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Magnetic properties of mixed Ising system with random field*

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A spin-1/2 and spin-3/2 mixed Ising system in a random field is studied by the use of effective-field theory with correlations. The phase diagrams and thermal behaviours of magnetizations are investigated numerically for the honeycomb lattice (z=3) and square lattice (z=4) respectively. The tricritical behaviours for both honeycomb and square lattices, as well as the reentrant behaviour for the square lattice are found.

Keywords: mixed spins, random field, phase diagram, magnetizations
PACC: 7510H, 7510D, 7550G

1. Introduction

The random field Ising model (RFIM) for mono-atomic lattices has been one of subjects of theoretical[1−5] and experimental[6−8] studies in the statistical physics of random and frustrated systems. This is because it is a rather complicated problem and its difficulty challenges many investigators to find a solution. One of the main points concerning the RFIM is the occurrence of a tricritical behaviour. Indeed, the mean-field theory[9] predicted that the existence of a tricritical point depended on the form of the random field distribution. In particular, the bimodal distribution yields a tricritical point, while for a model with a Gaussian distribution of random field the phase transition remains to be of the second-order. The existence of a tricritical behaviour has been examined by the use of various techniques, such as mean-field theory,[10] Monte Carlo simulations, [11] renormalization-group calculations,[12] Bethe–Peieris approximation[13] and effective-field theories (EFT).[14] It is worth noting that the analyses of the RFIM have been almost limited to the simple spin-1/2 systems. Some interest has been directed to the understanding of more complicated systems in the presence of random fields using different models, such as the transverse Ising model,[15] the amorphous Ising ferromagnet,[16] the site-dilute Ising model,[17] the semi-infinite Ising model,[18] the decorated Ising model[19] and the Blume–Capel model.[20] On the other hand, some authors[21,22] extended the study of the random field effects to the spin-3/2. The obtained phase diagram is qualitatively similar to those of the spin-1/2 systems.

Recently, attention[23−28] has been paid to the study of the magnetic properties of the two-sublattice mixed Ising systems. These systems are of interest because they consist of two interpenetrating inequivalent sublattices and have less translational symmetry than their single-spin counterparts. The former property is very important to study a certain type of ferrimagnetism, namely molecule-based magnetic materials. As far as we know, no work has been done on magnetic properties of the spin-1/2 and spin-3/2 mixed Ising system in a longitudinal random field. Therefore, in this work we are to study the random field effects on the spin-1/2 and spin-3/2 mixed Ising system by the use of the effective-field theory with correlations.

The paper is organized as follows. In Section 2 the

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general formulation of the problem within the framework of EFT is presented. In Section 3 the phase diagram and the temperature dependence of the magnetization are examined in detail for the honeycomb lattice and square lattice. Finally, a brief discussion is given in Section 4.

2. Formulation

We consider a mixed system consisting of spin-1/2 and spin-3/2 in a longitudinal random field. The Hamiltonian of the system is given by

$$H = -\sum_{\langle i,j\rangle} J_{ij}\sigma_i S_j - \sum_i H_i\sigma_i - \sum_j H_j S_j, \qquad (1)$$

where σ_i takes the value $\frac{1}{2}$ or $-\frac{1}{2}$, S_j can be $\pm\frac{1}{2}$ and $\pm\frac{3}{2}$. J_{ij} is the exchange interaction which couples the nearest neighbours only. We shall consider that each spin is subjected to a random magnetic field H_i according to the following distribution:

$$P(H_i) = \frac{1}{2}\left[\delta(H_i - H) + \delta(H_i + H)\right]. \qquad (2)$$

The starting point for the statistics of our spin system is the relation proposed by SaBarreto *et al*,[28] in which the sublattice magnetizations for the mixed Ising system are given by

$$M_a = \langle\sigma_i\rangle = \left[\cosh(J\eta\nabla) + \frac{M_b}{\eta}\sinh(J\eta\nabla)\right]^Z F_A(x)\,|_{x=0}\ , \qquad (3)$$

$$M_b = \langle S_j\rangle = \left[\cosh\left(\frac{1}{2}J\nabla\right) + 2M_a\sinh\left(\frac{1}{2}J\nabla\right)\right]^z F_B(x)\,|_{x=0}\ , \qquad (4)$$

and the parameter q is defined as

$$q = \eta^2 = \left\langle (S_j)^2\right\rangle = \left[\cosh\left(\frac{1}{2}J\nabla\right) + 2M_a\sinh\left(\frac{1}{2}J\nabla\right)\right]^z G(x)\,|_{x=0}\ , \qquad (5)$$

where $\nabla = \partial/\partial x$ is differential operator and functions $F_A(x)$, $F_B(x)$ and $G(x)$ are

$$F_A(x) = \frac{1}{4}\left\{\tanh\left[\frac{1}{2}\beta(x+h)\right] + \tanh\left[\frac{1}{2}\beta(x-h)\right]\right\}, \qquad (6)$$

$$F_B(x) = \frac{1}{2}\left\{\frac{3\sinh\left[\frac{3}{2}\beta(x+h)\right] + \sinh\left[\frac{1}{2}\beta(x+h)\right]}{2\cosh\left[\frac{3}{2}\beta(x+h)\right] + 2\cosh\left[\frac{1}{2}\beta(x+h)\right]} + \frac{3\sinh\left[\frac{3}{2}\beta(x-h)\right] + \sinh\left[\frac{1}{2}\beta(x-h)\right]}{2\cosh\left[\frac{3}{2}\beta(x-h)\right] + 2\cosh\left[\frac{1}{2}\beta(x-h)\right]}\right\}, \qquad (7)$$

$$G(x) = \frac{1}{2}\left\{\frac{9\cosh\left[\frac{3}{2}\beta(x+h)\right] + \cosh\left[\frac{1}{2}\beta(x+h)\right]}{4\cosh\left[\frac{3}{2}\beta(x+h)\right] + 4\cosh\left[\frac{1}{2}\beta(x+h)\right]} + \frac{9\cosh\left[\frac{3}{2}\beta(x-h)\right] + \cosh\left[\frac{1}{2}\beta(x-h)\right]}{4\cosh\left[\frac{3}{2}\beta(x-h)\right] + 4\cosh\left[\frac{1}{2}\beta(x-h)\right]}\right\}, \qquad (8)$$

where $\beta = 1/k_{\rm B}T$.

Equations (3)–(5) can be evaluated for any lattice structure characterized by the coordination number z. In particular, for the case of a honeycomb lattice (z=3) and the case of a square lattice (z=4) one finds that

$$M_a = A_{z1}M_b + A_{z2}M_b^3, \qquad (9)$$

$$M_b = B_{z1}M_a + B_{z2}M_a^3, \qquad (10)$$

$$\eta^2 = q_{z1} + q_{z2}M_a^2 + q_{z3}M_a^4. \qquad (11)$$

The coefficients A_{zi}, B_{zi}, q_{zi} (z=3,4, and i=1, 2, 3) can be easily calculated by applying a mathematical relation $e^{\alpha\nabla}f(x) = f(x+\alpha)$ and they are given in the Appendix. Then the average of total magnetization per site is given by

$$\frac{M}{N} = \frac{M_a + M_b}{2}. \qquad (12)$$

The sublattice magnetization M_a in the vicinity of transition point can be written as

$$M_a^2 = \frac{1-a}{b}. \qquad (13)$$

The parameters a and b are obtained as follows

$$a = A_{z1}B_{z1}, \qquad (14)$$

$$b = \frac{A'_{z1}B_{z1}q_{z2}}{2\eta_0} + B_{z2}A_{z1} + A_{z2}B_{z1}^3, \qquad (15)$$

where A_{zi} (i=1, 2) are defined as a function of η_0, which is the value of η at $T = T_{\rm c}$, and A'_{z1} denotes

the first derivative of A_{z1} with respect to η_0. According to Ref.[30], when $a = 1$ and $b < 0$ the transition is of the second order; when $a = 1$ and $b > 0$ the transition is of the first order; and the tricritical point at which the phase transition changes from the second order to the first order is determined by $a = 1$ and $b = 0$.

3. Numerical results and discussions

3.1. Magnetic properties of the honeycomb lattice

In this section we are interested in studying the transition temperature and thermal variations of magnetization for the honeycomb lattice. At first we examine the finite-temperature phase diagram of the mixed-spin system for the honeycomb lattice.

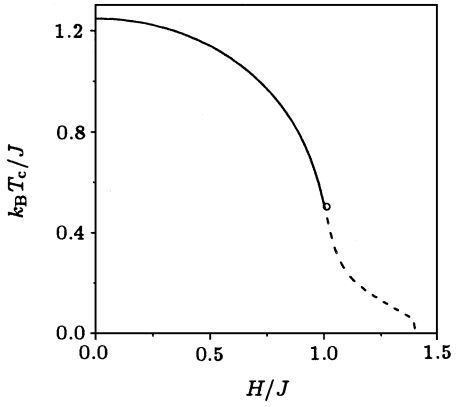


Fig.1. Phase diagram in the T–H plane of the mixed Ising system in a random field for the honeycomb lattice (z=3). The solid and dashed lines represent the second- and the first-order phase transitions, respectively. The white circle indicates the tricritical point.

In Fig.1 is presented the phase diagram in the (k_BT_C/J, H/J) plane for the honeycomb lattice. The solid line and the dashed line are used for the second- and first-order transitions, respectively, while the white circle with coordinates (H_t/J, k_BT_t/J) $\approx$(1.0063, 0.4902) indicates the tricritical point. In particular, the value of T_C at $H/J = 0$ is given from k_BT_C/J=1.246, which is very similar to k_BT_C/J=1.216 obtained in Refs.[29–31]. From the figure, we obtain the critical random field, as H_C/J = 1.401 at which the transition temperatures reduce to zero. From Fig.1 we can see that the system exhibits a different behaviour depending on the random-field strength H/J. Namely, for $0 \leq H/J < 1.0063$ we can see the second-order phase transition, while for $1.0063 \leq H/J < 1.401$ the first-order phase transition appears.

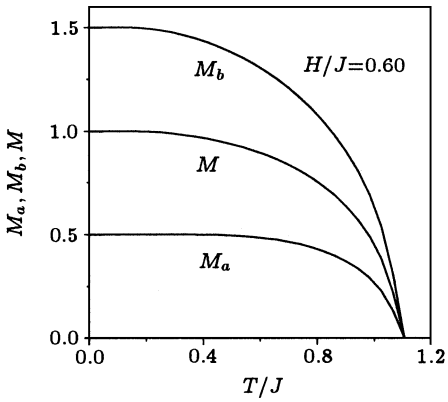


Fig.2. The temperature dependence of the sublattice magnetizations M_a, M_b and the total magnetization M of the mixed Ising system for the honeycomb lattice (z=3), when the random field is set as $H/J = 0.60$.

Now, let us show some typical temperature dependence of magnetizations in the spin-1/2 and spin-3/2 mixed system in the presence of the random field. The results are given in Figs.2 and 3. Figure 2 shows typical temperature dependence of sublattice magnetizations M_a, M_b and the total magnetization M for $H/J = 0.60$. We can find the magnetization curves monotonically decrease from the saturation values M_a=0.50, M_b=1.50, M=1.00 at T=0K and vanish at the critical temperature. In fact, this corresponds to the second-order transition. On the other hand, when H/J is in a range of $1.0063 \leq H/J < 1.401$, the system undergoes a first-order transition, and the magnetizations M_a, M_b and M exhibit a gap at that transition. Such thermal behaviours are depicted in Fig.3 when we set the value of the random field H/J as $H/J = 1.10$.

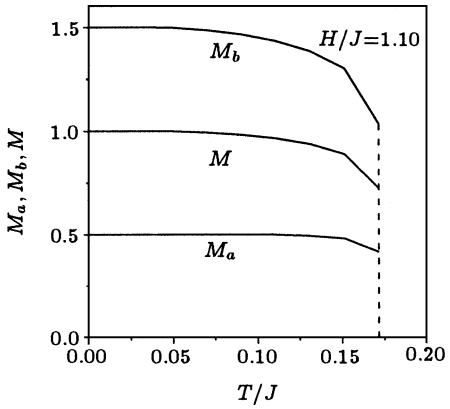


Fig.3. The temperature dependence of the sublattice magnetizations M_a, M_b and the total magnetization M of the mixed spin Ising system for the honeycomb lattice (z=3), when the random field is set as $H/J = 1.10$.

3.2. Magnetic properties of the square lattice

In this section, we show some typical results for the spin-1/2 and spin-3/2 mixed random-field Ising system for the square lattice. Figure 4 shows the behaviour of the critical temperature as a function of H/J. In contrast to the case of z=3 (for Fig.1), the phase transition line exhibits the behaviour similar to that in Fig.1. The solid line and the dashed line are used for the second- and first-order transitions, respectively, while the white circle with coordinates (H_t/J, k_BT_t/J ≈1.3878, 0.8018) indicates the tricritical point. On the other hand, the phase transition line exhibits different behaviours from those in Fig.1. The curve exhibits a bulge in a region of $1.101 \leq H/J \leq 1.411$, which implies the occurrence of reentrant phenomenon due to the effect of the random-field. Particularly, the T_C value at $H/J = 0$ is given from k_BT_C/J=1.806 for the square lattice. From Fig.4 we can see that the system exhibits a different behaviour depending on the random-field strength H/J. Namely, for $0 \leq H/J < 1.101$ we can see the second-order phase transition only, while for $1.101 \leq H/J < 1.3878$ the phase transition line shows the second-order phase transition at higher temperature and the first-order one in a low-temperature region. Finally, in a very narrow region $1.3878 \leq H/J < 1.4110$ the transition line shows the first-order phase transition at both higher temperature and lower temperature.

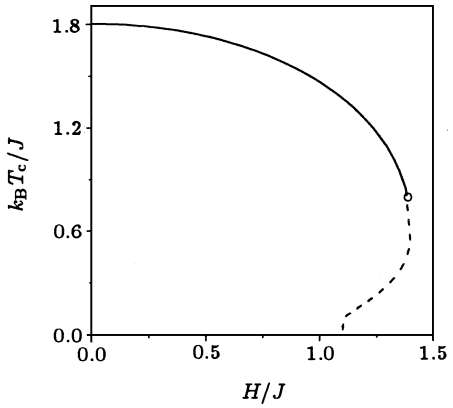


Fig.4. Phase diagram in T–H plane of the mixed Ising system in a random field for the square lattice (z=4). The solid and dashed lines represent the second- and the first-order phase transitions, respectively. The white circle indicates the tricritical point.

In Figs.5, 6 and 7 we show the temperature dependence of sublattice magnetizations M_a, M_b and the total magnetization M for the square lattice. In Fig.5 the sublattice magnetizations M_a, M_b and the total magnetization M are plotted versus temperature for $H/J = 0.60$. We can find the magnetization curves monotonically decrease from the saturation values $M_a = 0.50$, $M_b = 1.50$,$M = 1.00$ at T=0K and vanish at the critical temperature. In fact, this corresponds to the second-order transition. The temperature dependence shown in Fig.6 is the same as that in Fig.5, but for $H/J = 1.25$ where both first- and second-order phase transitions are clearly visible. As the value of H/J increases, the first-order phase transition moves to the higher temperature region. Consequently, the region with nonzero magnetization is rapidly reduced. The effect is illustrated in Fig.6(b) with $H/J = 1.36$. Moreover, a different behaviour of the magnetization can be found near the first-order phase transition temperature. If the value of H/J is weak enough ($1.10 \leq H/J \leq 1.25$), then the magnetizations show a monotonical behaviour near the first-order phase transition temperature. On the other hand, for $H/J > 1.25$ a broad maximum can be seen in a region close to the first-order phase transition temperature. Finally, a very interesting case is presented in Fig.7. For $H/J = 1.40$ the nonzero sublattice magnetizations M_a, M_b and the total magnetization M first decrease monotonically from the first critical temperature and then reach certain values at the second critical temperature. This result is consistent with the phase transition line. In this case two phase transitions of first-order can be observed.

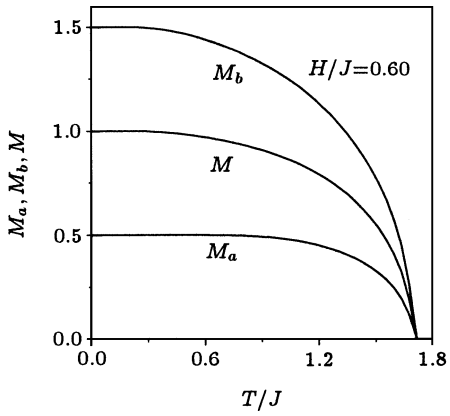


Fig.5. The temperature dependence of the sublattice magnetizations M_a, M_b and the total magnetization M of the mixed Ising system for the square lattice (z=4), when the random field is set as $H/J = 0.60$.

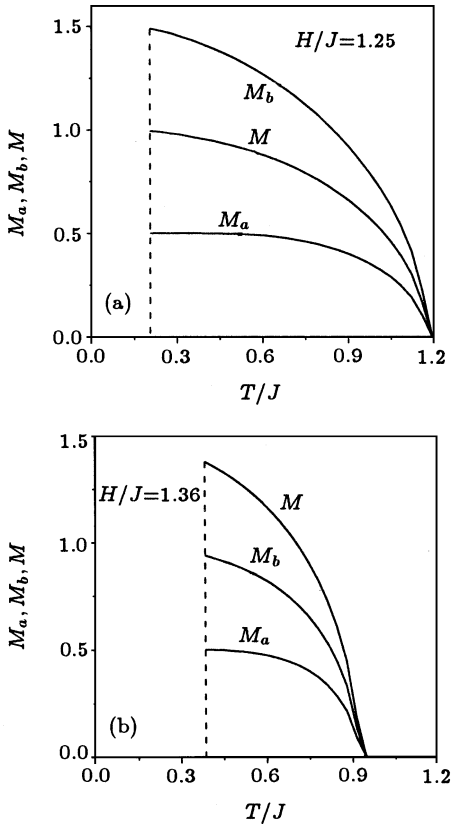


Fig.6. The temperature dependence of the sublattice magnetizations M_a, M_b and the total magnetization M of the mixed Ising system for the square lattice (z=4), when the random field is set respectively as $H/J = 1.25$ (a) and $H/J = 1.36$ (b).

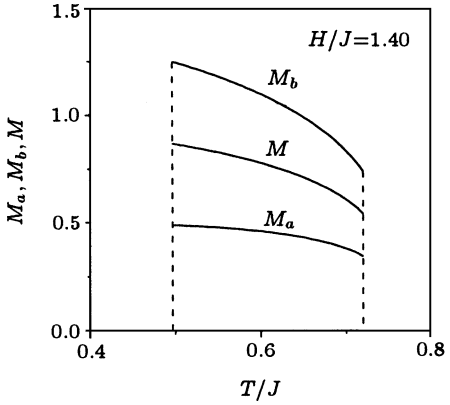


Fig.7. The temperature dependence of the sublattice magnetizations M_a, M_b and the total magnetization M of the mixed spin Ising system for the square lattice (z=4), when the random field is set as $H/J = 1.40$.

4. Conclusion

In this work, we have studied mixed Ising systems in a longitudinal random field by the use of the effective-field theory with correlations. We have discussed the influence of the random field on the transition temperature and the magnetizations. A number of interesting phenomena have been found, which are due to the longitudinal random field. We have found that the tricritical behaviour appears for both the honeycomb lattice and the square lattice. On the other hand, the reentrant behaviour occurs for the square lattice. It yields a lot of reasonable results in comparison with those in the other literatures.

Finally, we hope that the present method will be potentially useful for studying more complicated systems in the presence of a random field.

Appendix

For the honeycomb lattice (z=3):

$$A_{31} = \frac{3}{4\eta}\left[F_A(3J\eta) + F_A(J\eta)\right],$$
$$A_{32} = \frac{1}{4\eta^3}\left[F_A(3J\eta) - 3F_A(J\eta)\right],$$
$$B_{31} = \frac{3}{2}\left[F_B\left(\frac{3}{2}J\right) + F_B\left(\frac{1}{2}J\right)\right],$$
$$B_{32} = 2\left[F_B\left(\frac{3}{2}J\right) - 3F_B\left(\frac{1}{2}J\right)\right],$$
$$q_{31} = \frac{1}{4}\left[G\left(\frac{3}{2}J\right) + 3G\left(\frac{1}{2}J\right)\right],$$
$$q_{32} = 3\left[G\left(\frac{3}{2}J\right) - G\left(\frac{1}{2}J\right)\right],$$
$$q_{33} = 0.$$

For the square lattice (z=4):

$$A_{41} = \frac{1}{2\eta}\left[F_A(4J\eta) + 2F_A(2J\eta)\right],$$
$$A_{42} = \frac{1}{2\eta^3}\left[F_A(4J\eta) - 2F_A(2J\eta)\right],$$
$$B_{41} = F_B(2J) + 2F_B(J),$$
$$B_{42} = 4F_B(2J) - 8F_B(J),$$
$$q_{41} = \frac{1}{8}\left[G(2J) + 4G(J) + 3G(0)\right],$$
$$q_{42} = 3\left[G(2J) - G(0)\right],$$
$$q_{43} = 2G(2J) - 8G(J) + 6G(0).$$

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