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Calculation of geometrical birefringence from two-dimensional refractive-index profile in single-mode fibres*

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We present two numerical approaches to calculate the geometrical birefringence from two-dimensional refractive-index profile in single-mode fibres. One is the improved perturbation model theory, the other is based on the general solution of noncircular uniform optical waveguide in an arbitrary coordinate system. Using these methods, we can obtain not only the magnitude of geometrical birefringence, but also the orientation of birefringent axes. We report the calculation results of geometrical birefringence in a real single-mode fibre.

Keywords: birefringence, refractive-index profile, polarization mode dispersion
PACC: 4280M, 4281C, 4281F

1. Introduction

Polarization-mode dispersion (PMD) becomes important in very high bit-rate optical communication systems.[1−4] PMD is caused by the local birefringence in fibres. The PMD has been modelled by using the dynamic equation[5]

$$\frac{\partial \Omega}{\partial z} = \frac{\partial \Delta\boldsymbol{\beta}}{\partial \omega} + \Delta\boldsymbol{\beta} \times \Omega, \tag{1}$$

where Ω is the PMD vector, $\Delta\boldsymbol{\beta} = (\Delta\beta_1\ \Delta\beta_2\ \Delta\beta_3)^{\mathrm{T}}$ is the local birefringence vector, z and ω are fibre length and angular frequency respectively. From this equation, we know that PMD distribution in fibres depends not only on the magnitude of the local birefringence vector, but also on the orientation of the fibre's birefringent axes. In commercial telecommunication fibres, the circular birefringence β_3 is negligible, so local birefringence can be expressed as[6]

$$\Delta\boldsymbol{\beta}(z,\omega) = \Delta\beta_l \begin{pmatrix} \cos 2\alpha \\ \sin 2\alpha \\ 0 \end{pmatrix}, \tag{2}$$

where $\Delta\beta_l$ is the magnitude of the linear local birefringence, α is the angle between the birefringent axes and the x-axis of the coordinate system. If we want to determine PMD distribution along the fibre, we must know $\Delta\beta_l$ and α distributions simultaneously. $\Delta\beta_l$ includes intrinsic birefringence and induced birefringence. The former includes geometrical birefringence and stress birefringence. In this paper, we only take into account the geometrical birefringence, which comes from the noncircularity of refractive-index profile in real fibres. So the problem that we need to solve is how to calculate the local geometrical birefringence using two-dimensional (2D) refractive-index profile.

In the past 20 years, several methods have been developed to calculate birefringence in weakly-guiding optical waveguides.[7−9] In the analytical method, the authors only analysed the simple profiles which are different from those of real fibres. In the numerical method, both scalar perturbation method and vectorial perturbation method presented can only evaluate the magnitude of linear birefringence, not the orientation of the birefringent axes; and the symmetry of the index profile about x-axis or y-axis must be assumed. But strictly speaking, the 2D index profile in real fibres has no symmetry at all.

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This paper is divided into four sections. Firstly, in Section 2 we will present an improved vectorial perturbation model method which can calculate the geometrical birefringence, both its magnitude and the orientation of its birefringent axes, from the 2D index profile measured directly from a single-mode fibre. Then we also present another novel method based on the general solution of the noncircular uniform waveguides in an arbitrary coordinate system. Finally, in Section 4 we calculate the geometrical birefringence in a real single-mode fibre from its refractive-index profiles measured on the preform.

2. Method I: Improved vectorial perturbation model

In Ref.[10], a perturbation model for computing optical fibre birefringence from a 2D refractive-index profile has been presented, in which the author has almost obtained the full information about the birefringence vector, what he left out needs to explain further.

An arbitrary 2D index profile can be expressed as $n_{r,\theta} = n_r + \Delta n_{r,\theta}$. We consider this profile to be axially uniform. As we know, in a uniform waveguide, there always exists a pair of orthogonal linearly polarized modes $\boldsymbol{E}_{Tp}$ with $p = 1, 2$,[11] which are given by the equation

$$(\nabla_T^2 + k^2 n_{r,\theta}^2 - \beta_p^2)\boldsymbol{E}_{Tp} + \nabla_T(\boldsymbol{E}_{Tp} \cdot \nabla_T \ln n_{r,\theta}^2) = 0, \tag{3}$$

where k is the propagation constant in free space, β_p is the propagation constant in the fibre and $\nabla_T = \frac{\partial}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial}{\partial \theta}\hat{\theta}$ is the operator in the cylindrical coordinate system. Generally, fibres are weakly-guiding, so the longitudinal field component $E_z \approx 0$. The total field can be expressed as

$$\boldsymbol{E}_T = a_1 \boldsymbol{E}_{T1} \exp(\mathrm{j}\beta_1 z) + a_2 \boldsymbol{E}_{T2} \exp(\mathrm{j}\beta_2 z). \tag{4}$$

The birefringence is defined as $\Delta\beta_l = |\beta_1 - \beta_2|$.

When $n_{r,\theta} = n_r$, Eq.(3) becomes

$$(\nabla_T^2 + k^2 n_r^2 - \beta_0^2)\boldsymbol{e}_{Tq} + \nabla_T(\boldsymbol{e}_{Tq} \cdot \nabla_T \ln n_r^2) = 0. \tag{5}$$

From Eq.(5), we can obtain the two orthogonal degenerate linearly polarized modes $\boldsymbol{e}_{Tq}$, $q = 1, 2$. The transverse field $\boldsymbol{E}_{Tp}$ is the linear combination of the two degenerate modes

$$\boldsymbol{E}_{Tp} = \sum_{q=1}^{2} c_{pq} \boldsymbol{e}_{Tq}. \tag{6}$$

We calculate $\iint \{\boldsymbol{e}_{Tq}^* \cdot Eq.(3) - \boldsymbol{E}_{Tp} \cdot \mathrm{Eq.}(5)^*\}\mathrm{d}s$ and consider the orthonormality condition to be $\iint \boldsymbol{e}_{Tp} \cdot \boldsymbol{e}_{Tq}^* \mathrm{d}s = \delta_{pq}$, where δ_{pq} is the Kronecker delta function. Thus we have

$$\begin{aligned}
&\iint (\boldsymbol{e}_{Tq}^* \cdot \nabla_T^2 \boldsymbol{E}_{Tp} - \boldsymbol{E}_{Tp} \cdot \nabla_T^2 \boldsymbol{e}_{Tq}^*)\mathrm{d}s \\
&+ \iint [k^2(n_{r,\theta}^2 - n_r^2) - (\beta_p^2 - \beta_0^2)]\boldsymbol{e}_{Tq}^* \cdot \boldsymbol{E}_{Tp}\mathrm{d}s \\
&+ \iint [\boldsymbol{e}_{Tq}^* \cdot \nabla_T(\boldsymbol{E}_{Tp} \cdot \nabla_T \ln n_{r,\theta}^2) \\
&- \boldsymbol{E}_{Tp} \cdot \nabla_T(\boldsymbol{e}_{Tq} \cdot \nabla_T \ln n_r^2)]\mathrm{d}s = 0.
\end{aligned} \tag{7}$$

Considering

$$\begin{aligned}
n_{r,\theta}^2 &= n_r^2 + 2n_r \Delta n_{r,\theta} + \Delta n_{r,\theta}^2 \\
&\approx n_r^2 + 2n_r \Delta n_{r,\theta} = n_r^2(1 + 2\delta_{r,\theta})
\end{aligned}$$

and $\delta_{r,\theta} = \Delta n_{r,\theta}/n_r$,

$$\begin{aligned}
&\iint (\boldsymbol{e}_{Tq}^* \cdot \nabla_T^2 \boldsymbol{E}_{Tp} - \boldsymbol{E}_{Tp} \cdot \nabla_T^2 \boldsymbol{e}_{Tq}^*)\mathrm{d}s \\
&+ \iint 2k^2 n_r^2 \delta_{r,\theta} \boldsymbol{e}_{Tq}^* \cdot \boldsymbol{E}_{Tp}\mathrm{d}s - c_{pq}(\beta_p^2 - \beta_0^2) \\
&+ \iint \boldsymbol{e}_{Tq}^* \cdot \nabla_T(\boldsymbol{E}_{Tp} \cdot \nabla_T \ln(1 + 2\delta_{r,\theta}))\mathrm{d}s \\
&+ \iint [\boldsymbol{e}_{Tq}^* \cdot \nabla_T(\boldsymbol{E}_{Tp} \cdot \nabla_T \ln n_r^2) \\
&- \boldsymbol{E}_{Tp} \cdot \nabla_T(\boldsymbol{e}_{Tq} \cdot \nabla_T \ln n_r^2)]\mathrm{d}s = 0.
\end{aligned} \tag{8}$$

According to the Stokes theorem, the first and the last items are equal to zero in Eq.(8).[10] Hence we obtain

$$\begin{aligned}
&c_{pq}(\beta_p^2 - \beta_0^2) \\
&= -\sum_{l=1}^{2} c_{pl} \left\{ \iint 2k^2 n_r^2 \delta_{r,\theta} \boldsymbol{e}_{Tq}^* \cdot \boldsymbol{e}_{Tl}\mathrm{d}s \right. \\
&\quad \left. - \iint 2\boldsymbol{e}_{Tq}^* \cdot \nabla_T(\boldsymbol{e}_{Tl} \cdot \nabla_T \delta_{r,\theta})\mathrm{d}s \right\} \\
&= \sum_{l=1}^{2} c_{pl} M_{ql}.
\end{aligned} \tag{9}$$

By calculating the eigenvalues of the matrix M, we can calculate the two propagation constants β_1 and β_2. Finally, the geometrical birefringence can be deduced.

To determine the birefringent axes, we substitute the known β_p into Eq.(9), then we obtain a ratio $c_{p1}/c_{p2} = \tan\alpha$. This α is the angle between the x-axis and the birefringent axes related to β_p.

To compute the vector solution $\boldsymbol{e}_{Tq}$, $q = 1, 2$ of unperturbed field, we use the numerical method in Ref.[12].

3. Method II: A novel method based on the general solution of the noncircular uniform waveguides

We have obtained a new form of Helmholtz equation in Ref.[11] as

$$\nabla_T^2 \boldsymbol{E}_T + \{k^2 n_{r,\theta}^2 \boldsymbol{I} - [\bar{\beta}^2 + (\Delta\beta)^2]\boldsymbol{I} - 2\bar{\beta}\Delta\beta \boldsymbol{V}\}\boldsymbol{E}_T = -\nabla_T(\boldsymbol{E}_T \cdot \nabla_T \ln n_{r,\theta}^2), \tag{10}$$

where $\bar{\beta} = \dfrac{\beta_1 + \beta_2}{2}$, $\Delta\beta = \dfrac{\beta_1 - \beta_2}{2}$, $\boldsymbol{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $\boldsymbol{V} = \begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{pmatrix}$. For an unperturbed fibre, $\Delta\beta = 0$ and Eq. (10) can be changed into

$$(\nabla_T^2 + k^2 n_r^2 - \bar{\beta}^2)\boldsymbol{e}_T + \nabla_T(\boldsymbol{e}_T \cdot \nabla_T \ln n_r^2) = 0. \tag{11}$$

Based on the numerical method described in Ref.[12], we can compute the $\bar{\beta}$ and unperturbed electric field $\boldsymbol{e}_T$.

To calculate the birefringence $\Delta\beta$ and birefringent axes, we consider an elliptical core step-index fibre and assume that its $\bar{\beta}$ and $\Delta\beta$ are equal to those of our noncircular fibre, but its major axis and minor axis are superposed with x-axis and y-axis respectively. Then, it meets the following equation:

$$\nabla_T^2 \boldsymbol{e}_e + \left\{k^2 n_e^2 - (\bar{\beta}^2 + \Delta\beta^2) - 2\bar{\beta}\Delta\beta \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}\right\}\boldsymbol{e}_e = -\nabla_T(\boldsymbol{e}_e \cdot \nabla_T \ln n_e^2), \tag{12}$$

where $\boldsymbol{n}_{\mathrm{e}}$ is the refractive index of elliptical core fibre, $\boldsymbol{e}_{\mathrm{e}}$ is its transverse electric field. We make the manipulation $\boldsymbol{e}_{\mathrm{e}}^* \cdot (10) - \boldsymbol{E} \cdot (12)^*$ and then integrate both sides over the infinite cross-section. After simplification, we obtain

$$\begin{aligned} &-2\bar{\beta}\Delta\beta \iint \left[\boldsymbol{e}_e^* \cdot \nabla \boldsymbol{E}_T - \boldsymbol{E}_T \cdot \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}\boldsymbol{e}_e^*\right] \mathrm{d}s + k \iint (n_{r,\theta}^2 - n_e^2)\boldsymbol{e}_e^* \cdot \boldsymbol{E}_T \mathrm{d}s \\ &= \iint [(\boldsymbol{E}_T \cdot \nabla_T \ln n_{r,\theta}^2)(\nabla_T \cdot \boldsymbol{e}_e^*) - (\boldsymbol{e}_e^* \cdot \nabla_T \ln n_e^2)(\nabla_T \cdot \boldsymbol{E}_T)] \mathrm{d}s. \end{aligned} \tag{13}$$

In Eq. (13), if $\boldsymbol{E}_T = \boldsymbol{E}_{T1} = \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix} E_{T1}$ and $\boldsymbol{e}_e = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e_x$. Here e_x is the mode field of elliptical core fibre and can always be expressed as[13]

$$e_x = \begin{cases} \dfrac{J_0(U\frac{r}{a})}{J_0(U)} + e^2 \dfrac{W^2}{8} \dfrac{J_0(U)K_2(W)J_2(U\frac{r}{a})}{J_1^2(U)K_0(W)} \cos 2\alpha & r < a, \\ \dfrac{K_0(W\frac{r}{a})}{K_0(W)} + e^2 \dfrac{W^2}{8} \dfrac{J_0(U)J_2(U)K_2(W\frac{r}{a})}{J_1^2(U)K_0(W)} \cos 2\alpha & r > a, \end{cases} \tag{14}$$

where J_n and K_n are the nth-order Bessel functions, U and W are the mode parameters in the core and cladding respectively, e is the ellipticity, a is the radius of the core. Then Eq.(13) becomes

$$b_1 \cos\alpha + b_2 \sin\alpha = 0, \tag{15}$$

where

$$\begin{cases} b_1 = \iint \left[\left(\dfrac{\partial \ln n_{r,\theta}^2}{\partial x}\right)\left(\dfrac{\partial e_x^*}{\partial x}\right) E_{T1} - \left(\dfrac{\partial \ln n_e^2}{\partial x}\right)\left(\dfrac{\partial E_{T1}}{\partial x}\right) e_x^* - k(n_{r,\theta}^2 - n_e^2) e_x^* E_{T1} \right] \mathrm{d}s, \\ b_2 = \iint \left[\left(\dfrac{\partial \ln n_{r,\theta}^2}{\partial y}\right)\left(\dfrac{\partial e_x^*}{\partial x}\right) E_{T1} - \left(\dfrac{\partial \ln n_e^2}{\partial x}\right)\left(\dfrac{\partial E_{T1}}{\partial y}\right) e_x^* \right] \mathrm{d}s. \end{cases}$$

From Eq.(15), we can compute the orientations of birefringent axes which are represented by the angle α.

Now, in Eq.(13), if we let $\boldsymbol{E}_T = \boldsymbol{E}_{T2} = \begin{pmatrix} -\sin\alpha \\ \cos\alpha \end{pmatrix} E_{T2}$ and $\boldsymbol{e}_e = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e_x$, we can obtain

$$-4\bar{\beta}\Delta\beta \sin\alpha \iint e_x^* E_{T2} \mathrm{d}s = d_1 \cos\alpha - d_2 \sin\alpha, \tag{16}$$

where

$$\begin{cases} d_1 = \iint \left[\left(\dfrac{\partial \ln n_{r,\theta}^2}{\partial y} \right) \left(\dfrac{\partial e_x^*}{\partial x} \right) E_{T2} - \left(\dfrac{\partial \ln n_e^2}{\partial x} \right) \left(\dfrac{\partial E_{T2}}{\partial y} \right) e_x^* \right] \mathrm{d}s, \\ d_2 = \iint \left[\left(\dfrac{\partial \ln n_{r,\theta}^2}{\partial x} \right) \left(\dfrac{\partial e_x^*}{\partial x} \right) E_{T2} - \left(\dfrac{\partial f \ln n_e^2}{\partial x} \right) \left(\dfrac{\partial E_{T2}}{\partial x} \right) e_x^* - k(n_{r,\theta}^2 - n_e^2) e_x^* E_{T2} \right] \mathrm{d}s. \end{cases}$$

From Eq.(16), we have the magnitude of birefringence $\Delta\beta_l = 2|\Delta\beta|$. But, the above equations have been found under the precondition that $\bar{\beta}$, $\Delta\beta$ of the elliptical fibre are equal to those of the noncircular fibre. So, we have to recur to iterative method to achieve the final result. The major procedures are:

1) Using the numerical method in Ref.[12] to calculate $\bar{\beta}$, mode field $\boldsymbol{e}_T$ of a circular fibre.

2) Taking the mode field of the circular core step-index fibre[13]

$$e_0 = \begin{cases} J_0\left(U\dfrac{r}{a}\right)/J_0(U) & r < a, \\ K_0\left(W\dfrac{r}{a}\right)/K_0(W) & r > a, \end{cases} \tag{17}$$

as the first-order approximation of mode fields e_x and $\boldsymbol{e}_T$ as the first-order approximation of $\boldsymbol{E}_T$, we can find the first-order approximation of α_1 and $\Delta\beta_1$. Then we can obtain the second-order approximation of e_x and $\boldsymbol{E}_T$.

3) Using these second-order approximation of e_x and $\boldsymbol{E}_T$ to obtain the second-order approximation of α_2 and $\Delta\beta_2$, then we obtain the third-order approximation of e_x and $\boldsymbol{E}_T$.

Repeating this process until $|\alpha_i - \alpha_{i-1}| \leq \varepsilon_1$ and $|\Delta\beta_i - \Delta\beta_{i-1}| \leq \varepsilon_2$, ε_1 and $\varepsilon_2$are the calculating accuracy requirements we need.

4. Results and conclusion

Fifty of two-dimensional refractive-index profiles in a single-mode fibre have been provided by Yangtze Optical Fibre and Cable Company Ltd, one of which is shown in Fig.1.

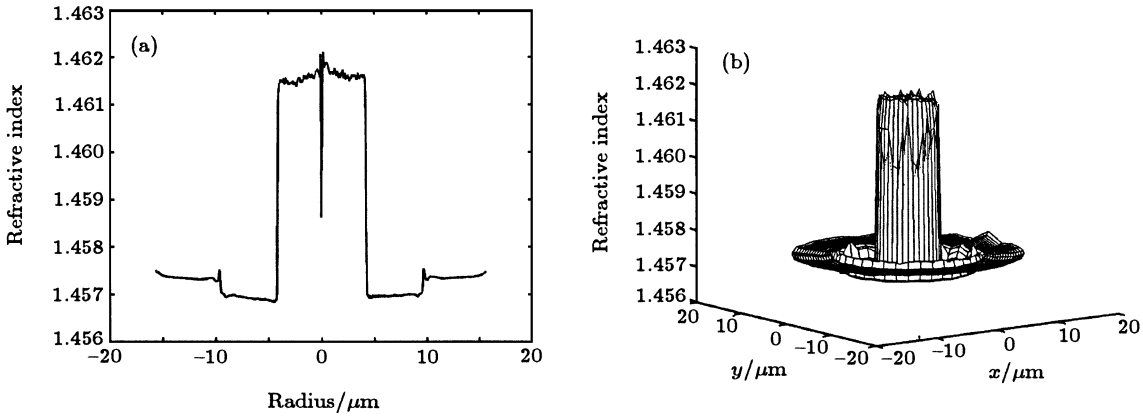


Fig.1. A 2D refractive-index profile measured in a single-mode fibre (a) along x direction (b) the 3D profile.

Using the above two methods, we have calculated the geometrical birefringences for different profiles. Both results are almost the same with very little differences, as shown in Fig.2.

From Fig.1, we know the refractive-index profiles in a real fibre are deviated from perfect circular symmetry, and the deviation causes the geometrical birefringence. Furthermore, the deviation is random so it cannot be simply described as elliptical symmetry. Then we must find out a method to calculate the geometrical birefringence from the arbitrary refractive-index profile. Compared with the calculation of the magnitude of birefringence, how to determine the birefringent axis is a more complicated problem. We present two methods to accomplish this work. One is to improve the former work to make it possible to calculate not only the magnitude but also the birefringent axis. The other method is based on the concept of the general solution of the noncircular uniform waveguides in an arbitrary coordinate system we have presented. With the two methods, we calculate 50 of the 2D refractive-index profiles we have measured in a fibre preform. The results show that the two methods have very good consistency. We can learn the geometrical birefringence distribution in a real fibre from Fig.2.

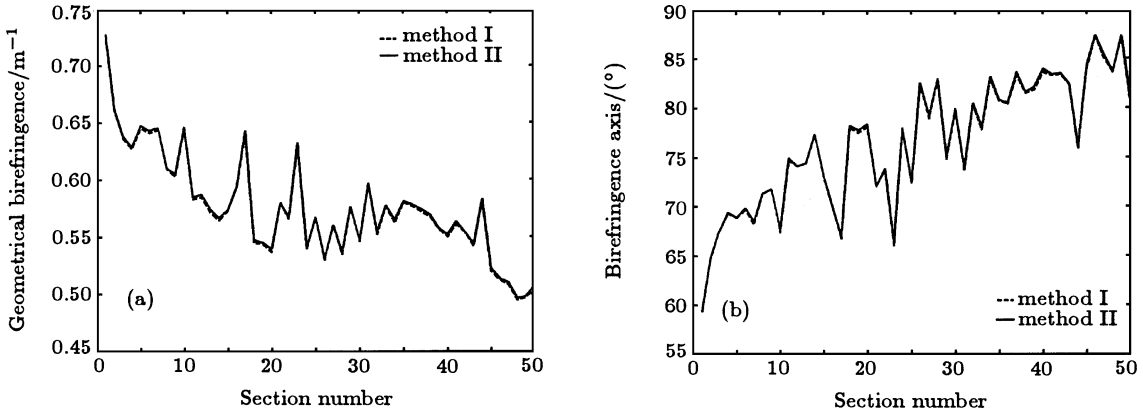


Fig.2. The magnitude (a) and orientation (b) of geometrical birefringence for 50 sections.

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