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Probabilistic remote state preparation by W states *

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In this paper we consider a scheme for probabilistic remote state preparation of a general qubit by using W states. The scheme consists of the sender, Alice, and two remote receivers Bob and Carol. Alice performs a projective measurement on her qubit in the basis spanned by the state she wants to prepare and its orthocomplement. This allows either Bob or Carol to reconstruct the state with finite success probability. It is shown that for some special ensembles of qubits, the remote state preparation scheme requires only two classical bits, unlike the case in the scheme of quantum teleportation where three classical bits are needed.

Keywords: remote state preparation, W state, probabilistic, entanglement
PACC: 0365, 0367

One of the striking phenomena in quantum information theory is quantum teleportation, originally proposed by Bennett *et al*,[1] which allows the transmission of an unknown quantum state from a sender (Alice) to a receiver (Bob) by a maximally entangled quantum channel and two classical bit communication. Recently, Lo,[2] Pati,[3] and Bennett *et al*[4] have presented an interesting new topic, remote state preparation (RSP), which is very similar to teleportation. RSP has the same task as quantum teleportation, i.e. Alice helps Bob in his laboratory to prepare a quantum state by means of a previously shared entanglement and some classical information. The main difference between RSP and teleportation is that in the former Alice is assumed to have complete classical knowledge of the state to be prepared by Bob, while in the latter neither Alice nor Bob has any classical knowledge of the teleported state. In RSP, the central issue is to investigate whether the required entanglement and the classical communication cost can be reduced with Alice's knowledge of the teleported state.

As is pointed out by Lo [2] and Pati,[3] for some special ensemble of qubits, the classical communication cost in RSP is less than that in standard teleportation. However, for general states, Lo [2] conjectured that RSP requires as much classical communication cost as quantum teleportation. Furthermore, Bennett *et al*[4] have considered the tradeoff between the required entanglement and classical communication, and have shown that the asymptotic classical communication cost of RSP of any qubits is one bit per qubit if a large amount of prior entanglement exists. Since then, there are a number of theoretical protocols for generalization of RSP, such as RSP in the higher-dimension case,[5] oblivious RSP,[6] RSP for two parties,[7] optimal RSP,[8] low-entanglement RSP,[9] and continuous variable RSP in phase space.[10] What is more, Peng *et al*[11] have reported an experimental implementation of RSP by the method of nuclear magnetic resonance. In addition, Shi and Tomita [12] have considered a scheme for preparing remotely a qubit using a three-particle Greenberge–Horne–Zeilinger (GHZ) state. As we know, for tripartite entanglement, there are two nonequivalent classes of tripartite entangled states,[13] the GHZ class and the W class, even under stochastic local operation and classical communication. Particularly, an interesting property of W state is that it remains bipartite entanglement when any one of the three qubits is traced out. Therefore, is it possible to remotely prepare a general quantum state by using W states as the quantum channel? How many classical communication bits are needed in the process of RSP?

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The goal of this paper is to give a scheme for probabilistic remote state preparation of a general qubit based on shared W states. We find that for some specially chosen qubits from either an equatorial or a polar great circle on Bloch sphere, the receiver Bob (Carol) needs only two classical bits in the protocol of RSP, one bit from Alice, the other from Carol (Bob), which is one bit less than that in the teleportation scheme of Ref.[14], and the successful probability of the two tasks would be equal. However, for general states, the probability of successful RSP is only half of that in the scheme of quantum teleportation.

Suppose that sender Alice and two receivers Bob and Carol share a three-particle entangled W state $|W\rangle_{ABC} = \frac{1}{\sqrt{3}}(|100\rangle_{ABC} + |010\rangle_{ABC} + |001\rangle_{ABC})$ (to avoid confusion, we call it the special W state in this paper), where particles A, B and C belong to Alice, Bob and Carol respectively. Now Alice wishes to help Bob or Carol prepare remotely an arbitrary single qubit state $|\phi\rangle = \alpha|0\rangle + \beta|1\rangle$, which is known to Alice but unknown to Bob and Carol. We also assume that the parameter α is real and β is a complex number, $|\alpha|^2 + |\beta|^2 = 1$. In order to help Bob or Carol prepare the qubit $|\phi\rangle$, since Alice knows the parameters α and β exactly, she can perform a projective measurement on the particle A in the qubit basis $\{|\psi\rangle, |\psi_\perp\rangle\}$, which is related to the computation basis $\{|0\rangle, |1\rangle\}$. The relation between the qubit basis $\{|\psi\rangle, |\psi_\perp\rangle\}$ and the computation basis $\{|0\rangle, |1\rangle\}$ is given by

$$\begin{aligned} |\psi\rangle_A &= \alpha|0\rangle_A + \beta|1\rangle_A, \\ |\psi_\perp\rangle_A &= \alpha|1\rangle_A - \beta^*|0\rangle_A. \end{aligned} \tag{1}$$

Then, the special W state can be rewritten as

$$\begin{aligned} |W\rangle_{ABC} =& \frac{1}{\sqrt{3}}(|100\rangle_{ABC} + |010\rangle_{ABC} + |001\rangle_{ABC}) \\ =& \frac{1}{\sqrt{3}}[(\beta^*|\psi\rangle_A + \alpha|\psi_\perp\rangle_A)|00\rangle_{BC} \\ &+ (\alpha|\psi\rangle_A - \beta|\psi_\perp\rangle_A)|10\rangle_{BC} \\ &+ (\alpha|\psi\rangle_A - \beta|\psi_\perp\rangle_A)|01\rangle_{BC}] \\ =& \frac{1}{\sqrt{3}}[|\psi\rangle_A(\alpha|10\rangle_{BC} + \alpha|01\rangle_{BC} \\ &+ \beta^*|00\rangle_{BC}) + |\psi_\perp\rangle_A(\alpha|00\rangle_{BC} \\ &- \beta|10\rangle_{BC} - \beta|01\rangle_{BC})]. \end{aligned} \tag{2}$$

Now a selective measurement is performed on particle A, and the outcome $|\psi_\perp\rangle$ reduces particles B and C to $\frac{1}{\sqrt{1+|\beta|^2}}[\alpha|00\rangle_{BC} - \beta|10\rangle_{BC} - \beta|01\rangle_{BC}]$. On the one hand, if Bob wants to prepare the original state $|\phi\rangle$, he can ask Carol to make the projection measurement on particle C in the basis $\{|0\rangle, |1\rangle\}$, then we have

$$\begin{aligned} &\frac{1}{\sqrt{1+|\beta|^2}}[\alpha|00\rangle_{BC} - \beta|10\rangle_{BC} - \beta|01\rangle_{BC}] \\ =& \frac{1}{\sqrt{1+|\beta|^2}}[(\alpha|0\rangle_B - \beta|1\rangle_B)|0\rangle_C - \beta|01\rangle_{BC}]. \end{aligned} \tag{3}$$

If the result of Carol's measurement is $|0\rangle_C$, the particle B will be projected to the state $\alpha|0\rangle_B - \beta|1\rangle_B$. After having received the classical information about the measurement result from Carol, Bob performs a unitary operation σ_z on particle B, (in this paper σ_x, σ_y, σ_z stand for Pauli matrices), thus he can construct the original state $|\phi\rangle$ since $\sigma_z(\alpha|0\rangle_B - \beta|1\rangle_B) = \alpha|0\rangle_B + \beta|1\rangle_B$. Therefore, the successful probability for remote preparation of general qubits corresponding to the outcome $|\psi_\perp\rangle$ on particle A is $\frac{1+|\beta|^2}{3} \cdot \frac{1}{1+|\beta|^2} = 1/3$. In this case, Bob needs two classical bits to prepare the state $|\phi\rangle$ successfully, one bit from Alice and the other from Carol, which is one bit less than that in the scheme for teleportation using the entangled W state.[14,15] In contrast, if Carol obtains the state $|1\rangle_C$, the particle B will have collapsed to $|0\rangle_B$, and clearly the scheme has failed. On the other hand, suppose that Carol hopes to construct the state $|\phi\rangle$, what Bob needs to do is to make the projection measurement on particle B in the basis $\{|0\rangle, |1\rangle\}$, then

$$\begin{aligned} &\frac{1}{\sqrt{1+|\beta|^2}}[\alpha|00\rangle_{BC} - \beta|10\rangle_{BC} - \beta|01\rangle_{BC}] \\ =& \frac{1}{\sqrt{1+|\beta|^2}}[|0\rangle_B(\alpha|0\rangle_C - \beta|1\rangle_C) - \beta|10\rangle_{BC}]. \end{aligned} \tag{4}$$

Similarly, if the result measured by Bob is $|0\rangle_B$, Carol can prepare the state $|\phi\rangle$ by making the operation σ_z on particle C. The probability of success is also $1/3$, which does not depend on the parameters α and β of the single qubit state.

In addition, from Eq.(2), if Alice finds the state $|\psi\rangle_A$ in a single-particle projective measurement, the special W state will collapse to $\frac{1}{\sqrt{1+\alpha^2}}[\alpha|10\rangle_{BC} + \alpha|01\rangle_{BC} + \beta^*|00\rangle_{BC}]$, which can be reexpressed as $\frac{1}{\sqrt{1+\alpha^2}}[\alpha|10\rangle_{BC} + |0\rangle_B(\alpha|1\rangle_C + \beta^*|0\rangle_C)]$ or $\frac{1}{\sqrt{1+\alpha^2}}[(\alpha|1\rangle_B + \beta^*|0\rangle_B)|0\rangle_C + \alpha|01\rangle_{BC}]$. When the outcome of Bob's (Carol's) measurement result is $|0\rangle_B$ ($|0\rangle_C$), Carol (Bob) can only get the orthogonal-complement qubit $\alpha|1\rangle_1 - \beta^*|0\rangle_1$ of the single state $|\phi\rangle$ up to a corresponding rotation operator σ_z, and she (he) cannot convert the complement qubit unitarily

into the single state since it involves an antiunitary operation.[3,11] If the single state $|\phi\rangle$ is a real qubit chosen from the polar line or a qubit chosen from the equatorial line on Bloch sphere,[3] then Carol (Bob) can remotely construct the specially chosen qubits with the prior shared entanglement and the classical information about the measurement result from Alice and Bob (Carol). In this case, the successful probability for RSP is $\frac{1+|\alpha|^2}{3}\cdot\frac{1}{1+|\alpha|^2}=1/3$. Thus the total probability of successful remote state preparation of the special ensemble of qubits is 1/3+1/3=2/3, which is the same as the probability of the scheme of teleportation.[14,15]

Of course, we can achieve the probabilistic RSP scheme in another way: from the W state $|W\rangle_{ABC}$, one can obtain with probability 2/3 a maximally entangled state shared between parties A and C by means of a measurement in the eigenbasis of σ_z of particle B and keeping the result corresponding to $|0\rangle$ (or alternatively a maximally entangled state between A and B by measuring the particle C). This maximally entangled state can then be used for RSP based on the standard bipartite protocol.[3] So the total success probability is 2/3, which is independent of the original state $|\phi\rangle$ that Alice wishes to prepare remotely.

As we know, the general W state can be written as

$$|W^{'}\rangle_{ABC} = a|100\rangle_{ABC} + b|010\rangle_{ABC} + c|001\rangle_{ABC} + d|000\rangle_{ABC}, \quad (5)$$

where the parameters $a, b, c > 0$, $d \geq 0$ and $a^2+b^2+c^2+d^2=1$. Now Alice wishes to use the general W state for the remote preparation of the state $|\phi\rangle$. The projection measurement made by Alice on particle A is also given by Eq.(1), and then

$$\begin{aligned}|W^{'}\rangle_{ABC} =& |\psi\rangle_A(a\beta^*|00\rangle_{BC} + b\alpha|10\rangle_{BC} \\ &+ c\alpha|01\rangle_{BC} + d\alpha|00\rangle_{BC}) \\ &+ |\psi_\perp\rangle_A(a\alpha|00\rangle_{BC} - b\beta|10\rangle_{BC} \\ &- c\beta|01\rangle_{BC} - d\beta|00\rangle_{BC}). \end{aligned} \quad (6)$$

In virtue of the theoretical analysis on Eq.(6), we find that if the coefficient $d \neq 0$, the standard remote state preparation via the general W entangled state as quantum channel cannot be realized. Thus, in the following, we will only discuss the special case of $d = 0$. Furthermore, without loss of generality, we assume that the parameters satisfy $a > b > c$. Firstly, we consider the case that Alice, together with Carol, helps, Bob prepare remotely the original state $|\phi\rangle$. If the result of Alice's measurement on particle A is $|\psi_\perp\rangle$, the general W state will have collapsed into

$$\frac{1}{\sqrt{|\beta|^2+a^2(\alpha^2-|\beta|^2)}} \times [a\alpha|00\rangle_{BC} - b\beta|10\rangle_{BC} - c\beta|01\rangle_{BC}]. \quad (7)$$

The probability that Alice measures an outcome $|\psi_\perp\rangle$ is

$$p_1 = |_A\langle\psi_\perp|W^{'}\rangle_{ABC}|^2 = |\beta|^2 + a^2(\alpha^2-|\beta|^2).$$

Since Eq.(7) can be rewritten as

$$\frac{1}{\sqrt{|\beta|^2+a^2(\alpha^2-|\beta|^2)}} \times [(a\alpha|0\rangle_B - b\beta|1\rangle_B)|0\rangle_C - c\beta|01\rangle_{BC}],$$

if Carol's measurement outcome on particle C is $|0\rangle_C$, the measurement probability is $\frac{a^2\alpha^2+b^2|\beta|^2}{|\beta|^2+a^2(\alpha^2-|\beta|^2)}$ and particle B will be projected to $\frac{1}{\sqrt{a^2\alpha^2+b^2|\beta|^2}}(a\alpha|0\rangle_B - b\beta|1\rangle_B)$, which can be transformed into the state $\alpha|0\rangle_B - \beta|1\rangle_B$ according to the following procedure. Bob introduces an auxiliary particle F with the original state $|0\rangle_F$, and makes a collective unitary transformation [16,17] U_1 under the basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}_{BF}$,

$$U_1 = \begin{pmatrix} \frac{b}{a} & -\sqrt{1-\frac{b^2}{a^2}} & 0 & 0 \\ \sqrt{1-\frac{b^2}{a^2}} & \frac{b}{a} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (8)$$

Then the joint state of Bob and the auxiliary particle will be transformed into

$$\begin{aligned} & U_1\frac{1}{\sqrt{a^2\alpha^2+b^2|\beta|^2}}(a\alpha|0\rangle_B - b\beta|1\rangle_B)|0\rangle_F \\ =& \frac{b}{\sqrt{a^2\alpha^2+b^2|\beta|^2}}(\alpha|0\rangle_B - \beta|1\rangle_B)|0\rangle_F \\ &+ \sqrt{\frac{(a^2-b^2)\alpha^2}{a^2\alpha^2+b^2|\beta|^2}}|0\rangle_B|1\rangle_F. \end{aligned} \quad (9)$$

Next a measurement on the auxiliary particle is performed. If $|1\rangle_F$ is obtained, the transformation fails. But if the measurement outcome is $|0\rangle_F$, the transformation will succeed since Bob can turn the state $(\alpha|0\rangle - \beta|1\rangle)_B$ into $(\alpha|0\rangle + \beta|1\rangle)_B$ by making a σ_z operation. Thus the probability of successful state

preparation corresponding to the measurement result $|\psi_\perp\rangle$ on particle A is $[|\beta|^2 + a^2(\alpha^2 - |\beta|^2)] \times \frac{a^2\alpha^2 + b^2|\beta|^2}{|\beta|^2 + a^2(\alpha^2 - |\beta|^2)} \frac{b^2}{a^2\alpha^2 + b^2|\beta|^2} = b^2$. Besides, from Eq.(6), if Alice finds the state $|\psi\rangle_A$ in the projective measurement, the state of particles B and C becomes $\frac{1}{\sqrt{\alpha^2 + a^2(|\beta|^2 - \alpha^2)}}[(a\beta^*|0\rangle_B + b\alpha|1\rangle_B)|0\rangle_C + c\alpha|01\rangle_{BC}]$. Using the above methods and introducing the same unitary transformation U_1, Bob can get the state $\beta^*|0\rangle_B + \alpha|1\rangle_B$, which cannot be converted into the single state $|\phi\rangle$ since it is unknown to him. Thus, for general arbitrary qubits, the process of RSP fails. However, if α and β are real numbers or the state $|\phi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + e^{i\varphi}|1\rangle)$, Bob can construct the original single state with the help of Carol. In this case, the successful probability is $[\alpha^2 + a^2(|\beta|^2 - \alpha^2)] \times \frac{a^2|\beta|^2 + b^2\alpha^2}{\alpha^2 + a^2(|\beta|^2 - \alpha^2)} \frac{b^2}{a^2|\beta|^2 + b^2\alpha^2} = b^2$. Therefore, for the specially chosen qubits, the total successful probability of remote state preparation on Bob's side is $2b^2$, which is the same as the probability of quantum teleportation.[14] Clearly, in the scheme of remotely preparing the specially chosen qubits, Bob requires only two classical communication bits, one bit from Alice, the other from Carol, which is one bit less than that in the teleportation case.[14]

Secondly, with the help of Bob, Carol can also prepare the state $|\phi\rangle$ remotely. In the case of Alice's measurement outcome $|\psi_\perp\rangle$, since Eq.(7) can be given by $\frac{1}{\sqrt{|\beta|^2 + a^2(\alpha^2 - |\beta|^2)}}[|0\rangle_B(a\alpha|0\rangle_C - c\beta|1\rangle_C) - b\beta|10\rangle_{BC}]$, if the state $|1\rangle_B$ is obtained by Bob, the scheme fails. Furthermore, if the measurement result on particle B by Bob is $|0\rangle_B$, the state of particle C is $\frac{1}{\sqrt{a^2\alpha^2 + c^2|\beta|^2}}(a\alpha|0\rangle_C - c\beta|1\rangle_C)$. By introducing an auxiliary qubit with the state $|0\rangle_F$ and constructing a collective transformation U_2 on the basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}_{CF}$,

$$U_2 = \begin{pmatrix} \frac{c}{a} & -\sqrt{1 - \frac{c^2}{a^2}} & 0 & 0 \\ \sqrt{1 - \frac{c^2}{a^2}} & \frac{c}{a} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad (10)$$

Carol can transform the combined state of particle C and the auxiliary particle F into

$$\begin{aligned} & U_2 \frac{1}{\sqrt{a^2\alpha^2 + c^2|\beta|^2}}(a\alpha|0\rangle_C - c\beta|1\rangle_C)|0\rangle_F \\ =& \frac{c}{\sqrt{a^2\alpha^2 + c^2|\beta|^2}}(\alpha|0\rangle_C - \beta|1\rangle_C)|0\rangle_F \\ & + \sqrt{\frac{(a^2 - c^2)\alpha^2}{a^2\alpha^2 + c^2|\beta|^2}}|0\rangle_C|1\rangle_F. \end{aligned} \quad (11)$$

If the result of measurement on the auxiliary qubit is $|0\rangle_F$, Carol can prepare the original state $|\phi\rangle$ successfully by making a σ_z operation on the particle C. In this case, the successful probability of RSP for Carol as the receiver is $[|\beta|^2 + a^2(\alpha^2 - |\beta|^2)] \times \frac{a^2\alpha^2 + c^2|\beta|^2}{|\beta|^2 + a^2(\alpha^2 - |\beta|^2)} \frac{c^2}{a^2\alpha^2 + c^2|\beta|^2} = c^2$. Moreover, based on Eq.(6), if Alice finds $|\psi\rangle_A$ in the projective measurement, the state of particles B and C will become $\frac{1}{\sqrt{\alpha^2 + a^2(|\beta|^2 - \alpha^2)}}[|0\rangle_B(a\beta^*|0\rangle_C + c\alpha|1\rangle_C) + b\alpha|10\rangle_{BC}]$. In similar ways and by introducing the same unitary transformation U_2, Carol can get the state $\beta^*|0\rangle_C + \alpha|1\rangle_C$, which cannot be converted into the original state $|\phi\rangle$ since it is unknown to her. Thus, for general arbitrary qubits, the process of RSP fails. However, if α and β are real numbers or the state $|\phi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + e^{i\varphi}|1\rangle)$, Carol can construct the original single state with the help of Bob. In this regard, the successful probability for RSP corresponding to the measurement result $|\psi\rangle_A$ is $[\alpha^2 + a^2(|\beta|^2 - \alpha^2)] \frac{a^2|\beta|^2 + c^2\alpha^2}{\alpha^2 + a^2(|\beta|^2 - \alpha^2)} \frac{c^2}{a^2|\beta|^2 + c^2\alpha^2} = c^2$. Therefore, for the specially chosen qubits, the scheme of remote state preparation needs only two classical bits and the total successful probability of RSP for the receiver Carol is $2c^2$, which is the same as the quantum teleportation.[14] Thus we have proved that the general W state can also be used to achieve probabilistic remote state preparation.

In fact, we can also apply the technique of generalized measurement to deal with the transformation from the state $\frac{1}{\sqrt{a^2\alpha^2 + b^2|\beta|^2}}(a\alpha|0\rangle_B - b\beta|1\rangle_B)$ $\left(\text{or } \frac{1}{\sqrt{a^2\alpha^2 + c^2|\beta|^2}}(a\alpha|0\rangle_C - c\beta|1\rangle_C)\right)$ to $\alpha|0\rangle + \beta|1\rangle$, and from $\frac{1}{\sqrt{a^2|\beta|^2 + b^2\alpha^2}}(a\beta^*|0\rangle_B + b\alpha|1\rangle_B)$ $\left(\text{or } \frac{1}{\sqrt{a^2|\beta|^2 + c^2\alpha^2}}(a\beta^*|0\rangle_C + c\alpha|1\rangle_C)\right)$ to $\beta^*|0\rangle + \alpha|1\rangle$. For instance, Bob now introduces an auxiliary qubit $|0\rangle_F$, and makes a controlled-not (CNOT) unitary operation between particles B and F, where the

particle B is the controller and the particle F the target, then

$$\frac{1}{\sqrt{a^2\alpha^2+b^2|\beta|^2}}(a\alpha|0\rangle_B-b\beta|1\rangle_B)|0\rangle_F \xrightarrow{\text{CNOT}} \frac{1}{\sqrt{a^2\alpha^2+b^2|\beta|^2}}(a\alpha|00\rangle_{BF}-b\beta|11\rangle_{BF})$$

$$=\frac{1}{2}\sqrt{\frac{a^2+b^2}{a^2\alpha^2+b^2|\beta|^2}}[(\alpha|0\rangle_B+\beta|1\rangle_B)(x_1|0\rangle_F-y_1|1\rangle_F) + (\alpha|0\rangle_B-\beta|1\rangle_B)(x_1|0\rangle_F+y_1|1\rangle_F)], \quad (12)$$

where we assume $x_1=\frac{a}{\sqrt{a^2+b^2}}$ and $y_1=\frac{b}{\sqrt{a^2+b^2}}$. To realize remote preparation of the state $|\phi\rangle$ on Bob's side, we need to discriminate between the two nonorthogonal states $x_1|0\rangle_F+y_1|1\rangle_F$ and $x_1|0\rangle_F-y_1|1\rangle_F$. It is clear that the two nonorthogonal quantum states cannot be distinguished with certainty, since the scalar product of the two states is $|x_1^2-y_1^2|$. As we know, optimal state discrimination between two nonorthogonal states can be made by means of generalized measurement, also known as positive operator-valued measurement (POVM), which has recently been demonstrated experimentally using free-space interferometry.[18] The optimal POVM operation [19,20] corresponding to the two quantum states $x_1|0\rangle_F+y_1|1\rangle_F$ and $x_1|0\rangle_F-y_1|1\rangle_F$ is given by

$$A_1=\frac{1}{2x_1^2}\begin{pmatrix} y_1^2 & x_1y_1 \\ x_1y_1 & x_1^2 \end{pmatrix},$$

$$A_2=\frac{1}{2x_1^2}\begin{pmatrix} y_1^2 & -x_1y_1 \\ -x_1y_1 & x_1^2 \end{pmatrix},$$

$$A_3=\begin{pmatrix} 1-\frac{y_1^2}{x_1^2} & 0 \\ 0 & 0 \end{pmatrix}. \quad (13)$$

Since we have known $x_1>y_1$, the probability of the optimal identification of the two states from such a generalized measurement is $1-(x_1^2-y_1^2)=2y_1^2$. Based on the results of the state discrimination, Bob can construct the qubit state $|\phi\rangle$ with certain probability. If Bob confirms that after the POVM measurement, the state of the auxiliary particle F is $x_1|0\rangle_F-y_1|1\rangle_F$ $(x_1|0\rangle_F+y_1|1\rangle_F)$; of course, he can also confirm that particle B is in the state $\alpha|0\rangle_B+\beta|1\rangle_B$ $(\alpha|0\rangle_B-\beta|1\rangle_B)$. When the state of the particle B is $\alpha|0\rangle_B+\beta|1\rangle_B$, the scheme of remote state preparation is successfully achieved. For the state $\alpha|0\rangle_B-\beta|1\rangle_B$, Bob performs a σ_z unitary operation on particle B, then transforms it into $\alpha|0\rangle_B+\beta|1\rangle_B$, which is the desirable state for Bob. In this case, the probability of successful remote state preparation for the receiver Bob is $[|\beta|^2+a^2(\alpha^2-|\beta|^2)]\frac{a^2\alpha^2+b^2|\beta|^2}{|\beta|^2+a^2(\alpha^2-|\beta|^2)}$ $\times\frac{1}{4}\cdot\frac{a^2+b^2}{a^2\alpha^2+b^2|\beta|^2}\cdot 2\cdot 2y_1^2=b^2$. Likewise, using the POVM operation Eq.(13), Bob can also transform the state $\frac{1}{\sqrt{a^2|\beta|^2+b^2\alpha^2}}(a\beta^*|0\rangle_B+b\alpha|1\rangle_B)$ into the state $\beta^*|0\rangle+\alpha|1\rangle$ with the same probability of state discrimination $2y_1^2$.

Similarly, in the case of the state

$$\frac{1}{\sqrt{a^2\alpha^2+c^2|\beta|^2}}(a\alpha|0\rangle_C-c\beta|1\rangle_C),$$

Carol can also prepare the single qubit $|\phi\rangle$ probabilistically by introducing an auxiliary qubit in a state $|0\rangle_F$ and using the following generalized measurement for discrimination between the two nonorthogonal states $x_2|0\rangle_F+y_2|1\rangle_F$ and $x_2|0\rangle_F-y_2|1\rangle_F$:

$$B_1=\frac{1}{2x_2^2}\begin{pmatrix} y_2^2 & x_2y_2 \\ x_2y_2 & x_2^2 \end{pmatrix},$$

$$B_2=\frac{1}{2x_2^2}\begin{pmatrix} y_2^2 & -x_2y_2 \\ -x_2y_2 & x_2^2 \end{pmatrix},$$

$$B_3=\begin{pmatrix} 1-\frac{y_2^2}{x_2^2} & 0 \\ 0 & 0 \end{pmatrix}, \quad (14)$$

where $x_2=\frac{a}{\sqrt{a^2+c^2}}$, $y_2=\frac{c}{\sqrt{a^2+c^2}}$. In this way, the optimal probability of the state identification is $2y_2^2$ and the successful probability of RSP for Carol is c^2. With the same state discrimination probability $2y_2^2$, Carol can also turn $\frac{1}{\sqrt{a^2|\beta|^2+c^2\alpha^2}}(a\beta^*|0\rangle_C+c\alpha|1\rangle_C)$ into $\beta^*|0\rangle+\alpha|1\rangle$ by using the matrices (14).

In conclusion, we have presented a scheme for probabilistic remote state preparation of a general qubit by W states. It has been shown that in the protocol of successful RSP, the receiver Bob (Carol) requires only two classical bits, one bit from Alice, the other from Carol (Bob), which is one bit less than that in the corresponding teleportation scheme.[14] In addition, for some specially chosen qubits from either an equatorial or a polar great circle on Bloch sphere, the successful probabilities of the two tasks would be equal. However, in the case of general states, regardless of the special W state or the general W being

used as the quantum channel of RSP, the successful probability is only half of that in the teleportation scheme.[14] It should be mentioned that for some special ensembles of qubits, the remote state preparation scheme not only reduces the cost of classical communication bits, but does not require Bell-state measurement which is difficult to perform experimentally. Our results about RSP may be helpful to consider more carefully the classical communication cost in quantum information processing, and may be useful in studying quantum communication complexity. Note that the W state can be generated via cavity quantum electrodynamics,[21] thus our scheme may be realized with current experimental technology.

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