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Higher-order squeezing of quantum electromagnetic fields and higher-order uncertainty relations in two-mode squeezed states*

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It is found that the two-mode output quantum electromagnetic field in two-mode squeezed states exhibits higher-order squeezing to all even orders, and the degree of higher-order squeezing is greater than that of the second-order. The higher-order squeezed parameter and squeezed limit due to the modulation frequency are investigated. The smaller the modulation frequency is, the stronger the degree of higher-order squeezing becomes. Furthermore, the higher-order uncertainty relations in two-mode squeezed states are presented for the first time. The product of higher-order noise moments is related to even order number N and the squeeze factor r.

Keywords: higher-order squeezing, higher-order uncertainty relations, quantum fields, two-mode squeezed states
PACC: 4250, 0365, 5225G, 0365C

1. Introduction

The subject of noise squeezing[1−7] of the quantum radiation field has received a great deal of attention, because of the important applications in low-noise communications, high-precision interferometric measurement, quantum teleportation and cryptography with continuous variables.[8−11]

The quantum electromagnetic fields can be decomposed into two quadrature components. The fields in squeezed states have fewer fluctuations in one quadrature phase than that in a coherent state at the expense of increased fluctuations in the other quadrature phase. The ordinary second-order squeezing of quantum fields and second-order uncertainty relations in single-mode squeezed states have been studied and reviewed in many articles [1−4,12−15] and books[16,17].

In 1985, Caves and Schumaker studied the second-order squeezing and second-order uncertainty relations of quantum electromagnetic fields in two-mode squeezed states. They introduced a new formulism of two-photon quantum optics,[18,19] suited to analysing two-photon processes, such as parametric down-conversion and four-wave mixing, in which photons are generated in output modes two at a time.

With the development of techniques for making higher-order correlation measurement in quantum optics, the concept of higher-order squeezing of the single-mode quantum electromagnetic field was first introduced and applied to several processes by Hong and Mandel in 1985.[5−7] In this paper we generalize all the above work to the higher-order squeezing of quantum electromagnetic fields in two-mode squeezed states. The properties of this higher-order squeezing are investigated and the higher-order uncertainty relations among higher-order noise moments of quadrature components in two-mode squeezed states are given for the first time. By applying the combined two-mode squeezed annihilation operators, which we introduce and define, the higher-order noise moment in two-mode squeezed states can be calculated easily.

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2. Definition of higher-order squeezing in two-mode squeezed states

Before introducing the definition of the higher-order squeezing in two-mode squeezed states, it may be useful to summarize the properties of the two-mode squeezed states.

The definition of two-mode squeezed states was given by Caves and Schumaker:[18]

$$|\alpha_+,\alpha_-;\zeta\rangle = \hat{S}(\zeta)|\alpha_+,\alpha_-\rangle, \tag{1}$$

where $\hat{S}(\zeta)$ is the two-mode squeezed operator,

$$\hat{S}(\zeta) = \exp\left[\frac{1}{2}(\zeta^*\hat{a}_+\hat{a}_- - \zeta\hat{a}_+^\dagger\hat{a}_-^\dagger)\right], \tag{2}$$

$|\alpha_+,\alpha_-\rangle$ is the two-mode coherent state, $\hat{a}_\pm$ are two-mode annihilation operators, $\alpha_\pm$ are eigenvalues of $\hat{a}_\pm$ in $|\alpha_+,\alpha_-\rangle$, $\zeta = r\mathrm{e}^{\mathrm{i}\theta}$ is the squeeze parameter, and r is the squeeze factor.

Define the two-mode squeezed annihilation operators $\hat{A}_\pm$ by

$$\hat{A}_\pm = \hat{S}(\zeta)\hat{a}_\pm\hat{S}^\dagger(\zeta) = \mu\hat{a}_\pm + \nu\hat{a}_\mp^\dagger, \tag{3}$$

where

$$\mu = \cosh r, \qquad \nu = \mathrm{e}^{\mathrm{i}\theta}\sinh r. \tag{4}$$

Then the two-mode squeezed states are the eigenstates of $\hat{A}_\pm$,

$$\hat{A}_\pm|\alpha_+,\alpha_-;\zeta\rangle = \alpha_\pm|\alpha_+,\alpha_-;\zeta\rangle, \tag{5}$$

and $\alpha_\pm$ are the eigenvalues of $\hat{A}_\pm$.

The real two-mode output field $\hat{E}$ can be decomposed into two quadrature components $\hat{E}_1$ and $\hat{E}_2$, which are canonical conjugates. The output field $\hat{E}$ exhibits higher-order squeezing to Nth order in $\hat{E}_1$ for even number N, if there exists such a phase angle ϕ for which the higher-order moment $\langle(\Delta\hat{E}_1)^N\rangle$ in a two-mode squeezed state is smaller than its value in a completely two-mode coherent state, namely

$$\langle(\Delta\hat{E}_1)^N\rangle_{\mathrm{S.S.two-mode}} < \langle(\Delta\hat{E}_1)^N\rangle_{\mathrm{Coh.S.two-mode}}.$$

This is the definition of higher-order squeezing in two-mode squeezed states.

3. The quadrature components of the two-mode output field $\hat{E}$

The electric field operator for the combined two-mode output field has the form

$$\hat{E}(x,t) = \hat{E}^{(+)}(x,t) + \hat{E}^{(-)}(x,t), \tag{6}$$

$$\hat{E}^{(+)}(x,t) = \lambda\sqrt{\frac{\omega_+}{2}}\hat{a}_+\mathrm{e}^{-\mathrm{i}\omega_+(t-x)} + \lambda\sqrt{\frac{\omega_-}{2}}\hat{a}_-\mathrm{e}^{-\mathrm{i}\omega_-(t-x)}, \tag{7}$$

$$\hat{E}^{(-)}(x,t) = \lambda\sqrt{\frac{\omega_+}{2}}\hat{a}_+^\dagger\mathrm{e}^{\mathrm{i}\omega_+(t-x)} + \lambda\sqrt{\frac{\omega_-}{2}}\hat{a}_-^\dagger\mathrm{e}^{\mathrm{i}\omega_-(t-x)}, \tag{8}$$

where $\hat{E}^{(+)}(x,t)$ and $\hat{E}^{(-)}(x,t)$ are the positive and negative frequency parts of the field, ω_+ and ω_- are angular frequencies of the two-mode field, $\lambda = \sqrt{\frac{\hbar}{V\varepsilon_0}} = \sqrt{\frac{1}{V\varepsilon_0}}$ and V is the mode volume.

The units are chosen so that $\hbar = c = 1$.

Now we construct two Hermitian quadrature components $\hat{E}_1$ and $\hat{E}_2$ of the electric field, namely

$$\hat{E}_1(x,t) = \hat{E}^{(+)}\mathrm{e}^{\mathrm{i}[\Omega(t-x)-\phi]} + \hat{E}^{(-)}\mathrm{e}^{-\mathrm{i}[\Omega(t-x)-\phi]}, \tag{9}$$

$$\hat{E}_2(x,t) = \hat{E}^{(+)}\mathrm{e}^{\mathrm{i}[\Omega(t-x)-(\phi+\frac{\pi}{2})]} + \hat{E}^{(-)}\mathrm{e}^{-\mathrm{i}[\Omega(t-x)-(\phi+\frac{\pi}{2})]}. \tag{10}$$

Then $\hat{E}(x,t)$ can be decomposed into two quadrature components $\hat{E}_1$ and $\hat{E}_2$,

$$\hat{E}(x,t) = \hat{E}_1\cos[\Omega(t-x)-\phi] + \hat{E}_2\sin[\Omega(t-x)-\phi], \tag{11}$$

where Ω is the carrier frequency (or mid-frequency of frequencies ω_+ and ω_-),

$$\Omega = \frac{\omega_+ + \omega_-}{2}, \tag{12}$$

and ϕ is an arbitrary phase angle that may be chosen at will.

The quadrature components $\hat{E}_1$ and $\hat{E}_2$ are canonical conjugates and obey the commutation relations

$$[\hat{E}_1,\hat{E}_2] = 2\mathrm{i}C_0, \tag{13}$$

where C_0 stands for the commutator $[\hat{E}^{(+)},\hat{E}^{(-)}]$.

Substituting Eqs.(7) and (8) into (9), we obtain

$$\hat{E}_1(x,t) = g_+\hat{a}_+ + g_-\hat{a}_- + g_+^*\hat{a}_+^\dagger + g_-^*\hat{a}_-^\dagger, \tag{14}$$

where

$$g_\pm = \lambda\sqrt{\frac{\Omega\pm\epsilon}{2}}\mathrm{e}^{-\mathrm{i}[\phi\pm\epsilon(t-x)]}, \tag{15}$$

and

$$\epsilon = \omega_+ - \Omega = \Omega - \omega_- \tag{16}$$

is the modulation frequency.

From Eq.(3), we obtain

$$\hat{a}_{\pm} = \mu^* \hat{A}_{\pm} - \nu \hat{A}^{\dagger}_{\mp}. \tag{17}$$

Substituting (17) into (14), we obtain $\hat{E}_1$ in terms of $\hat{A}_{\pm}$, therefore

$$\hat{E}_1(x,t) = (h_+ \hat{A}_+ + h_- \hat{A}_-) + (h^*_+ A^{\dagger}_+ + h^*_- \hat{A}^{\dagger}_-), \tag{18}$$

where

$$h_{\pm} = g_{\pm}\mu^* - g^*_{\mp}\nu^*. \tag{19}$$

Now we introduce a new operator $\hat{B}$, defined by

$$\hat{B} = h_+ \hat{A}_+ + h_- \hat{A}_-, \tag{20}$$

then we can express $\hat{E}_1$ in terms of $\hat{B}$ and $\hat{B}^{\dagger}$ operators in the form

$$\hat{E}_1 = \hat{B} + \hat{B}^{\dagger}. \tag{21}$$

We call $\hat{B}$ the combined two-mode squeezed annihilation operator, and it can be proved that the two-mode squeezed states are also the eigenstates of $\hat{B}$,

$$\hat{B}|\alpha_+, \alpha_-; \zeta\rangle = \langle\hat{B}\rangle|\alpha_+, \alpha_-; \zeta\rangle. \tag{22}$$

The operator $\hat{B}$ can be used to calculate the higher-order noise moments in two-mode squeezed states.

4. Higher-order noise moment $\langle(\Delta\hat{E}_1)^N\rangle$ and higher-order squeezing

The calculation of the higher-order noise moment $\langle(\Delta\hat{E}_1)^N\rangle$ is very complex and difficult, since the quantum field is a two-mode field. However, applying the new combined two-mode squeezed annihilation operator $\hat{B}$, we can calculate the higher-order noise moment in two-mode squeezed states easily.

Using the Campbell–Baker–Hausdorff formula, we express the higher-order noise moment of ΔE_1 in the following relation:[5−7]

$$\begin{aligned}\langle(\Delta\hat{E}_1)^N\rangle =& \langle:(\Delta\hat{E}_1)^N:\rangle + \frac{N^{(2)}}{1!}\left(\frac{1}{2}C_0\right)\langle:(\Delta\hat{E}_1)^{N-2}:\rangle + \frac{N^{(4)}}{2!}\left(\frac{1}{2}C_0\right)^2\langle:(\Delta\hat{E}_1)^{N-4}:\rangle \\ &+\cdots+\begin{cases}(N-1)!!C_0^{N/2} & (N \text{ is even}),\\ \dfrac{N!}{3!2^{(1/2)N-3/2}\left(\frac{1}{2}N-\frac{3}{2}\right)!}\dfrac{C_0^{(1/2)N-3/2}}{\left(\frac{1}{2}N-\frac{3}{2}\right)!}\langle:(\Delta\hat{E}_1)^3:\rangle & (N \text{ is odd}),\end{cases}\end{aligned} \tag{23}$$

where $N^{(r)} = N(N-1)\cdots(N-r+1)$, $C_0 = [\hat{E}^{(+)}, \hat{E}^{(-)}] = \frac{1}{2\mathrm{i}}[\hat{E}_1, \hat{E}_2] = [\hat{B}, \hat{B}^{\dagger}]$, and "$::$" denotes normal ordering with respect to $\hat{B}$ and $\hat{B}^{\dagger}$.

Now we take the two-mode squeezed states, then from Eq.(22) we obtain

$$\begin{aligned}\langle:(\Delta\hat{E}_1)^n:\rangle &= \langle\alpha_+, \alpha_-; \zeta|:(\Delta\hat{E}_1)^n:|\alpha_+, \alpha_-; \zeta\rangle \\ &= \sum_{\gamma=0}^{n}\begin{bmatrix} n \\ \gamma \end{bmatrix}\langle:(\Delta\hat{B}^{\dagger})^{\gamma}(\Delta\hat{B})^{n-\gamma}:\rangle \\ &= \sum_{\gamma=0}^{n}\begin{bmatrix} n \\ \gamma \end{bmatrix}\langle:(\Delta\hat{B}^{\dagger})^{\gamma}(\Delta\hat{B})^{n-\gamma-1}(\hat{B}-\langle\hat{B}\rangle):\rangle = 0.\end{aligned} \tag{24}$$

This shows that the normal ordered moments $\langle:(\Delta\hat{E}_1)^n:\rangle$ all vanish for $n \le N$ in two-mode squeezed state. Then from Eqs.(23), (24) and (19) we get a simple result:

$$\langle(\Delta\hat{E}_1)^N\rangle = (N-1)!!C_0^{N/2} \quad (N \text{ is even}), \tag{25}$$

where

$$\begin{aligned}C_0 =& [\hat{B}, \hat{B}^+] = |h_+|^2 + |h_-|^2 \\ =& (|g_+|^2 + |g_-|^2)(|\mu|^2 + |\nu|^2) \\ &- \Omega\sqrt{1-\frac{\epsilon^2}{\Omega^2}}(\mu^*\nu \mathrm{e}^{-2\mathrm{i}\phi} + \mu\nu^* \mathrm{e}^{2\mathrm{i}\phi}).\end{aligned} \tag{26}$$

Substituting Eq.(26) into (25), and from Eq.(4) and (15) we get

$$\begin{aligned}\langle(\Delta\hat{E}_1)^N\rangle =& (N-1)!!\lambda^N\Omega^{N/2}[\cosh(2r) \\ &- \sqrt{1-\frac{\epsilon^2}{\Omega^2}}\sinh(2r)\cos(\theta-2\phi)]^{N/2}.\end{aligned} \tag{27}$$

This is the higher-order noise moment in two-mode squeezed states. It is shown that the higher-order noise moment is dependent on the angle $(\theta-2\phi)$, even order number N and the squeeze factor r.

If ϕ is chosen to satisfy $\cos(\theta - 2\phi) = 1$, Eq.(27) leads to the result

$$\langle(\Delta\hat{E}_1)^N\rangle = (N-1)!!\lambda^N\Omega^{N/2}[\cosh(2r) - \sqrt{1-\frac{\epsilon^2}{\Omega^2}}\sinh(2r)]^{N/2}. \quad (28)$$

When $\cosh r < \frac{\Omega}{\epsilon}$, the right-hand side is smaller than $(N-1)!!\Omega^{N/2}$, which is the corresponding Nth order moment for two-mode coherent states. It follows that the two-mode output field exhibits higher-order squeezing to all even orders.

When $\cosh(2r) = \frac{\Omega}{\epsilon}$, $\langle(\Delta\hat{E}_1)^N\rangle$ has the minimum value

$$\langle(\Delta\hat{E}_1)^N\rangle_{\min} = (N-1)!!\lambda^N\epsilon^{\frac{N}{2}}. \quad (29)$$

This shows that there exists limit of higher-order squeezing in two-mode squeezed states due to modulation frequency ϵ.

5. Higher-order squeezed parameter in two-mode squeezed states

Higher-order squeeze parameter q_N for measuring the degree of Nth-order squeezing can be expressed by

$$q_N = \frac{\langle(\Delta\hat{E}_1)^N\rangle_{\text{S.S. two-mode}} - \langle(\Delta\hat{E}_1)^N\rangle_{\text{Coh.S. two-mode}}}{\langle(\Delta\hat{E}_1)^N\rangle_{\text{Coh.S. two-mode}}}, \quad (30)$$

$$= \left[\cosh(2r) - \sqrt{1-\frac{\epsilon^2}{\Omega^2}}\sinh(2r)\right]^{N/2} - 1. \quad (31)$$

The numerical results of the higher-order squeeze parameter q_N versus squeeze factor r are illustrated in Fig.1 (a), (b), (c).

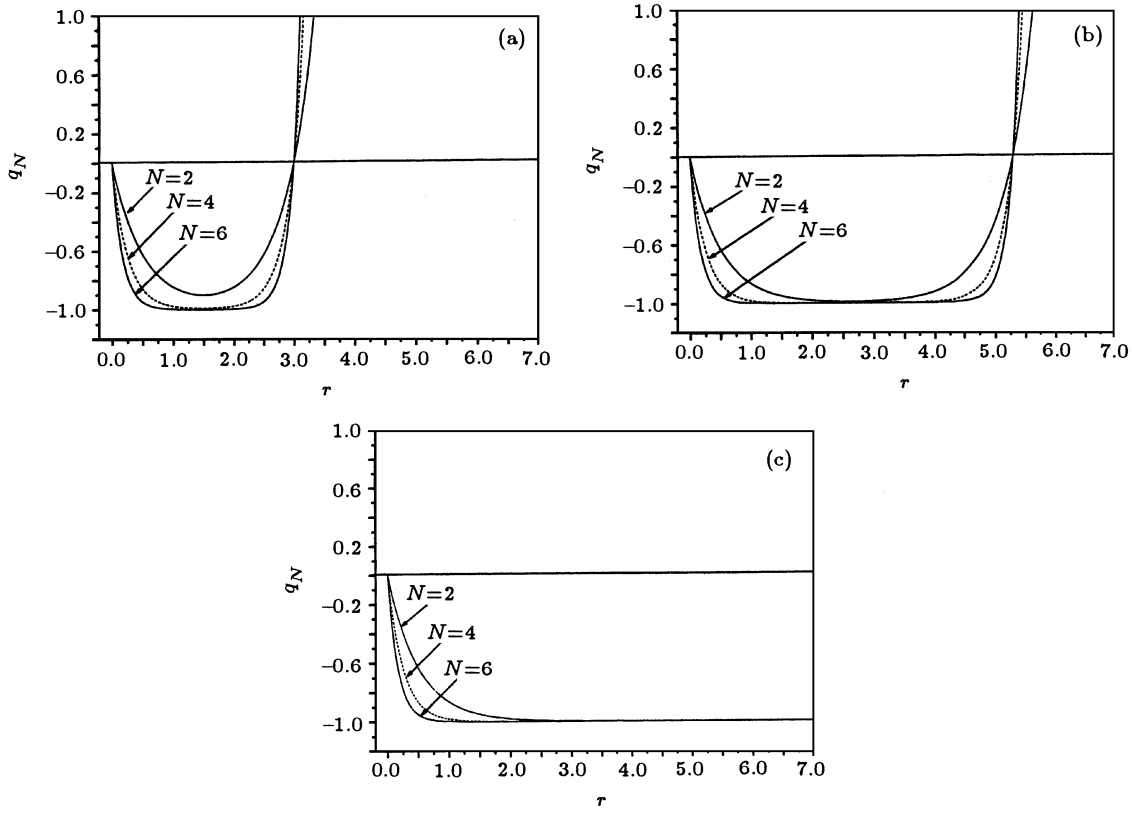


Fig.1. Higher-order squeeze parameter q_N versus squeeze factor r for (a) $\frac{\epsilon}{\Omega}$=0.1, (b) $\frac{\epsilon}{\Omega}$ = 0.01, (c) $\frac{\epsilon}{\Omega}$=0.

From Eq.(31) and Fig.1(a), (b), (c), it is found that

(1) For certain value of $\frac{\epsilon}{\Omega}$, the higher-order squeezed parameter q_N is a function of squeeze factor r and even order number N.

(2) When $\cosh(2r) = \frac{\Omega}{\epsilon}$, q_N has minimum values,

$$q_{N\min} = \left[\frac{\epsilon}{\Omega}\right]^{N/2} - 1. \quad (32)$$

Since $\frac{\epsilon}{\Omega} < 1$, $q_{N\min}$ is negative, and may approach the maximum negative value -1 when $\epsilon = 0$.

(3) $|q_N|$ increases with N, this leads to the conclusion that the degree of higher-order squeezing is greater than that of the second order. This confirms that the higher-order squeezing of quantum fields is particularly attractive for applications from the point of view of noise reduction.

(4) The smaller the modulation frequency ϵ is, the larger the degree of higher-order squeezing becomes.

6. The higher-order uncertainty relations

We now derive and discuss the higher-order uncertainty relations among higher-order noise moments of quadrature components $\hat{E}_1$ and $\hat{E}_2$ in two-mode squeezed states.

6.1. Higher-order noise moment $\langle(\Delta\hat{E}_2)^N\rangle$

$\hat{E}_2$ can be regarded as a special case of $\hat{E}_1$, in which if ϕ is replaced by $\phi + \pi/2$, then from Eq.(27) it follows that

$$\langle(\Delta\hat{E}_2)^N\rangle = (N-1)!!\lambda^N\Omega^{N/2}\left[\cosh(2r) + \sqrt{1-\frac{\epsilon^2}{\Omega^2}}\sinh(2r)cos(\theta-2\phi)\right]^{N/2}. \tag{33}$$

If ϕ is chosen to satisfy $\cos(\theta - 2\phi) = 1$, then

$$\langle(\Delta\hat{E}_2)^N\rangle = (N-1)!!\lambda^N\Omega^{N/2} \times \left[\cosh(2r) + \sqrt{1-\frac{\epsilon^2}{\Omega^2}}\sinh(2r)\right]^{N/2}. \tag{34}$$

When $\cosh r < \frac{\Omega}{\epsilon}$, the right-hand side is greater than $(N-1)!!\lambda^N\Omega^{N/2}$.

6.2. Higher-order uncertainty relations

From Eqs.(28) and (34), we obtain

$$\langle(\Delta\hat{E}_1)^N\rangle \cdot \langle(\Delta\hat{E}_2)^N\rangle = [(N-1)!!]^2\lambda^{2N}\Omega^N\left[1+\frac{\epsilon^2}{\Omega^2}\sinh^2(2r)\right]^N. \tag{35}$$

This shows that the product of the two higher-order noise moments is related to squeeze factor r and even order number N. For given values of r and N, the product is a constant. So Eq.(35) shows that $\langle(\Delta\hat{E}_1)^N\rangle$ and $\langle(\Delta\hat{E}_2)^N\rangle$ in two-mode squeezed states cannot be made arbitrarily small simultaneously. We call Eq.(35) the higher-order uncertainty relations in two-mode squeezed states, which are the generalization of the ordinary second-order uncertainty relations in two-mode quantum field.

Since

$$1 + \frac{\epsilon^2}{\Omega^2}\sinh^2(2r) > 1,$$

so

$$\langle(\Delta\hat{E}_1)^N\rangle \cdot \langle(\Delta\hat{E}_2)^N\rangle > [(N-1)!!]^2\lambda^{2N}\Omega^N. \tag{36}$$

If $N = 2$, we obtain

$$\langle(\Delta\hat{E}_1)^2\rangle \cdot \langle(\Delta\hat{E}_2)^2\rangle = \lambda^4\Omega^2\left[1+\frac{\epsilon^2}{\Omega^2}\sinh^2(2r)\right]^2. \tag{37}$$

This is just the second-order uncertainty relations in two-mode quantum fields.[18]

7. Conclusions

The higher-order squeezing of the quantum electromagnetic field and the higher-order uncertainty relations in two-mode squeezed states have been studied. The major achievements or findings of this paper are as follows:

1. After introducing the definition of higher-order squeezing of the two-mode output radiation field, we define a new combined two-mode squeezed annihilation operator $\hat{B}$, which is the linear combination of operators $\hat{A}_+$ and $\hat{A}_-$. By applying this operator, we can calculate the higher-order noise moment $\langle(\Delta\hat{E}_1)^N\rangle$ in two-mode squeezed states easily.

2. We have established the quantum theory of the higher-order squeezing of the quantum electromagnetic field in two-mode squeezed states. We found that if ϕ is chosen to satisfy $\cos(\theta - 2\phi) = 1$, the two-mode output field exhibits higher-order squeezing to all even orders when $\cosh r < \frac{\Omega}{\epsilon}$.

3. The limit of the higher-order squeezing due to modulation frequency ϵ has been discussed. For a given ϵ, the minimum value of $\langle(\Delta\hat{E}_1)^N\rangle$ occurs when $\cosh(2r) = \frac{\Omega}{\epsilon}$. The minimum value is $\langle(\Delta\hat{E}_1)^N\rangle_{\min} = (N-1)!!\lambda^N\epsilon^{\frac{N}{2}}$.

4. We have studied the squeeze parameter q_N for measuring the degree of N'th order squeezing. The minimum value of the squeeze parameter is $q_{N\min} = \left[\frac{\epsilon}{\Omega}\right]^{N/2} - 1$, and $q_{N\min}$ is negative. But the absolute value of the squeeze parameter $|q_N|$ increases with N. This leads to the conclusion that the degree of higher-order squeezing of two-mode field is greater than that of the second order.

5. We have presented the higher-order uncertainty relations between higher-order noise moments of quadrature components in two-mode squeezed states for the first time. The product of the two higher-order noise moments is related to squeeze factor r and even order number N. The higher-order uncertainty relations are the generalization of ordinary second order uncertainty relations in two-photon quantum optics. The discovery of this higher-order uncertainty relations enables people to have a deeper understanding of the indeterminacy principle in quantum mechanics.

All the above results should be possible to apply to non-degenerate two-photon nonlinear optical processes, in which the two-mode output field exhibits higher-order squeezing. They will provide further evidence for the validity of quantum theory.

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