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Controlling chaos by a delayed continuous feedback in a gas discharge plasma*

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Control of chaos by a delayed continuous feedback is studied experimentally in a gas discharge plasma. The power spectrum, the maximum of Lyapunov exponents and the time series of the signals all indicate that the period-1 unstable periodic orbit is controlled successfully. The dependence of the control on the delay time and the feedback gain as well as the strength of white noise is also investigated in detail. The experimental results show that the scaling index of the control versus the strength of white noise is 1.995, which is very close to that obtained from the simple logistic map.

Keywords: controlling chaos, delayed continuous feedback
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1. Introduction

Nonlinear dynamic phenomena are abundant in nature and laboratory plasmas. Various nonlinear phenomena have been observed in laboratory plasmas, such as self-oscillation, periodic doubling bifurcation, intermittency, quasi-periodicity, and chaos. In many real physical systems, chaos is an undesirable process because it could have harmful consequences. For example, chaos in magnetically confined plasma may evolve into fully developed turbulence and lead to anomalous energy and particle cross-field transport. Therefore, the ability to control chaos is of much practical importance. Recently, several feedback schemes have been proposed for controlling chaos, such as the Ott–Grebogi–Yorke method,[1] the occasional proportional feedback,[2] the delayed continuous feedback (DCF)[3] and other alternative approaches.[4] The main idea of these methods relies on the fact that unstable periodic orbits (UPOs) embedded in a chaotic attractor can be stabilized by applying a time-dependent perturbation in the form of feedback to some accessible system parameters. These schemes have been successfully utilized to control chaos in various dynamic systems.[5−8] The DCF method first proposed by Pyragas seems more applicable to experimental systems because it does not require any *a priori* analytical knowledge of the dynamic system and hence avoids laborious data processing. The basic idea of the method is to realize an active continuous control of a dynamic system by applying a feedback signal which is proportional to the difference between an dynamical variable $y(t)$ and its delayed value:

$$F(t) = K\left[y(t) - y(t-\tau)\right] = KD(t), \qquad (1)$$

where τ is the delay time and K the feedback gain. If the delay time τ coincides with the period τ_k of the period-k UPO, i.e. $\tau = \tau_k$, the feedback vanishes on this UPO. This means that the feedback in the form of Eq.(1) does not change the solution of the system that corresponds to the period-k UPO. Choosing an appropriate feedback gain K, one can achieve the stabilization of the desired UPO. Because this control process synchronizes the system to its state at one previous period by continuously perturbing an accessible control parameter, it is also referred to as time-delay auto-synchronization (TDAS). This method has been applied successfully to experimental chaotic systems, including discharge plasmas.[9−12]

In this paper, we report our experimental study

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on controlling chaos with the DCF approach in a gas discharge plasma. Not only the influence of the delay time and the feedback gain but also the impact of white noise on the stabilization are experimentally investigated.

2. Experiments and results

2.1. Experimental arrangement

Our experiment was performed in an unmagnetized steady-state plasma device.[13] This device consists of an electron-emitting cathode (six parallel tungsten filaments) at one end and a current-collecting anode (a rectangular stainless steel plate) at the other end, as shown in Fig.1. The cathode is connected to the chamber wall and is electrically grounded. The anode is mounted on the shaft of a probe so that the distance between the cathode and anode can be varied. The argon plasma is produced by a dc discharge between the anode and cathode. The discharge is controlled by the argon pressure (P_a), the filament current (I_f), the discharge voltage (V_D), and the distance between the cathode and anode (d). The typical parameters of the steady-state plasma obtained with a Langmuir probe are: the electron density $n_e = 10^8 - 10^9\,\mathrm{cm}^{-3}$, the electron temperature $T_e = 1-3\,\mathrm{eV}$, and the ion temperature $T_i \ll T_e$. Both the ion saturation current signal $I_{si}(t)$ of the probe and the discharge current I_d are recorded by digitizers (12 bit, N=2^{15} data points, sampling interval Δt=16μs). The ion saturation current is proportional to the electron density. The power spectra, the information dimensions as well as the Lyapunov exponents calculated from the time series of the signals are used to detect periodic and chaotic motions.

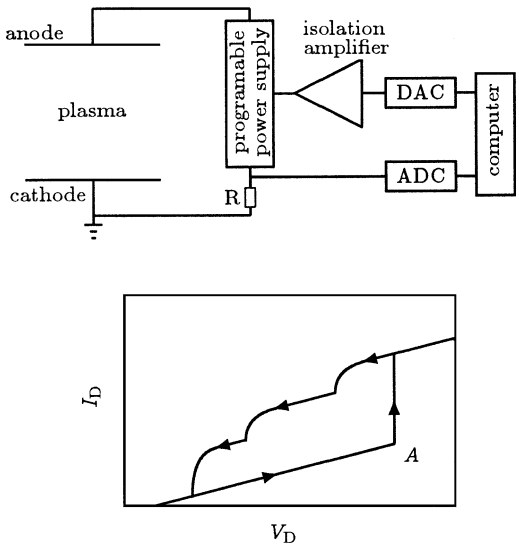


Fig.1. Schematic of experimental setup and discharge current–voltage characteristic curve.

We have previously studied this discharge plasma extensively and found that the nonlinear dynamic phenomena are always associated with the occurrence of hysteresis in the current–voltage ($I-V$) characteristic of the discharge.[13] The chaotic behaviour always occurs in the region near the turning point either on the lower or on the upper branch of the $I-V$ curve (see Fig.1), where the plasma potential is negative with respect to the anode. In this regime, the plasma can be driven into chaos by varying the discharge parameters (P_a, I_f, V_D and d). The most convenient parameter for controlling chaos is found to be the discharge voltage. Moreover, the chaotic state on the lower branch can be maintained over a rather wide range of the discharge parameters. Therefore, in our experiment, a dc voltage V_{dc} is applied to the anode, which is fixed below the turning point A on the lower branch of the discharge $I-V$ curve. A sinusoidal voltage with the adjustable amplitude V_s and the frequency f_s is added to the dc voltage V_{dc} to drive the plasma into a chaotic state. Hence the total discharge voltage consists of a dc part and a sinusoidal part. The discharge voltage is provided by a programable power supply: the sinusoidal signals and control signals generated by a computer are firstly converted to analogue signals with a 12 bit digital/analogue converter (DAC), and then are amplified with an isolation amplifier. A sampling resistor R is used to detect the discharge current $I_D(t)$, a state variable which is proportional to the electron density. The signal $I_D(t)$ is digitized with an analog/digital converter (ADC). In order to implement the control in real time, both the ADC and DAC are designed to be able to work in a continuous mode.[14]

2.2. Control of chaos via DCF

In the experiment, a proper choice of the discharge condition leads to the easy onset of the chaotic oscillations. The experimental conditions are $P_a = 1.6\times10^{-1}$Pa, $I_f = 25.0$A, $V_s = 8$V, $f_s = 3.89$kHz, $d = 9$cm, and $V_{dc} = 11.5$V, respectively. As reported previously,[13] when the chaotic regime is not set far from the threshold boundary, chaos is characterized by a low-dimensional chaotic attractor ($2 < D_2 < 3$) and one positive Lyapunov exponent, indicating that the plasma is in a 'weakly' chaotic regime.

Shown in Fig.2 are (a) the time series of discharge

current I_d, and (b) its power spectrum before the feedback is switched on. The broad power spectrum of the signal illustrates that the plasma is being driven in the chaotic regime. The correlation dimension and the Lyapunov exponents can be obtained with the method of the time delay logarithm.[15] They are $D_2 = 2.6 \pm 0.1$ and $(1.26, -0.03, -1.56)f_s$. As the feedback is switched on, the system becomes stable if the delay time τ and feedback gain K are adjusted to appropriate values. The feedback signal can be added either to the dc discharge voltage V_{dc} or to the sinusoidal modulation V_s. We will present in the following the results when the feedback signal is added to the dc discharge voltage.

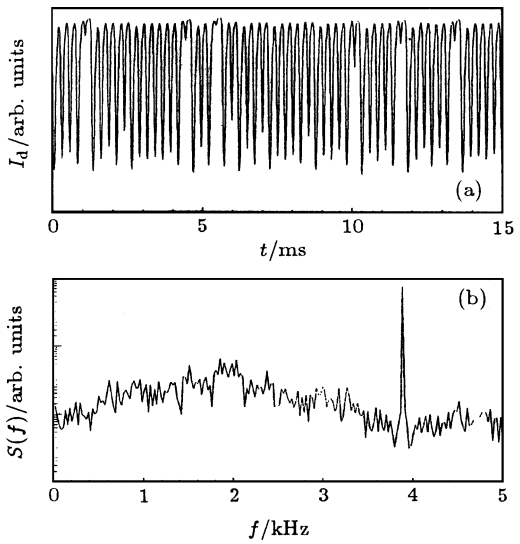


Fig.2. Time series of discharge current and its power spectrum before feedback is switched on.

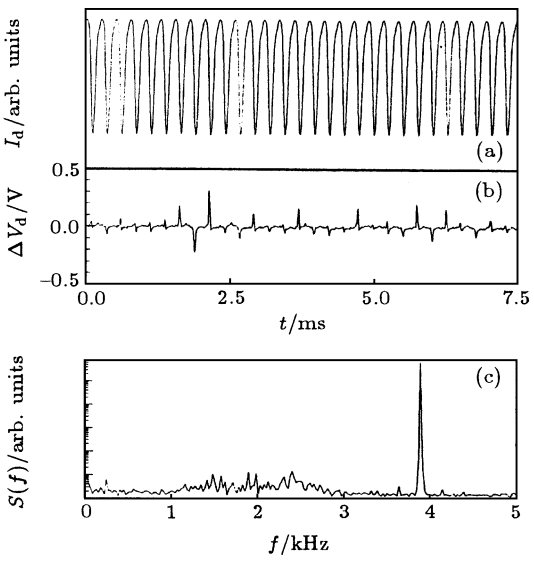


Fig.3. Characteristics of discharge current after feedback is switched on: (a) time series of discharge current , (b) the feedback signal, and (c) the power spectrum of discharge current.

The time series and the power spectrum of the controlled signal as well as the corresponding feedback signal are shown in Fig.3, where the delay time is $\tau = 1/f_s = 257\mu$s, the feedback gain is $K = 0.4$, and the Lyapunov exponents are $(0.05, -1.03, -4.11)f_s$, respectively. The time series, the power spectrum and the Lyapunov exponents of the signal all indicate that the period-1 UPO is stabilized. From Fig.3, we can also see that the feedback signal in this case is lower than 0.5V which is much smaller than V_{dc} and V_s.

The key adjustable parameters for controlling chaos via DCF are the delay time τ and the feedback gain K. Only when the delay time τ is close to the period of an UPO, can this UPO be stabilized. We adjust the delay time τ close to the period of the period-1 UPO, and measure its influence on the root-mean-square (rms) of the state variable, i.e. $D(\tau) = \langle (x(t) - x(t-\tau))^2 \rangle^{1/2}$, and the maximum of the Lyapunov exponents λ_1. Our experimental results are shown in Fig.4. Although the Lyapunov exponent spectrum indicates that the UPO can be stabilized as the delay time varies from 248 to 274μs, there exists a resonance valley for $D(\tau)$ near $\tau = T = 257\mu$m, indicating that $D(\tau)$ reaches a minimum only when the delay time coincides with the period of the UPO. The dependence of $D(\tau)$ and λ_1 on the feedback gain K is also measured and presented in Fig.5. As seen in this figure, only positive feedback can stabilize the UPO, and the system is controlled when the feedback gain is operated in the range of $0.3 < K < 0.6$.

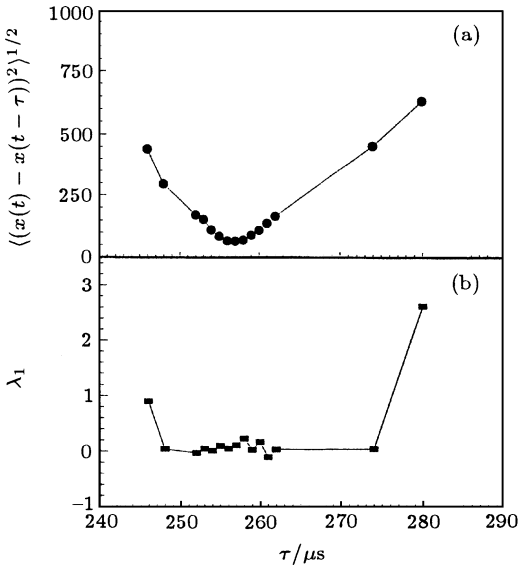


Fig.4. Dependence on the delay time τ of (a) the root-mean-square of discharge current $D(\tau) =< (x(t) - x(t-\tau))^2 >^{1/2}$, and (b) the maximum of the Lyapunov exponent λ_1 , where the feedback gain is 0.3.

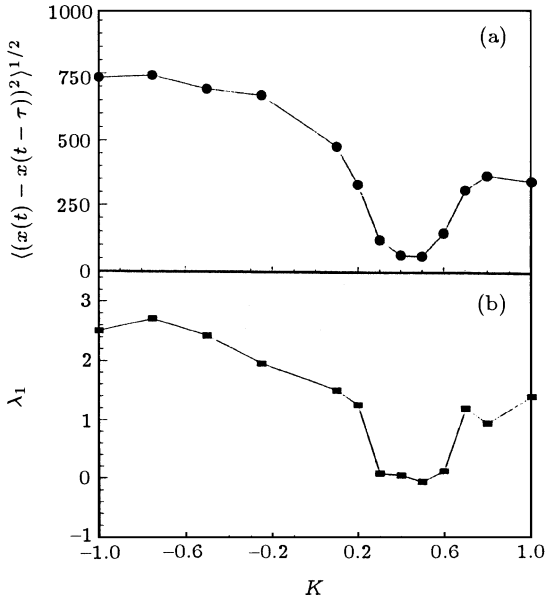


Fig.5. Dependence on the weight of the feedback K of (a) the mean square root of discharge current $D(\tau) =< (x(t) - x(t-\tau))^2 >^{1/2}$, and (b) the maximum of the Lyapunov exponent λ_1 , where the delay time of the feedback is T.

2.3. Influence of noise on DCF

Noise prevails in any real physical system. Therefore, the influence of noise on the control should be studied. To our best knowledge, there has been no previous report on the subject of how DCF responds to white noise. A well characterized Gaussian white noise is used in our experiment. The bandwidth of the noise is about 200kHz which is much higher than that of the oscillation. The rms of the amplitude $(V_{\rm n})_{\rm rms}$ of the noise is about 4V. The noise is attenuated and added to the dc discharge voltage $V_{\rm dc}$. In the experiment, we first stabilize the period-1 UPO by setting the delay time $\tau = 1/f_{\rm s}$ and the feedback gain $K = 0.4$. Then, we increase the strength of the noise ϵ by adjusting the attenuation factor ϵ until the control becomes failure. The dependence of the quantity $D^2(\tau)$ on the parameter ϵ is plotted in Fig.6. A least rms fitting gives the index of the scaling law $\alpha = 1.995$, where a curve of $D^2 = A + B\epsilon^{\alpha}$ is assumed.

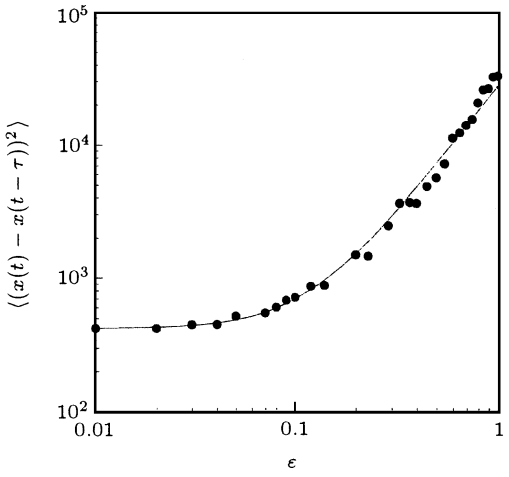


Fig.6. Dependence of the quantity $D^2(\tau)$ on the noise level, where the solid line is the fitting line.

The scaling law we obtained can be explained by the simple logistic map, which is given by

$$X_{n+1} = \mu X_n(1 - X_n). \tag{2}$$

The period-1 fix point is

$$X_{\rm F} = \frac{\mu - 1}{\mu}. \tag{3}$$

With DCF, the dynamic system becomes

$$X_{n+1} = \mu X_n(1 - X_n) + K(X_n - X_{n-1}). \tag{4}$$

Denoting $A_n = X_n - X_{\rm F}$ and $B_n = X_{n-1} - X_{\rm F}$, we can linearize the dynamic system in the vicinity of the fix point $X = X_{\rm F}$,

$$\begin{pmatrix} A_{n+1} \\ B_{n+1} \end{pmatrix} = \begin{pmatrix} 2 - \mu + K & -K \\ 1 & 0 \end{pmatrix} \begin{pmatrix} A_n \\ B_n \end{pmatrix}. \tag{5}$$

The system can be stabilized when the feedback gain is operated in the range of

$$K \in \left(\frac{\mu - 3}{2}, 2 \right).$$

If the feedback is added to the system parameter μ, we have

$$X_{n+1} = \left(1 + K\left(X_n - X_{n-1}\right)\right) \mu X_n \left(1 - X_n\right).$$

Linearizing the system near the fix point $X = X_{\rm F}$, we again obtain

$$\begin{pmatrix} A_{n+1} \\ B_{n+1} \end{pmatrix} = \begin{pmatrix} 2 - \mu + \dfrac{\mu - 1}{\mu} K & -\dfrac{\mu - 1}{\mu} K \\ 1 & 0 \end{pmatrix} \begin{pmatrix} A_n \\ B_n \end{pmatrix}. \tag{6}$$

Denoting $K' = K(\mu - 1)/\mu$, Eqs.(5) and (6) are in the same form. So, we use Eq.(5) for the discussion below.

With the inclusion of a Gaussian white noise, Eq.(5) is rewritten as

$$A_{n+1} = (2 - \mu + K)A_n - KA_{n-1} + \epsilon N_n, \quad (7)$$

where N is the normalized white noise with $\langle N \rangle = 0$ and $\langle N^2 \rangle = 1$. Multiplying Eq.(7) with A_{n+1} and A_n respectively, and averaging the equation with respect of time, we obtain

$$\langle A_n^2 \rangle = \frac{1 + K}{(2K + 3 - \mu)(1 - K)(\mu - 1)} \epsilon^2, \quad (8)$$

where we use the equation $\langle N_n \cdot A_n \rangle = 0$. Therefore, the index of the scaling law is 2, which is very close to our experimental result 1.995.

3. Conclusion

We studied experimentally the control of chaos by DCF, and its stability under white noise. From the time series, the power spectrum and the maximum of the Lyapunov exponents, we conclude that the period-1 UPO is successfully stabilized. The influences of the delay time and the feedback gain on the control are also investigated in detail. Experimental results show that $D^2(\tau)$ depends sensitively on the delay time, and reaches a minimum when the delay time is set equal to the period of the UPO. The stability of the control is further researched by adding a Gaussian white noise on the dc discharge voltage. The index of the scaling law of the relation of $D^2(\tau)$ and the strength of the noise is obtained, which is very close to the value inferred from the logistic map.

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