Vol 13 No 12, December 2004
1009-1963/2004/13(12)/2025-05

Chinese Physics

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The negative energy density for a three-single-electron state in the Dirac field*

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(Received 4 September 2003; revised manuscript received 12 May 2004)

We examine the energy density produced by a state vector which is the superposition of three single electron states in the Dirac field in the four-dimensional Minkowski spacetime. We derive the conditions on which the energy density can be negative. We then show that the energy density satisfies two quantum inequalities in the ultrarelativistic limit.

Keywords: negative energy density, energy condition, quantum inequality
PACC: 0420, 0490

1. Introduction

In classical physics, the energy density is positive-definite. However, in quantum field theory, the renormalized energy density can be negative at a spacetime point due to quantum coherence effects.[1] Thus the weak energy condition and other pointwise energy conditions can be violated. Phenomena associated with negative energy densities, such as the Casimir effect[2,3] and squeezed states of light,[4] have been studied in laboratory. On the other hand, if the energy density was negative for an arbitrarily long time, exotic consequences could occur. These include violation of the second law of thermodynamics[5] or of cosmic censorship[6,7] "warp drive"[8] and time machines.[9] Hence much work has been done in order to find restrictions on the extent of energy condition breakdown. One famous approach is that of the quantum inequality (QI). For the scalar and electromagnetic fields in flat and curved spacetimes, QIs have been derived.[10−15] It was shown by Ford and Roman[11] that for a free quantized massless scalar field in four-dimensional Minkowski spacetime, the sampled energy density satisfies

$$\hat{\rho} = \frac{t_0}{\pi}\int_{-\infty}^{\infty}\frac{\langle{:}T_{00}{:}\rangle}{t^2+t_0^2}\mathrm{d}t \geq -\frac{3}{32\pi^2 t_0^4} \qquad (1)$$

for the sampling time t_0. Here $\langle{:}T_{00}{:}\rangle$ is the renormalized expectation value of the energy density evaluated in an arbitrary quantum state by an arbitrary inertial observer whose proper time coordinate is t. This QI indicates that the more negative the energy density becomes, the shorter it lasts.

As far as the Dirac field is concerned, there has not been so much work until recently. Vollick examined a state which is the superposition of two single electron states and proved that the QI of the form of Eq.(1) holds for this state,[16] and derived a QI for the massless Dirac field in two-dimensional spacetimes.[17] Fewster and Verch[18] established the existence of QI for the Dirac field in general four-dimensional globally hyperbolic spacetimes. In Ref.[19], we discussed an arbitrary state that is the superposition of two multi-electron–positron states. We obtained the conditions on which energy density can become negative and derived a QI of the form

$$E \triangle t < 1. \qquad (2)$$

In this paper, we investigate a state that is the superposition of three single electron states using methods in Ref.[19–21]. We study the energy density produced by the state and search for the conditions of the energy density being negative in the next section. In Section 3 it is verified that Eq.(2) still holds. Finally we prove that the energy density also satisfies the QI of the form of Eq.(1) in Section 4. The last section is for the conclusion.

*Project supported by the National Natural Science Foundation of China (Grant Nos 10075019 and 10375023).
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We work in units where $c = \hbar = 1$ and use the metric tensor $g_{\mu\nu} = g^{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$.

2. Conditions of states with energy densities being negative

Before addressing the conditions, let us recall the basic knowledge we are going to use about the Dirac field. The equation of motion is

$$(\mathrm{i}\gamma^\mu \partial_\mu - m)\psi = 0, \tag{3}$$

and the symmetrized stress tensor[22] is

$$T_{\mu\nu} = \frac{\mathrm{i}}{4}\big[\overline{\psi}\gamma^\mu \overleftrightarrow{\partial}_\nu \psi + \overline{\psi}\gamma^\nu \overleftrightarrow{\partial}_\mu \psi\big]. \tag{4}$$

The field operator can be expanded as

$$\psi(x) = \sum_k \sum_{\alpha=1,2} \Big[b_\alpha(k)u^\alpha(k)\mathrm{e}^{\mathrm{i}k\cdot x} + d^\dagger_\alpha(k)v^\alpha(k)\mathrm{e}^{-\mathrm{i}k\cdot x}\Big], \tag{5}$$

where the mode functions are taken to be

$$u^\alpha(k) = \begin{pmatrix} \sqrt{\dfrac{\omega+m}{2\omega V}}\phi^\alpha \\ \dfrac{\sigma\cdot\boldsymbol{k}}{\sqrt{2\omega(\omega+m)V}}\phi^\alpha \end{pmatrix}$$

and

$$v^\alpha(k) = \begin{pmatrix} \dfrac{\sigma\cdot\boldsymbol{k}}{\sqrt{2\omega(\omega+m)V}}\phi^\alpha \\ \sqrt{\dfrac{\omega+m}{2\omega V}}\phi^\alpha \end{pmatrix}. \tag{6}$$

Note that $\phi^{1\dagger} = (1, 0)$, $\phi^{2\dagger} = (0, 1)$ and α labels the two independent spin states. Here $b_\alpha(k)$ and its Hermitian conjugate $b^\dagger_\alpha(k)$ are the annihilation and creation operators for the electron respectively, while $d_\alpha(k)$ and $d^\dagger_\alpha(k)$ are the annihilation and creation operators for the positron. The four operators anticommute except in the cases $\{b_\alpha(k), b^\dagger_{\alpha'}(k')\} = \{d_\alpha(k), d^\dagger_{\alpha'}(k')\} = \delta_{\alpha,\alpha'}\delta_{k,k'}$. The renormalized expectation value of the energy density, i.e. $\langle{:}T_{00}{:}\rangle$, for an arbitrary quantum state is

$$\begin{aligned}\langle\rho\rangle =& \frac{1}{2}\sum_{k,k'}\sum_{\alpha,\alpha'}(\omega_k + \omega_{k'}) \\ &\times [\langle b^\dagger_\alpha(k)b_{\alpha'}(k')\rangle u^{\dagger\alpha}(k)u^{\alpha'}(k')\mathrm{e}^{-\mathrm{i}(k-k')\cdot x} \\ &+ \langle d^\dagger_{\alpha'}(k')d_\alpha(k)\rangle v^{\dagger\alpha}(k)v^{\alpha'}(k')\mathrm{e}^{\mathrm{i}(k-k')\cdot x}] \\ &+ \frac{1}{2}\sum_{k,k'}\sum_{\alpha,\alpha'}(\omega_{k'} - \omega_k) \\ &\times [\langle d_\alpha(k)b_{\alpha'}(k')\rangle v^{\dagger\alpha}(k)u^{\alpha'}(k')\mathrm{e}^{\mathrm{i}(k+k')\cdot x} \\ &- \langle b^\dagger_\alpha(k)d^\dagger_{\alpha'}(k')\rangle u^{\dagger\alpha}(k)v^{\alpha'}(k')\mathrm{e}^{-\mathrm{i}(k+k')\cdot x}],\end{aligned} \tag{7}$$

where $\langle\rho\rangle$ is defined to be $\langle{:}T_{00}{:}\rangle$.

Now we explore the energy density in the Dirac field produced by a state which is the superposition of three single particle electron states and discuss whether it can be negative. We choose such a state vector as

$$|\Psi\rangle = \frac{1}{\sqrt{1+\lambda^2+\gamma^2}}\Big[\lambda|k_x, 2\rangle + \gamma|k_y, 1\rangle + |k_z, 1\rangle\Big]. \tag{8}$$

Then the energy density can be worked out:

$$\langle\rho\rangle = \frac{1}{(\lambda^2+\gamma^2+1)V}[\lambda^2\omega_{k_x} + \lambda A + \gamma^2\omega_{k_y} + \gamma B + \omega_{k_z}], \tag{9}$$

where

$$A = \frac{k_x k_z(\omega_{k_x} + \omega_{k_z})}{2\sqrt{\omega_{k_x}(\omega_{k_x}+m)\omega_{k_z}(\omega_{k_z}+m)}}\cos[(k_x - k_z)\cdot x] \tag{10}$$

and

$$\begin{aligned}B =& \sqrt{\frac{(\omega_{k_y}+m)(\omega_{k_z}+m)}{\omega_{k_y}\omega_{k_z}}}\frac{(\omega_{k_y}+\omega_{k_z})}{2} \\ &\times \cos[(k_y - k_z)\cdot x].\end{aligned} \tag{11}$$

We can see that $\langle\rho\rangle$ will be negative if

$$\Delta_\lambda > 0 \tag{12}$$

and

$$\lambda_- < \lambda < \lambda_+, \tag{13}$$

where

$$\begin{aligned}\Delta_\lambda =& A^2 - 4\omega_{k_x}(\gamma^2\omega_{k_y} + \gamma B + \omega_{k_z}) \\ =& -\gamma^2\cdot 4\omega_{k_x}\omega_{k_y} - \gamma\cdot 4\omega_{k_x}B + A^2 - 4\omega_{k_x}\omega_{k_z},\end{aligned} \tag{14}$$

and

$$\lambda_\pm = \frac{1}{2\omega_{k_x}}\big[-A \pm \sqrt{\Delta_\lambda}\big]. \tag{15}$$

For Eq.(12) to exist, the following conditions should be satisfied:

$$\Delta_\gamma > 0 \quad \text{and} \quad \gamma_- < \gamma < \gamma_+, \tag{16}$$

where

$$\begin{aligned}\Delta_\gamma =& (-4\omega_{k_x}B)^2 + 4\cdot 4\omega_{k_x}\omega_{k_y}(A^2 - 4\omega_{k_x}\omega_{k_z}) \\ =& 16\omega_{k_x}(\omega_{k_x}B^2 + \omega_{k_y}A^2 - 4\omega_{k_x}\omega_{k_y}\omega_{k_z}),\end{aligned} \tag{17}$$

and

$$\gamma_\pm = \frac{1}{8\omega_{k_x}\omega_{k_y}}\big[-4\omega_{k_x}B \pm \sqrt{\Delta_\gamma}\big]. \tag{18}$$

To show the negative energy density is possible, let us consider the ultrarelativistic limit, i.e. $k \gg m$.

Now the above conditions on which the energy density can become negative are

$$\Delta'_\gamma > 0, \tag{19}$$

$$\gamma'_- < \gamma < \gamma'_+, \tag{20}$$

and

$$\lambda'_- < \lambda < \lambda'_+, \tag{21}$$

where

$$\begin{aligned}\Delta'_\gamma =&4\omega_{k_x}\{\omega_{k_x}(\omega_{k_y}+\omega_{k_z})^2\cos^2[(k_y-k_z)\cdot x]\\ &+\omega_{k_y}(\omega_{k_x}+\omega_{k_z})^2\cos^2[(k_x-k_z)\cdot x]\\ &-16\omega_{k_x}\omega_{k_y}\omega_{k_z}\},\end{aligned} \tag{22}$$

$$\begin{aligned}\gamma'_\pm =&\frac{1}{8\omega_{k_x}\omega_{k_y}}\{-2\omega_{k_x}(\omega_{k_y}+\omega_{k_z})\\ &\times\cos[(k_y-k_z)\cdot x]\pm\sqrt{\Delta'_\gamma}\},\end{aligned} \tag{23}$$

and

$$\begin{aligned}\lambda'_\pm =&\frac{1}{2\omega_{k_x}}\Big[-\frac{1}{2}(\omega_{k_x}+\omega_{k_z})\\ &\times\cos[(k_x-k_z)\cdot x]\pm\sqrt{\Delta'_\lambda}\Big].\end{aligned} \tag{24}$$

Here

$$\begin{aligned}\Delta'_\lambda =&-\gamma^2\cdot 4\omega_{k_x}\omega_{k_y}-\gamma\cdot 2\omega_{k_x}(\omega_{k_y}+\omega_{k_z})\\ &\times\cos[(\omega_{k_y}-\omega_{k_z})\cdot x]\\ &+\frac{1}{4}(\omega_{k_x}+\omega_{k_z})^2\cos^2[(\omega_{k_x}-\omega_{k_z})\cdot x]\\ &-4\omega_{k_x}\omega_{k_z}.\end{aligned} \tag{25}$$

If we choose ω_{k_x}, ω_{k_y} and ω_{k_z} properly, Eq.(19) can be satisfied. For example, clearly we have

$$\begin{aligned}\Delta'_\gamma \geq&4\omega_{k_x}\{\omega_{k_y}(\omega_{k_x}+\omega_{k_z})^2\cos^2[(k_x-k_z)\cdot x]\\ &-16\omega_{k_x}\omega_{k_y}\omega_{k_z}\}\equiv C.\end{aligned} \tag{26}$$

If $C>0$, $\Delta'_\gamma>0$. To make $C>0$ it is needed that

$$\cos^2[(k_x-k_z)\cdot x] > \frac{16\omega_{k_x}\omega_{k_z}}{(\omega_{k_x}+\omega_{k_z})^2}. \tag{27}$$

Equation (27) can be satisfied if

$$16\omega_{k_x}\omega_{k_z} < (\omega_{k_x}+\omega_{k_z})^2. \tag{28}$$

For the above inequality to exist we can choose ω_{k_x} and ω_{k_z} such as $\omega_{k_x} > (7+\sqrt{48})\omega_{k_z}$ or $\omega_{k_z} > (7+\sqrt{48})\omega_{k_x}$. Similarly, one can show that Eq.(19) is obeyed if ω_{k_y} and ω_{k_z} satisfy that $\omega_{k_y} > (7+\sqrt{48})\omega_{k_z}$ or $\omega_{k_z} > (7+\sqrt{48})\omega_{k_y}$. At the same time, if γ and λ are selected to meet Eq.(20) and Eq.(21) respectively, the energy density can be negative at a space-time point.

3. Quantum inequality of negative energy density

In the following we want to show that the QI of the form Eq.(2) is satisfied. Let us consider the case where $\omega_{k_x}=\omega_{k_y}$ at the spatial point $x=0$. Then in the ultrarelativistic limit, i.e. $k\gg m$, the energy density in Eq.(9) can be simplified as follows:

$$\begin{aligned}\langle\rho\rangle =&\frac{1}{(\lambda^2+\gamma^2+1)V}\\ &\times\Big[(\lambda^2+\gamma^2)\omega_{k_x}+\frac{1}{2}(\lambda+\gamma)(\omega_{k_x}+\omega_{k_z})\\ &\times\cos(\omega_{k_x}-\omega_{k_z})t+\omega_{k_z}\Big].\end{aligned} \tag{29}$$

The present case is a specific one of that in Section 2. With Eq.(19) we can show that the energy density can be negative if ω_{k_x} and ω_{k_z} are chosen as $\omega_{k_x} > (3+\sqrt{8})\omega_{k_z}$ or $\omega_{k_z} > (3+\sqrt{8})\omega_{k_x}$, and if γ and λ meet Eqs.(20) and (21) respectively. It needs to be mentioned that $\omega_{k_x} > (3+\sqrt{8})\omega_{k_z}$ or $\omega_{k_z} > (3+\sqrt{8})\omega_{k_x}$ is the necessary condition on which the energy density becomes negative and which we will use in the present section.

The minimum value of negative energy density satisfies

$$\langle\rho\rangle_{\min} \geq \frac{g}{(\lambda^2+\gamma^2+1)V}, \tag{30}$$

where

$$g=(\lambda^2+\gamma^2)\omega_{k_x}-\frac{1}{2}(|\lambda|+|\gamma|)(\omega_{k_x}+\omega_{k_z})+\omega_{k_z}. \tag{31}$$

And the duration for which the energy density remains negative is

$$\Delta t=\frac{2}{|\omega_{k_x}-\omega_{k_z}|}\cos^{-1}2\frac{\omega_{k_z}+(\lambda^2+\gamma^2)\omega_{k_x}}{|\lambda+\gamma|(\omega_{k_x}+\omega_{k_z})}, \tag{32}$$

where $\omega_{k_x}\neq\omega_{k_z}$. In the following we will derive a restriction on $V\langle\rho\rangle_{\min}\Delta t$. By Eqs.(30) and (32) one can obtain

$$V\langle\rho\rangle_{\min}\Delta t \geq \frac{g}{(\lambda^2+\gamma^2+1)|\omega_{k_x}-\omega_{k_z}|}\,\pi. \tag{33}$$

Now we discuss two cases where we always let the right-hand side in the above inequality remain negative.

1. if $\omega_{k_x}>\omega_{k_z}$,

$$V\langle\rho\rangle_{\min}\Delta t > \frac{g}{\omega_{k_x}-\omega_{k_z}}\,\pi. \tag{34}$$

For g is a two-variable function of $|\lambda|$ and $|\gamma|$ one can calculate the most negative value of g and obtain

$$g_{\min} = (-\omega_{k_x}^2 + 6\omega_{k_x}\omega_{k_z} - \omega_{k_z}^2)/8\omega_{k_x}. \tag{35}$$

So

$$V\langle\rho\rangle_{\min}\Delta t > \frac{-f_1^2 + 6f_1 - 1}{8(1-f_1)}\,\pi, \tag{36}$$

where $f_1 = \omega_{k_z}/\omega_{k_x}$. The right-hand side of Eq.(36) is always larger than $-\pi/8$ since it is an increasing function of f_1 in the region $(0, 3-\sqrt{8})$. Thus

$$V\langle\rho\rangle_{\min}\Delta t > -\frac{\pi}{8}. \tag{37}$$

2. if $\omega_{k_z} > \omega_{k_x}$,

$$V\langle\rho\rangle_{\min}\Delta t > \frac{g}{(\lambda^2+\gamma^2+1)(\omega_{k_z}-\omega_{k_x})}\,\pi. \tag{38}$$

Because $g/(\lambda^2+\gamma^2+1)$ reaches the least value when $|\lambda| = |\gamma| = (\omega_{k_z}+\omega_{k_x})/4\omega_{k_x}$, we acquire

$$V\langle\rho\rangle_{\min}\Delta t > -\frac{(f_2^2-6f_2+1)f_2}{(9f_2^2+2f_2+1)(1-f_2)}\pi \equiv -F\pi. \tag{39}$$

Here $f_2 = \omega_{k_x}/\omega_{k_z}$ and $0 < f_2 < 3-\sqrt{8}$. The function F is always less than $1/20$, so

$$V\langle\rho\rangle_{\min}\Delta t > -\frac{\pi}{20}. \tag{40}$$

With Eqs.(37) and (40), one can see

$$E\Delta t < 1, \tag{41}$$

where $E = V|\langle\rho\rangle_{\min}|$. Accordingly, the more negative the energy density is, the shorter the duration becomes. Comparing $E < 1/\Delta t$ with the quantum uncertainty principle, we know that the amount of negative energy that passes by a fixed point in time Δt is less than the quantum energy uncertainty on that time scale, Δt^{-1}. Therefore, the above QI forbids the bizarre consequences mentioned in the Introduction from appearing.

4. Ford and Roman's quantum inequality

Now we want to verify that Ford and Roman's QI of the form Eq.(1) is also satisfied using Vollick's method.[16] Define the sampled energy density $\hat{\rho}$ as[11]

$$\hat{\rho} = \frac{t_0}{\pi}\int_{-\infty}^{\infty}\frac{\langle\rho\rangle}{t^2+t_0^2}\mathrm{d}t. \tag{42}$$

Inserting Eq.(29) into the above, this integration results in

$$\hat{\rho} = \frac{2\pi}{(1+\lambda^2+\gamma^2)VL}\,h, \tag{43}$$

where

$$h = (\lambda^2+\gamma^2)n_x + n_z + \frac{1}{2}(\lambda+\gamma)(n_x+n_z)e^{-\alpha|n_x-n_z|/2}, \tag{44}$$

and $\alpha = 4\pi t_0/L$ are defined and $\omega_{k_{\tau_z}} = 2\pi n_z/L$ is used. Vollick[16] has proved that it is only required that

$$\hat{\rho} \geq -\frac{2\sqrt{3}\pi}{LV}\frac{e^{-\alpha}(1+e^{-\alpha})^2}{(1-e^{-\alpha})^4} \tag{45}$$

for $\hat{\rho}$ to satisfy the inequality of the form Eq.(1). So the task now is to prove

$$\frac{h}{1+\lambda^2+\gamma^2} \geq -\frac{\sqrt{3}e^{-\alpha}(1+e^{-\alpha})^2}{(1-e^{-\alpha})^4}. \tag{46}$$

Apparently Eq.(46) always exists if its left-hand side is positive or equal to zero. Thereby we just need to prove Eq.(46) when the left-hand side is negative, i.e. the inequality

$$-\frac{h}{1+\lambda^2+\gamma^2} \leq \frac{\sqrt{3}e^{-\alpha}(1+e^{-\alpha})^2}{(1-e^{-\alpha})^4}. \tag{47}$$

The minimum of h is

$$h_{\min} = n_z - \frac{(n_x+n_z)^2}{8n_x}e^{-\alpha|n_x-n_z|}. \tag{48}$$

1. if $n_x > n_z$,

$$-\frac{h}{1+\lambda^2+\gamma^2} < -h_{\min} < \frac{n}{8}e^{-\alpha n}, \tag{49}$$

where $n = n_x - n_z$ and n is a positive integer. The function $ne^{-\alpha n}/8$ is maximized at $\alpha n = 1$. For $\alpha \geq 1$, the maximum is $e^{-\alpha}/8$ when $n = 1$. Obviously,

$$\frac{1}{8}e^{-\alpha} < \frac{\sqrt{3}e^{-\alpha}(1+e^{-\alpha})^2}{(1-e^{-\alpha})^4}. \tag{50}$$

For $0 < \alpha < 1$ the maximum is $1/8\alpha e$. Then Eq.(47) will hold on condition that

$$\frac{1}{8e} < \frac{\sqrt{3}\alpha e^{-\alpha}(1+e^{-\alpha})^2}{(1-e^{-\alpha})^4}. \tag{51}$$

The right-hand side of Eq.(51) is a decreasing function and the minimum is $e^{-1}(1+e^{-1})^2(1+e^{-1})^{-4}$. So the inequality (47) is satisfied for the case $n_x > n_z$.

2. if $n_z > n_x$,

$$-\frac{h}{1+\lambda^2+\gamma^2} < -h_{\min} < \frac{n^2}{8}e^{-\alpha n} \equiv H. \tag{52}$$

Where $n = n_z - n_x$. The sign of the derivative of H is the same as that of $(2-\alpha n)$. For $\alpha \geq 2$, the function H decreases as n becomes bigger. Then the maximum is $e^{-\alpha}/8$ which is still less than the right-hand side of

Eq.(47). If $0 < \alpha < 2$, the function is maximized at $\alpha n = 2$. And the biggest value is $1/2\alpha^2 e^2$. Provided

$$\frac{1}{2e^2} < \frac{\sqrt{3}\alpha^2 e^{-\alpha}(1+e^{-\alpha})^2}{(1-e^{-\alpha})^4}, \tag{53}$$

Eq.(47) is satisfied. The right-hand side of Eq.(53) decreases as α increases and attains its minimum when $\alpha \to 2$. Then one knows that the above inequality is true. Accordingly, Eq.(47) is right if $n_z > n_x$.

To sum up, the QI of the form Eq.(1) is assured. The QI indicates that an inertial observer cannot see unboundedly large negative energy densities for an arbitrarily long period of time.

5. Conclusion

In this paper we have examined the energy density produced in the Dirac field by state vectors of the form Eq.(8) in the four-dimensional flat spacetime. Firstly, the conditions on which the energy density can be negative were obtained. Following the conditions one can create states where negative energy densities appear. Secondly, we showed that the negative energy density obeys the QI: $E\Delta t < 1$. Finally, we proved the sampled energy density satisfies the QI of the form Eq.(1). Recalling all the quantum inequalities derived before by different methods, we come to the conclusion that although quantum field allows negative energy to exist, it also has the mechanism to prevent the magnitude and duration of negative energy from being arbitrarily large at the same time.

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