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Generation and application of a controllable multi-atom entangled state*

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In this paper, we present a scheme to prepare a set of the multi-atom entangled states by the cavity quantum electrodynamics (QED) technology. The multi-atom entangled states have some particular entanglement properties. For example, the remaining reduced density matrices ρ_{ij} still retain entanglement or disentanglement when any N-2 atoms of the N atoms are traced out, which can be chosen freely according to our need, and the relative entanglement strength of any pair of atoms (measured by the concurrence) can be arbitrarily adjusted. In addition, they may be completely symmetric under the exchange of any two atoms, and perform certain quantum information tasks, such as telecloning, teleportation, secret sharing and so on.

Keywords: multi-atom entangled states, QED, telecloning
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1. Introduction

Quantum entanglement has played a key role in recent years as a potential resource for quantum communication and quantum information processing. An ongoing effort has been made to characterize quantitatively and qualitatively entanglement. For bipartite systems this problem has been completely solved, while it still remains open for multipartite entanglement. The properties, preparation and applications of multipartite entangled states are now a hot topic in the field of quantum information.[1−10] It has been shown that multipartite entangled states have an advantage over the two-particle Bell states in quantum cloning,[11−14] quantum teleportation,[15−17] superdense coding,[18−20] and quantum secret sharing.[21−22] In the case of tripartite entanglement Dur *et al*[3] have shown that there are two inequivalent kinds of genuine tripartite entanglement states: the GHZ (Greenberger–Horne–Zeilinger) class and the W class under stochastic local operation and classical communication (SLOCC). The GHZ state is maximally entangled in several senses[3] and has the feature that when any one of the three qubits is traced out, the remaining two qubits are completely disentangled. This means, in particular, that if one of the three parties sharing the system decides not to cooperate with the other two, then they cannot use the entanglement resource of the state at all. The GHZ state is very fragile under particle loss. Conversely, the W state maximizes the residual bipartite entanglement,[3] it is maximally robust under disposal of any one of the three qubits, which is argued as an important property for some practical problems in the sense of particle loss. Recently, some schemes have been proposed for the generation of the GHZ states and W states.[23−26] In this paper, we propose a cavity QED scheme for the multi-atom entanglement preparation; also we study its entanglement properties and point out its possible applications.

2. Generation of the multi-atom entangled states

In our previous work,[27] we have prepared a peculiar tripartite entangled state via cavity quantum electrodynamics. We consider the interaction of a ladder-type three level atom with a cavity. The atomic states

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are denoted by $|g\rangle$, $|e\rangle$, and $|f\rangle$. The transition frequency between levels $|e\rangle$ and $|g\rangle$is far from resonance with the cavity field mode. On the other hand, the transition frequency between the states $|e\rangle$and $|f\rangle$ is detuned from the cavity frequency enough so that only virtual transitions occur between these levels. Then the effective Hamiltonian in the interaction picture is[28−30]

$$H_{\text{eff}} = \frac{g^2}{\Delta} a^+ a(|f\rangle\langle f| - |e\rangle\langle e|), \tag{1}$$

where a^+ and a are the creation and annihilation operators for the cavity mode, and g is the atom–cavity coupling strength. Δ is the detuning between the atomic transition frequency ω_0 and the cavity frequency ω. We assume that the cavity field is initially prepared in a coherent state $|\alpha\rangle$ by a microwave source. The ith atom enters the cavity, which is initially prepared by laser excitation and microwave manipulation[31] in the superposition of the states $|g_i\rangle$ and $|e_i\rangle$, i.e. $c_{ei}|e_i\rangle + c_{gi}|g_i\rangle$, where $|c_{ei}|^2 + |c_{gi}|^2 = 1$. The tripartite entangled state can be obtained as follows:[27]

$$\begin{aligned}|\psi_3\rangle_{-\alpha} =& c_{g1}c_{g2}c_{e3}|g_1g_2e_3\rangle + c_{e1}c_{e2}c_{e3}|e_1e_2e_3\rangle \\ &+ c_{g1}c_{e2}c_{g3}|g_1e_2g_3\rangle + c_{e1}c_{g2}c_{g3}|e_1g_2g_3\rangle \\ =& c_{e3}|e_3\rangle|\psi_2\rangle_\alpha + c_{g3}|g_3\rangle|\psi_2\rangle_{-\alpha},\end{aligned} \tag{2}$$

$$\begin{aligned}|\psi_3\rangle_{\alpha} =& c_{g1}c_{g2}c_{g3}|g_1g_2g_3\rangle + c_{e1}c_{e2}c_{g3}|e_1e_2g_3\rangle \\ &+ c_{g1}c_{e2}c_{e3}|g_1e_2e_3\rangle + c_{e1}c_{g2}c_{e3}|e_1g_2e_3\rangle \\ =& c_{g3}|g_3\rangle|\psi_2\rangle_\alpha + c_{e3}|e_3\rangle|\psi_2\rangle_{-\alpha},\end{aligned} \tag{2'}$$

$$|\psi_2\rangle_\alpha = c_{g1}c_{g2}|g_1g_2\rangle + c_{e1}c_{e2}|e_1e_2\rangle, \tag{3}$$

$$|\psi_2\rangle_{-\alpha} = c_{g1}c_{e2}|g_1e_2\rangle + c_{e1}c_{g2}|e_1g_2\rangle. \tag{3'}$$

Using the same recipe as that in Ref.[27], we can obtain the four-atom entangled state as follows:

$$|\psi_4\rangle_\alpha = c_{g4}\,|g_4\rangle\,|\psi_3\rangle_\alpha + c_{e4}\,|e_4\rangle\,|\psi_3\rangle_{-\alpha}\,, \tag{4}$$

$$|\psi_4\rangle_{-\alpha} = c_{e4}\,|e_4\rangle\,|\psi_3\rangle_\alpha + c_{g4}\,|g_4\rangle\,|\psi_3\rangle_{-\alpha}\,. \tag{4'}$$

Generalizing the above case to the N successive atoms passing through the cavity one by one, we can obtain an N-atom entangled state similarly:

$$|\psi_N\rangle_\alpha = c_{g_N}\,|g_N\rangle\,|\psi_{N-1}\rangle_\alpha + c_{e_N}\,|e_N\rangle\,|\psi_{N-1}\rangle_{-\alpha}\,, \tag{5}$$

$$|\psi_N\rangle_{-\alpha} = c_{e_N}\,|e_N\rangle\,|\psi_{N-1}\rangle_\alpha + c_{g_N}\,|g_N\rangle\,|\psi_{N-1}\rangle_{-\alpha}\,, \tag{5'}$$

where $|\psi_N\rangle_\alpha$ and $|\psi_N\rangle_{-\alpha}$ are the unnormalized states. The requirements for our scheme can be realized using the current technology; for detailed discussion see Refs.[24,25].

3. Properties and application of the multi-atom entangled states

We can calculate the concurrence[2] of pairs of parties, as a measure of entanglement strength of the bipartite reduced density operators ρ_{ij}. Note that $C(\rho_{ij})$=0 corresponds to disentanglement, and $C(\rho_{ij})$=1 corresponds to maximal entanglement.

After some calculations for $|\psi_3\rangle_{-\alpha}$ or $|\psi_3\rangle_\alpha$, we obtain

$$C_{12} = 2k^2|c_{e_1}c_{g_1}c_{e_2}c_{g_2}| \cdot ||c_{g_3}|^2 - |c_{e_3}|^2|, \tag{6}$$

$$C_{13} = 2k^2|c_{e_1}c_{g_1}c_{e_3}c_{g_3}| \cdot ||c_{g_2}|^2 - |c_{e_2}|^2|, \tag{7}$$

$$C_{23} = 2k^2|c_{e_2}c_{g_2}c_{e_3}c_{g_3}| \cdot ||c_{g_1}|^2 - |c_{e_1}|^2|. \tag{8}$$

Similarly, for $|\psi_N\rangle_\alpha$, we have

$$C_{ij} = 2k^2|c_{e_i}c_{g_i}c_{e_j}c_{g_j}| \cdot \prod_{\substack{I=1 \\ (I\neq i,j)}}^{N} ||c_{g_I}|^2 - |c_{e_I}|^2|, \tag{9}$$

where k is a normalization factor and is determined by the corresponding state $|\psi_N\rangle_\alpha$. We denote $C_{ij} = C(\rho_{ij})$. Equations (6)–(9) show that the relative entanglement strength of any pair of atoms can be arbitrarily adjusted. The remaining reduced density matrices ρ_{ij} can retain entanglement or disentanglement, which only depends on the choice of parameters c_{ei} and c_{gi} of the corresponding atom, and the parameters c_{ei} and c_{gi} can be chosen independently for each atom. Hence, we can choose freely the entanglement and disentanglement of the remaining reduced density matrices ρ_{ij} according to our need. This property is very useful for the quantum information processing.

First, the state $|\psi_N\rangle_\alpha$ can apply to telecloning. For $|\psi_3\rangle_{-\alpha}$ when $|c_{e_1}| = |c_{e_2}| = |c_{e_3}| = |c_e|$, $|c_{g_1}| = |c_{g_2}| = |c_{g_3}| = |c_g|$, and $|c_e| \neq |c_g|$, one has $C_{12} = C_{13} = C_{23} = 2k^2|c_e^2c_g^2| \cdot ||c_g|^2 - |c_e|^2|$, where $k^2(3|c_e^2c_g^4| + |c_e|^6) = 1$. This shows that all atoms are equally entangled with each other. Let $c_e = \sin\theta$ and $c_g = \cos\theta, |\psi_3\rangle_{-\alpha}$ is the required three-particle entangled state for the 1→2 faithful telecloning of two linear-independent states of a one-qubit system.[14] Gu *et al*[14] also showed how to prepare the required three-particle entangled state under LOCC (local operation

and classical communication) using the GHZ state as a resource via POVM (positive operator valued measurement). However, our scheme is simpler and experimentally feasible. If $|c_g|^2 = 1/4$ and $|c_e|^2 = 3/4$, one obtains $C_{12} = C_{13} = C_{23} = 1/3$. This is the maximal value achievable for $|\psi_3\rangle_{-\alpha}$ in this case.

Likewise, for $|\psi_3\rangle_{-\alpha}$, when $|c_{e_1}|^2 = |c_{g_1}|^2 = 1/2, C_{23} = 0$, i.e. ρ_{23} is not entangled; if $|c_{g_2}| = |c_{g_3}| = |c_g|$, $|c_{e_2}| = |c_{e_3}| = |c_e|$ and $|c_e| \neq |c_g|$, one has $C_{12} = C_{13} = k^2|c_e c_g| \cdot ||c_g|^2 - |c_e|^2|$ where k^2=2. This shows that one atom is equally entangled with two others and the others are disentangled. This is the required three-particle entangled state for the 1→2 imperfect telecloning of an unknown one-qubit quantum state.[11] When $|c_g|^2 = 1/2 \pm \sqrt{2}/4$ and $|c_e|^2 = 1/2 \mp \sqrt{2}/4$, one has $C_{12} = C_{13} = 1/2$. This is the maximal value achievable for $|\psi_3\rangle_{-\alpha}$ in this case.

Murao *et al*[11] pointed out that the telecloning state should be symmetric under permutations of the output qubits; in particular, they should be all equally entangled with the port . Furthermore, in order to optimize the transfer of information the entanglement of the receiving and transmitting sides should be as large as possible. While the state $|\psi_N\rangle_\alpha$ or $|\psi_N\rangle_{-\alpha}$ prepared by our scheme just possesses the desired properties. Hence, it is useful for $1 \to N-1$ telecloning.

In addition, $|\psi_N\rangle_\alpha$ can also apply to teleportation. Lee *et al*[16] have considered entanglement teleportation of an entangled state unknown at all, such as $a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$, where a, b, c, and d are complex numbers. It may be the simplest protocol that employs EPR (Einstein–Podolsky–Rosen) pairs as a quantum channel. However, when a reservoir is present, it is important to study inseparable quantum channels because some inseparable channels can be more robust against decoherence than EPR pairs, and an inseparable quantum channel always leads to a total teleportation of entanglement.[16] The state $|\psi_4\rangle_\alpha$ can be regarded as an example of an inseparable quantum channel when parameters c_{ei} and c_{gi} have certain special values.[16] Thus, it has an advantage in the teleportation of entanglement over EPR pairs and GHZ state.

Recently, Bagherinezhad and Karimipour[21] have introduced a protocol for quantum secret sharing based on the reusable entangled states, namely $|\mathrm{GHZ}\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$ and $|E\rangle = \frac{1}{2}(|000\rangle + |110\rangle + |101\rangle + |011\rangle)$. These two states are transformed into each other by the local operations of Hadamard gates: that is, $|\mathrm{GHZ}\rangle = H \otimes H \otimes H|E\rangle$ and $|E\rangle = H \otimes H \otimes H|\mathrm{GHZ}\rangle$. They have used these two states that are shared by all three parties as carriers of information (the $|\mathrm{GHZ}\rangle$ state in the odd rounds and the $|E\rangle$ state in the even rounds). Let $c_{ei} = c_{gi} = \frac{1}{\sqrt{2}}$, then $|\psi_3\rangle_\alpha$ is the required three-particle entangled state $|E\rangle$ as carriers of information for quantum secret sharing.

The state $|\psi_N\rangle_\alpha$ possesses some special entanglement properties. It can have not only many-qubit entanglement as the GHZ state, but also pairing entanglement as the W state simultaneously when the parameters c_{ei} and c_{gi} are chosen appropriately: namely, the state $|\psi_N\rangle_\alpha$ may be highly entangled, yet particularly robust, hence it may be a spin-squeezed state.[32,33] Therefore, it has some possible applications in atomic interferometers and high-precision atomic clocks. Maybe it also has potential applications with respect to quantum error-prevention,[34] quantum remote state preparation,[35] and quantum games.

4. Conclusion

In summary, we have presented a scheme to prepare a set of the multi-atom entangled states by the cavity QED technology. The advantage of our scheme is whether the cavity field collapse into $|\alpha\rangle$ or $|-\alpha\rangle$, the N-atom entangled states $|\psi_N\rangle_\alpha$ and $|\psi_N\rangle_{-\alpha}$ created by selective measurements on the cavity field have identical entanglement properties due to symmetry. We have showed that the set of multi-atom entangled states has some particular entanglement properties, for example, the remaining reduced density matrices ρ_{ij} retain entanglement or disentanglement when any $N-2$ atoms of the N atoms are traced out, which can be chosen freely according to our need, and the relative entanglement strength of any pair of atoms can be arbitrarily adjusted. Also we have given the maximal amount of entanglement achievable for the two specific situations. The multi-atom entangled states can have not only many-qubit entanglement, but also pairing entanglement simultaneously when the parameters c_{ei} and c_{gi} are chosen appropriately, Besides, they may be completely symmetric under the exchange of any two atoms. Because of these entanglement properties, the multi-atom entangled states can perform certain quantum information tasks, such as telecloning, teleportation, and secret sharing ; they also have some potential applications in the aspects of atomic inter-

ferometers and high-precision atomic clocks, quantum error-prevention, quantum games and so on. In addition, we hope that our study is helpful for the understanding of the nature of multipartite entanglement.

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