these two metrics are physically not equivalent.

4. Exterior solutions for a spherical mass distribution

For a spherical static distribution of mass, there are other solutions other than the two mentioned above. They can be conveniently derived, for instance,

[1] The existence of a local Minkowski space and the following equations are clearly stated as a consequence of his equivalence principle in Eq.(106) of Einstein's book, "The Meaning of Relativity",[5] and in Eq.(71) of the 1916 initial paper.[4]

with the method provided by Weinberg.[7] Due to the principle of causality [1] in Refs.[16,18], the exterior gravitational field has the spherical static symmetry, which requires the metric of the form,[7]

$$\begin{aligned} \mathrm{d}s^2 =& F(r)c^2\mathrm{d}t^2 - D(r)\mathrm{d}r^2 \\ & - C(r)(r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\varphi^2). \end{aligned} \tag{12}$$

The frame for reference is attached to the resting mass with the spatial origin at the centre of mass. The metric is asymptotically flat such that a deflection angle can be defined. However, current theory cannot determine the functions $C(r)$, $D(r)$, and $F(r)$ uniquely. If $D(r) = C(r)$ is assumed, one obtains the isotropic solution (6). If $C(r)$=1 is assumed, one obtains another well-known solution, the Schwarzschild solution. Thus, metric (12) shows that they have the same frame of reference. Moreover, if the harmonic gauge is required, one obtains

$$\begin{aligned} \mathrm{d}s^2 =& [(1 - M\kappa/\rho')/(1 + M\kappa/\rho')]c^2\mathrm{d}t^2 \\ & - [(1 + M\kappa/\rho')/(1 - M\kappa/\rho')]\mathrm{d}\rho'^2 \\ & - (1 + M\kappa/\rho')^2\rho'^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2) \\ =& [(1 - M\kappa/\rho')/(1 + M\kappa/\rho')]\mathrm{d}t^2 \\ & - \sum_{i=1}^{3}\left(1 + \frac{M\kappa}{\rho'}\right)^2(\mathrm{d}x'^i)^2 \\ & - \left(\frac{1 + M\kappa/\rho'}{1 - M\kappa/\rho'}\right)\frac{M^2\kappa^2}{\rho'^4}\left[\sum_{i=1}^{3}x'^i\mathrm{d}x'^i\right]^2, \end{aligned} \tag{13a}$$

where

$$\begin{aligned} \rho'^2 &= x'^2 + y'^2 + z'^2, \\ x' &= \rho'\sin\theta\cos\varphi, \\ y' &= \rho'\sin\theta\sin\varphi, \\ z' &= \rho'\cos\theta \end{aligned} \tag{13b}$$

are defined in terms of the Euclidean-like structure. Moreover, metric (13) [2] is related to the Schwarzschild solution by $r = M\kappa + \rho'$. Again, the same first-order results on both gravitational red shifts and light bending are obtained from metric (13).

Now, let us consider the extreme case for no radius contraction, i.e. $D(r)$=1; for clarity r is denoted by R in the metric, and

$$\begin{aligned} \mathrm{d}s^2 =& H(R)c^2\mathrm{d}t^2 - \mathrm{d}R^2 \\ & - G(R)^2R^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2). \end{aligned} \tag{14}$$

Here $H(R)$ is independent of time because it must be compatible with the gravitational red shifts. This metric is also incompatible with the Maxwell–Newton approximation which requires $\mathrm{d}s^2 \approx (1 - 2M\kappa/R)\mathrm{d}t^2 - (1+2M\kappa/R)[\mathrm{d}R^2 + R^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2)]$.

In comparing metric (14) with the isotropic metric, one obtains

$$\begin{aligned} H(R) &= [(1 - M\kappa/2r)^2/(1 + M\kappa/2r)^2], \\ RG(R) &= (1 + M\kappa/2r)^2 r, \end{aligned} \tag{15a}$$

and

$$\mathrm{d}R = (1 + M\kappa/2r)^2\mathrm{d}r. \tag{15b}$$

From Eq.(15b), one obtains a monotonically increasing function of r,

$$\begin{aligned} R(r) = r - M\kappa\ln(M\kappa/2r) - (M\kappa)^2/4r + C_0, \\ r > M\kappa/2, \end{aligned} \tag{16}$$

where C_0 ($= R[M\kappa/2]$) is a non-negative constant, which corresponds to the "event horizon" for this new metric. Obviously, solution (16) satisfies the requirement of being asymptotically flat. However, one may wonder what is required to obtain the same light bending. It turns out that C_0 is not included in the final result.

Consider a light ray in the X-direction as Einstein did. Then, for the first-order approximation, the light speed would be

$$\gamma = c[-g_{tt}/g_{XX}]^{1/2}, \quad g_{tt} = H(R), \tag{17a}$$

and

$$\begin{aligned} -g_{XX} =& G^2 + (1 - G^2)X^2/R^2 \\ =& 1 + (G^2 - 1)(1 - X^2/R^2). \end{aligned} \tag{17b}$$

It thus follows that, for the first-order approximation (if C_0 is of the same order as that of $M\kappa$) of the light speed,

[1] The time-tested assumption that phenomena can be explained in terms of identifiable causes is called the principle of causality. This principle implies that the gravitational radiation must have sources and any parameter in a physical solution must be related to some physical causes.[16,18] Thus, symmetry is preserved unless some causes break it, and if an unphysical parameter exists in the metric, such a manifold would not be diffeomorphic to a physical space.

[2] The harmonic solution (13) and the Schwarzschild solution are recognized as distinct because the meaning of coordinates has been clarified. Experimental tests to find the realistic gravitational solution were attempted,[32] but the required accuracy has not been reached. Since Zhou [20] advocated the harmonic gauge and Yilmaz [33] advocated his isotropic metric solution, both of them have conjectured that the Schwarzschild solution is not valid in physics. However, their common theoretical problem comes from the fact that the physical meaning of space coordinates had not been identified.

$$\gamma/c \approx 1 - \frac{M\kappa}{R} + \left(\frac{C_0 - M\kappa}{R} - \frac{M\kappa}{R}\ln\frac{Mk}{2R}\right) \times \left(1 - \frac{X^2}{R^2}\right). \quad (18a)$$

Then

$$B \approx \frac{1}{c}\int_{-\infty}^{+\infty}\frac{\partial\gamma}{\partial Y}\mathrm{d}X = \int_{-\infty}^{+\infty}\frac{\partial}{\partial Y}\left(-\frac{M\kappa}{R} - \frac{M\kappa\Delta^2}{R^3}\ln\frac{M\kappa}{2R}\right)\mathrm{d}X, \quad (18b)$$

since

$$\int_{-\infty}^{+\infty}\frac{\partial}{\partial Y}\left(\frac{\Delta^2}{R^3}\right)\mathrm{d}X = 0, \quad (18c)$$

where Δ is the distance from the centre of the sun in terms of R. Surprisingly, it turns out that

$$\int_{-\infty}^{+\infty}\frac{\partial}{\partial Y}\left(\frac{1}{R}\right)\mathrm{d}X = \int_{-\infty}^{+\infty}\frac{\partial}{\partial Y}\left(\frac{\Delta^2}{R^3}\ln\frac{M\kappa}{2R}\right)\mathrm{d}X, \quad (19a)$$

therefore

$$B = \left(\frac{4M\kappa}{\Delta}\right). \quad (19b)$$

The trick is to observe that

$$\int \mathrm{d}X\frac{\partial}{\partial Y}\left(\frac{\Delta^2}{R^3}\ln\frac{1}{R}\right) = \int \mathrm{d}X\left[\left(-3\frac{\Delta^3}{R^5} + \frac{2\Delta}{R^3}\right)\ln\frac{1}{R} - \frac{\Delta^3}{R^5}\right] = \int \frac{\mathrm{d}(\sin\theta)}{\Delta}((-3\cos^2\theta + 2)\ln(\cos\theta) - \cos^2\theta), \quad (20a)$$

and

$$\frac{1}{2}\int \mathrm{d}x[-1 + 3x^2]\ln(1 - x^2) = \frac{1}{2}(-x + x^3)\ln(1 - x^2) - \int \mathrm{d}x x^2. \quad (20b)$$

Note that Eq.(20a) is valid due to Eq.(18c). This calculation greatly increases our confidence on the first order of the bending of light.[1)] The reason is that this calculation is on an integral effect not a local effect such as the local light speeds.

Now let us examine the physical validity of metric (14). From Eq.(15a) and solution (16), one obtains

$$\begin{aligned} G(R) =& (1 + M\kappa/2r)^2 r/R \\ =& (1 + 1/\alpha)^2[1 + 2(\ln\alpha)/\alpha \\ & - 1/\alpha^2 + 2C_0/\alpha M\kappa]^{-1} \\ \approx& 1 - 2(\ln\alpha + C_0/M\kappa - 1)/\alpha + 2/\alpha^2, \end{aligned} \quad (21a)$$

where $\alpha = 2r/M\kappa > 1$. Thus, $G(R)$ is smaller than 1 for sufficiently large α, and this means "space expansion" instead of the usual space contraction if Einstein's equivalence principle were valid for this metric. Moreover, the local light speeds at the θ- and φ-directions would be

$$\begin{aligned} & (1 - 1/\alpha)(1 + 1/\alpha)^{-3} \\ & \times [1 + 2(\ln\alpha)/\alpha - 1/\alpha^2 + 2C_0/\alpha M\kappa] \\ \approx& 1 + 2(\ln\alpha - 2 + C_0/M\kappa)/\alpha > 1, \end{aligned} \quad (21b)$$

for sufficiently large α. However, the responsible logarithm term in Eq.(16), as shown by Eqs.(19) and (20), is normalized after integration. Moreover, if the arbitrary non-negative constant C_0 in Eq.(16) is chosen to be zero, the radius of the "event horizon" for this new metric would be zero. Thus, the notion of black holes is also not gauge invariant since C_0 is arbitrary. (This is very different from the Schwarzschild, the harmonic and the isotropic metrics. Although they have different values for the event horizon, these values correspond to each other in a definite manner.)

Now, it is clear that metric (14) is not a physical metric, and that general relativity is not a gauge invariant theory. Experimentally, there are three different results of the local light speeds to be tested: namely, (1) according to the Maxwell–Newton approximation, the first-order local light speeds are isotropic, (2) according to the Schwarzschild solution, the horizontal light speed is faster than the vertical one, and (3) according to metric (14), the local light speeds are larger than the light speed in vacuum.

5. Remarks

In Einstein's theory,[4,5] the reality is modelled with a physical space (–time) that has a frame of reference and its local time rate for the descriptions of the physics. Einstein has shown that such a physical

[1)] The deflection angle of light is transformed from two dots in a photographic plate by referring to the coordinate system used to take the photos. A deflection is against the straight line,[4,5] which is defined as the approximate trajectory when the light ray is far away from the sun. However, for a uniformly rotating system, the light ray is not a straight line. Apparently, Einstein's "arbitrary choice of a system of coordinates" is among those that a deflection angle can be defined.

space is a Riemannian manifold with a Lorentz metric and a space–time coordinate system. However, since the physical meaning of space–time coordinates was not clear, it is difficult to see that the Schwarzschild and the isotropic solutions are incompatible although the mathematical diffeomorphism (11) implies that they are physically different. Moreover, if a favourite argument of advocates for the black holes is that coordinates have no physical meaning, they probably did not realize this would also mean that the notion of black holes has no meaning.

Now, the physical meaning of space–time coordinates has been clarified, and the physical incompatibility between the Schwarzschild solution and the isotropic solution is clear. Then, it is natural to ask what is the realistic gauge of the gravity of the earth. A related question is that if the realistic gauge of the sun is not known, would a different value for the bending of light be obtained when the realistic gauge is found. This calculation of another extreme case assures that this is very unlikely. The reason is that the bending of light is a result of integrated effects, and is different from the results of local effects such as gravitational red shift and local light speeds. It would boost our confidence on the validity of the connection between Einstein's predictions and his theory before the realistic gauge is identified.

A problem of Einstein equation is that it does not have a dynamic solution.[13,18] However, the Maxwell–Newton approximation, which is independent of the Einstein equation,[16] has assured the validity of light deflection. Moreover, two types of experiments support the Maxwell–Newton approximation: namely, the tracking of planets and spacecraft by the Einstein–Infeld–Hoffman equation (9), and the observations on binary pulsars radiation [1)] in Ref.[13]. Therefore, it is expected that an experiment on local light speeds would directly support the Maxwell–Newton approximation.[2]

Since the assumption that the bending of light is "gauge independent" is further supported, the theory of gravitational lensing [26] would not be affected by the result of the experiment on local light speeds and Stanford's Gravity Probe-B. However, since the theory of black holes depends on the presumed validity of the Schwarzschild solution,[26] it would have to go through a revision. In short, a major review of general relativity would necessarily take place.

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[1)] It has been shown that a result of the calculated gravitational radiation depends on the approach chosen.[17]

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