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Modulation instability of quasi-plane-wave optical beams in biased photorefractive-photovoltaic crystals*

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We investigate the modulation instability of quasi-plane-wave optical beams in biased photorefractive–photovoltaic crystals by globally treating the space-charge field. The modulation instability growth rate is obtained, which depends on the external bias field, on the bulk photovoltaic effect, and on the ratio of the optical beam's intensity to that of the dark irradiance. Our analysis indicates that this modulation instability growth rate is identical to the modulation instability growth rate studied previously in biased photorefractive–nonphotovoltaic crystals when the bulk photovoltaic effect is negligible for shorted circuits, and predicts the modulation instability growth rate in open- and closed-circuit photorefractive–photovoltaic crystals when the external bias field is absent.

Keywords: optical spatial solitons, modulation instability, photorefractive effect
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Spatial optical solitons[1−12] in photorefractive (PR) materials have been a topic of considerable research because of their possible applications for optical switching and routing. To date, two different types of steady-state PR solitons are known: screening solitons,[7,8] which occur when an external bias voltage is appropriately applied to a photorefractive–nonphotovoltaic (P-N) crystal, and photovoltaic (PV) solitons,[9,10] which exist in PR materials with appreciable PV coefficients. Of particular interest are solitons in biased photorefractive–photovoltaic (P-P) crystals, which change into screening solitons and PV solitons under appropriate conditions.[6,11,12] However, modulation instability (MI) is a general process inherent to most nonlinear systems.[13] It occurs in the same parameter region where solitons are observed, and it is considered as a precursor of soliton formation. In biased P-N crystals, experimental evidence of MI is provided by observing local effects in the space-charge field,[14] and soon afterwards the MI growth rate is obtained by globally treating the space-charge field.[15] In biased P-P crystals, we should also pay attention to MI.

In this paper, we study the modulation instability of quasi-plane-wave optical beams in biased P-P crystals under steady-state conditions. The MI growth rate is obtained by globally treating the space-charge field. The MI growth rate depends on the external bias field, on the bulk photovoltaic effect, and on the ratio of the optical beam's intensity to that of the dark irradiance. Moreover, our analysis indicates that when the bulk photovoltaic effect is negligible for shorted circuits, the MI growth rate is identical to the MI growth rate studied previously in biased P-N crystals, and when the external bias field is absent, this MI growth rate predicts the MI growth rate in open- and closed-circuit P-P crystals.

To study the modulation instability of a broad optical beam in a biased P-P crystal under steady-state conditions, let us first consider an optical beam that propagates in a P-P crystal along the z axis and is allowed to diffract only along the x direction. The

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P-P crystal is assumed to be $LiNbO_3$ with its optical c-axis oriented along the x direction. Moreover, let us assume that the optical beam is linearly polarized along x and that the external bias electric field is applied in the same direction. Under these conditions, the perturbed extraordinary refractive index is given by $n_{\rm e}'^2 = n_{\rm e}^2 - n_{\rm e}^4 r_{33} E_{\rm sc}$,[8] where $n_{\rm e}$ is the unperturbed extraordinary index of refraction, r_{33} is the electro-optic coefficient involved, and $E_{\rm sc} = E_{\rm sc}\boldsymbol{i}$ is the induced space-charge field, $\boldsymbol{i}$ is the unit vector pointing to the x direction. In typical PR media and for relatively broad beam configurations, the induced space-charge field can be obtained from the Vinetskii–Kukhtarev transport model and is given by[12]

$$E_{\rm sc} = \left(E_0 \frac{1}{1+|U|^2} - E_{\rm p}\frac{|U|^2}{1+|U|^2} + E_{\rm p}\frac{|U|^2}{1+|U|^2} f L_{\rm D}\frac{\partial}{\partial x}\frac{E_{\rm sc}}{E_{\rm t}}\right)\left(1 + L_{\rm D}\frac{\partial}{\partial x}\frac{E_{\rm sc}}{E_{\rm t}}\right)\left(1 - f L_{\rm D}\frac{\partial}{\partial x}\frac{E_{\rm sc}}{E_{\rm t}}\right)^{-1} - \frac{k_{\rm B}T}{e}\left\{\frac{\partial}{\partial x}\ln\left(1+|U|^2\right) - \left[\left(1 + L_{\rm D}\frac{\partial}{\partial x}\frac{E_{\rm sc}}{E_{\rm t}}\right)^{-1} + f\left(1 - f L_{\rm D}\frac{\partial}{\partial x}\frac{E_{\rm sc}}{E_{\rm t}}\right)^{-1}\right] L_{\rm D}\frac{\partial^2}{\partial x^2}\frac{E_{\rm sc}}{E_{\rm t}}\right\}, \quad (1)$$

where $U(x,z)$ is assumed to be a bright-like beam, i.e. $|U|^2 = 0$ at $x \to \pm\infty$, $L_{\rm D} = (\varepsilon_0\varepsilon_{\rm r}k_{\rm B}T/e^2N_{\rm A})^{1/2}$ is the Debye length, $N_{\rm A}$ is the density of negatively charged acceptor, e is the electronic charge, $k_{\rm B}$ is the Boltzmann constant, T is the absolute temperature, $\varepsilon_{\rm r}$ is the dielectric constant of the crystal, and ε_0 is the permittivity of vacuum. In Eq.(1), $E_{\rm t} = eN_{\rm A}L_{\rm D}/\varepsilon_0\varepsilon_{\rm r}$, $f = N_{\rm A}/(N_{\rm D} - N_{\rm A})$, E_0 represents the value of space-charge field in the dark regions of the crystal, and $E_{\rm p} = \kappa\gamma N_{\rm A}/e\mu$ is the PV field constant, where $N_{\rm D}$ is the total donor density, γ is the recombination rate coefficient, μ is the electron mobility, and κ is the PV constant. The constant field E_0 can be computed from the potential condition $V_0 = -\int_{-l/2}^{l/2} E_{\rm sc}{\rm d}x + RJS$ and is approximately given by $E_0 = -(V_0\chi + E_{\rm p}\delta\chi - RJS\chi)$,[16] where $\chi = 1/\int_{-l/2}^{l/2}[1/(1+|U|^2)]{\rm d}x$, $\delta = -\int_{-l/2}^{l/2}[|U|^2/(1+|U|^2)]{\rm d}x$, $\rho = \int_{-l/2}^{l/2}[\partial\ln(1+|U|^2)/\partial x]{\rm d}x$, S is the surface area of the electrodes, R is the external resistance, l is the width of the crystal between the electrodes, $\boldsymbol{J} = J\boldsymbol{i}$ is the electric current density, and V_0 is the electro-motive force of the source. By employing standard procedures, the optical beam envelope U is found to obey the following paraxial equation of diffraction:[8,12,16]

$${\rm i}\frac{\partial U}{\partial z} + \frac{1}{2k}\frac{\partial^2 U}{\partial x^2} - \frac{k_0}{2}(n_{\rm e}^3 r_{33}E_{\rm sc})U = 0, \quad (2)$$

where $k = k_0 n_{\rm e} = (2\pi/\lambda_0)n_{\rm e}$ is the wavevector and λ_0 is the free-space wavelength of the lightwave employed.

For quasi-plane-wave planar beams, the envelope U is expected to remain relatively constant over a large range of x. As a result, the diffraction term $(2k)^{-1}(\partial^2U/\partial x^2)$ in Eq.(2) and the spatial derivatives of $E_{\rm sc}$ and $|U|^2$ in Eq.(1) can be neglected. In this case, the space-charge field can be determined from Eq.(1) and is approximately given by $E_{\rm sc} = E_0/(1+|U|^2) - E_{\rm p}|U|^2/(1+|U|^2)$. When $(2k)^{-1}(\partial^2U/\partial x^2) = 0$, the solution of Eq.(2) is given by

$$U = r^{1/2}\exp\left[-{\rm i}\beta\left(E_0\frac{1}{1+r} - E_{\rm p}\frac{r}{1+r}\right)\right], \quad (3)$$

where $\beta = k_0 n_{\rm e}^3 r_{33}/2$, and the positive quantity r represents the ratio of the optical beam's intensity to that of the dark irradiance. To investigate the stability of this solution, let us perturb the stationary result of Eq.(3) in the following way:

$$U = [r^{1/2} + \sigma(x,z)]\exp\left[-{\rm i}\beta\left(E_0\frac{1}{1+r} - E_{\rm p}\frac{r}{1+r}\right)\right], \quad (4)$$

where $\sigma(x,z)$ represents the weak complex perturbation, and the magnitude of $\sigma(x,z)$ is much smaller than the amplitude of the quasi-plane-wave solution, i.e. $|\sigma(x,z)| \ll r^{1/2}$. We assume that the spatial perturbation $\sigma(x,z)$ is composed of two sideband plane waves, and takes the form

$$\sigma(x,z) = a(z)\exp({\rm i}px) + b(z)\exp(-{\rm i}px). \quad (5)$$

By substituting Eq.(4) into Eqs.(1) and (2) and by linearizing in σ, we find the following coupled set of equations:

$${\rm i}\frac{\partial\sigma}{\partial z} + \frac{1}{2k}\frac{\partial^2\sigma}{\partial x^2} - \beta E(r^{1/2} + \sigma) = 0, \quad (6)$$

$$E-\eta\frac{\partial E}{\partial x}-\alpha\frac{\partial^2 E}{\partial x^2} = -\frac{r^{1/2}}{1+r}\left[E_{01}(\sigma+\sigma^*)+\frac{k_{\rm B}T}{e}\left(\frac{\partial\sigma}{\partial x}+\frac{\partial\sigma^*}{\partial x}\right)\right], \quad (7)$$

where

$$E = E_{\rm sc}-E_0[1/(1+r)]+E_{\rm p}[r/(1+r)],$$

$$E_{01} = (E_0+E_{\rm p})/(1+r),$$

$$\eta = \{E_0[1/(1+r)]-E_{\rm p}[r/(1+r)]\}[\varepsilon_0\varepsilon_{\rm r}/(eN_{\rm A})],$$

and

$$\alpha = (k_{\rm B}T/e)[\varepsilon_0\varepsilon_{\rm r}/(eN_{\rm A})].$$

Notice that $f \ll 1$ have been assumed in Eq.(7). From Eqs.(7) and (5) the space-charge field $\hat{E}$ and the spatial perturbation $\hat{\sigma}$ in spatial-frequency domain k_x can be obtained by means of a Fourier transform and are given by

$$\hat{E} = -\frac{r^{1/2}}{1+r}\left\{\frac{E_{01}+E_{\rm p}\alpha+{\rm i}[E_{01}\eta k_x+k_x(k_{\rm B}T/e)(1+\alpha k_x^2)]}{(1+\alpha k_x^2)^2+\eta^2k_x^2}\right\}(\hat{\sigma}+\hat{\sigma}^*), \quad (8)$$

$$\hat{\sigma} = \int_{-\infty}^{+\infty}[a\exp({\rm i}px)+b\exp(-{\rm i}px)]\exp(-{\rm i}k_x x){\rm d}x. \quad (9)$$

From Eq.(9) we obtain that

$$\hat{\sigma}+\hat{\sigma}^* = 2\pi[(a+b^*)\delta(k_x-p)+(b+a^*)\delta(k_x+p)],$$

where $\delta(k_x)$ is a delta function. By substituting the expression we have found for $\hat{\sigma}+\hat{\sigma}^*$ into Eq.(8) and taking an inverse Fourier transform, the space-charge field in real space can be obtained and is given by

$$E = -\frac{1}{\beta r^{1/2}}[G(p)(a+b^*)\exp({\rm i}px) + G^*(p)(b+a^*)\exp(-{\rm i}px)], \quad (10)$$

where $G(p)$ is defined as

$$G(p) = \beta\frac{r}{1+r}\left\{\frac{E_{01}+E_{\rm p}\alpha+{\rm i}[(E_{01}\eta+k_{\rm B}T/e)p+\alpha(k_{\rm B}T/e)p^3]}{(1+\alpha p^2)^2+\eta^2p^2}\right\}. \quad (11)$$

Substituting Eqs.(5) and (10) into Eq.(6) and keeping only synchronous terms, we readily obtain the following coupled differential equations:

$${\rm i}\frac{{\rm d}a}{{\rm d}z}-\frac{p^2}{2k}a+G(p)(a+b^*) = 0, \quad (12a)$$

$${\rm i}\frac{{\rm d}b}{{\rm d}z}-\frac{p^2}{2k}b+G^*(p)(b+a^*) = 0. \quad (12b)$$

From Eq.(12) we find that

$$\frac{{\rm d}^2a}{{\rm d}z^2} = \left[\frac{p^2}{k}G(p)-\frac{p^4}{4k^2}\right]a, \quad (13a)$$

$$\frac{{\rm d}^2b}{{\rm d}z^2} = \left[\frac{p^2}{k}G^*(p)-\frac{p^4}{4k^2}\right]b. \quad (13b)$$

The global instability gain Γ_{gl} can be directly determined from Eq.(13) and is given by

$$\Gamma_{gl} = {\rm Re}\left\{\left[\frac{p^2}{k}G(p)-\frac{p^4}{4k^2}\right]^{1/2}\right\}, \quad (14)$$

where Re{} represents the real part of a complex variable.

To illustrate our results, let us consider the $LiNbO_3$ with the following parameters: $n_{\rm e}=2.2$, $r_{33}=30\times10^{-12}$m/V, $N_{\rm A}=10^{22}{\rm m}^{-3}$, $\varepsilon_{\rm r}=30$ and $E_{\rm p}$=40kV/cm. Moreover, we assume that $\lambda_0=0.5\mu$m, T=300K, R=0, and l=1cm. For this set of values, β and k are found to be $\beta=2.0\times10^{-3}{\rm V}^{-1}$ and $k=2.76\times10^7{\rm m}^{-1}$. Figure 1 depicts the global MI gain Γ_{gl} as a function of p/k and V_0 when r=5. Fig.2 shows the global MI gain Γ_{gl} as a function of p/k and r in $LiNbO_3$ when V_0=20000V. Figures 1 and 2 also illustrate that the global MI gains attain a maximum in the low spatial-frequency. On the other hand, some PV materials possess PV constants κ (i.e. $E_{\rm p}$) that change sign under polarization rotation.[10] The global MI gain Γ_{gl} can be changed by rotating the polarization of the light.

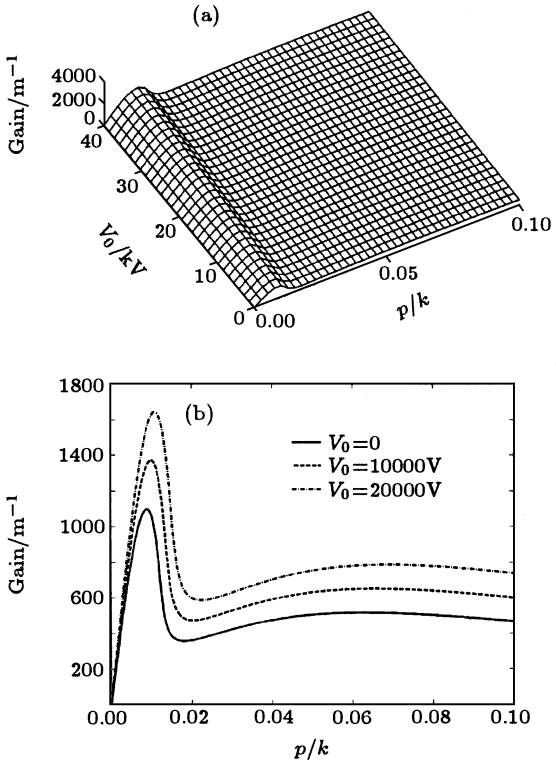


Fig.1. Global MI gain as a function of p/k and V_0 when $r = 5$.

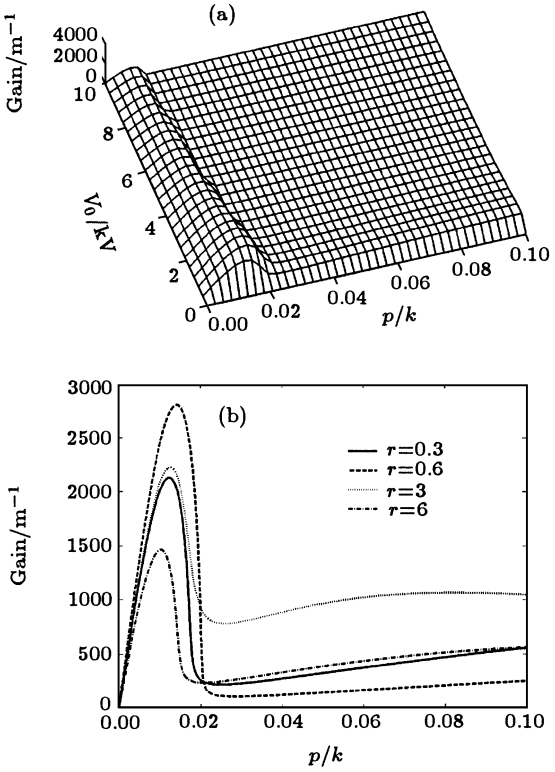


Fig.2. Global MI gain as a function of p/k and r when $V_0 = 20000V$.

The properties of the modulation instability of quasi-plane-wave optical beams in biased P-P crystals under steady-state conditions will now be discussed. When the bulk photovoltaic effect is negligible for short circuits (R=0), i.e. $E_{\rm p} = 0$ and $E_0 = E_{0\rm S} = -V_0\chi$, our physical system becomes that of the modulation instability in biased P-N crystals. In this case, Eqs.(11) and (14) yield

$$G_{\rm S}(p) = \beta\frac{r}{1+r}\left\{\frac{E_{\rm S} + {\rm i}[(E_{\rm S}\eta_{\rm S} + k_{\rm B}T/e)p + \alpha(k_{\rm B}T/e)p^3]}{(1+\alpha p^2)^2 + \eta_{\rm S}^2 p^2}\right\}, \tag{15}$$

$$\Gamma_{gl} = {\rm Re}\left\{\left[\frac{p^2}{k}G_{\rm S}(p) - \frac{p^4}{4k^2}\right]^{1/2}\right\}, \tag{16}$$

where

$$E_{\rm S} = E_{0\rm S}/(1+r)$$

and

$$\eta_{\rm S} = E_{0\rm S}[1/(1+r)]\ [\varepsilon_0\varepsilon_{\rm r}/(eN_{\rm A})].$$

Equations (15) and (16) are the same dimensionless parameters and of the same form as in Eqs.(10) and (13) in Ref.[15], in which MI of quasi-plane-wave optical beams in biased P-N crystals is investigated for short circuits. The modulation instability in open- and closed-circuit P-P crystals can be readily obtained from the modulation instability in biased P-P crystals. The absence of the external field in P-P crystals signifies either an open circuit ($J = 0$) or closed circuit ($J \neq 0$) in the absence of the external field.[12] Here, the electric current density is given by $J_{\rm p1} = E_{\rm p0}/E_{\rm p}$,[12] where $J_{\rm p1} = J/(\hat{s}I_{\rm d}N_{\rm D}\kappa)$, $E_{\rm p0} = -(E_{\rm p}\delta\chi - RJS\chi)$, and $\hat{s}$ is the photoexcitation cross section. Therefore, when the external bias field is absent, i.e. $E_0 = E_{\rm p0}$, our physical system becomes the physical system of the modulation instability in open- and closed-circuit P-P crystals. Thus, Eqs.(11) and (14) yield the following results:

$$G_{\rm p}(p) = \beta\frac{r}{1+r}\left\{\frac{E_{\rm p1} + {\rm i}[(E_{\rm p1}\eta_{\rm p} + k_{\rm B}T/e)p + \alpha(k_{\rm B}T/e)p^3]}{(1+\alpha p^2)^2 + \eta_{\rm p}^2 p^2}\right\}, \tag{17}$$

$$\Gamma_{gl} = {\rm Re}\left\{\left[\frac{p^2}{k}G_{\rm p}(p) - \frac{p^4}{4k^2}\right]^{1/2}\right\}, \tag{18}$$

where

$$E_{p1} = E_p(J_{p1} + 1)/(1 + r)$$

and

$$\eta_p = E_p(J_{p1} - r)[1/(1 + r)]\ [\varepsilon_0 \varepsilon_r/(eN_A)].$$

Equations (17) and (18) show the global instability gain in open- and closed-circuit P-P crystals by globally treating the space-charge field. Figure 3 depicts the global MI gain given by Eq.(18) as a function of the dimensionless quantity p/k in the open- and closed-circuit $LiNbO_3$ crystal when $r = 5$.

In conclusion, we have investigated the modulation instability of quasi-plane-wave optical beams in biased photorefractive–photovoltaic crystal. The modulation instability growth rate was obtained by globally treating the space-charge field, which depends on the external bias field, on the bulk photovoltaic effect, and on the ratio of the optical beam's intensity to that of the dark irradiance. Our analysis indicates that under appropriate conditions, this modulation instability growth rate is identical to the modulation instability growth rate studied previously in biased photorefractive–nonphotovoltaic crystals, and predicts the modulation instability growth rate in open- and closed-circuit photorefractive–photovoltaic crystals.

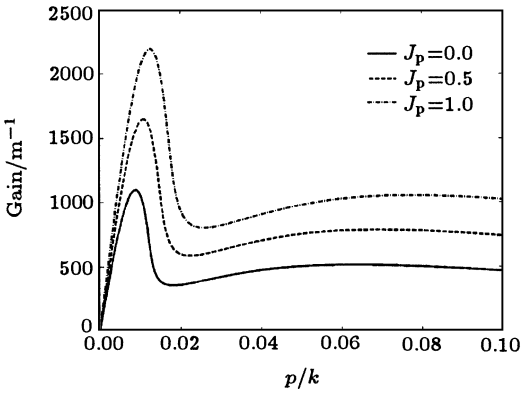


Fig.3. Global MI gain as a function of p/k in the open- and closed-circuit $LiNbO_3$ crystal when $r = 5$.

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