Vol 13 No 11, November 2004
1009-1963/2004/13(11)/1892-04

Chinese Physics

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One-step alternating direction implicit FDTD algorithm*

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(Received 27 November 2003; revised manuscript received 26 April 2004)

In this paper, a novel unconditionally stable alternating direction implicit finite-different time-domain method (ADI-FDTD) called the one-step ADI-FDTD method is presented, where the calculation for one discrete time step is performed using only one procedure, but not the original two sub-updating procedures. Consequently, the proposed one-step ADI-FDTD methods have consumed less computer memory and computation resources and have been faster than the conventional ADI-FDTD methods. We analytically and numerically verified that the new algorithm is unconditionally stable and free from the Courant condition.

Keywords: ADI-FDTD, FDTD methods, Courant condition, unconditionally stable
PACC: 5225, 5240, 5265

1. Introduction

The finite-difference time-domain (FDTD) method has been widely used to simulate the transient solutions of electromagnetic wave propagation. However, as the traditional FDTD method is based on an explicit finite difference algorithm, the Courant stability condition must be satisfied when this method is used. Therefore, the maximum time-step size is limited by minimum cell size. Recently, a novel FDTD method called the alternating-direction implicit (ADI-FDTD) method was put forth by Zheng *et al*,[1] Namiki[2,3] and Namiki and Ito.[4] In this method, the growth factor of the algorithm is always unity. Therefore, the method is unconditionally stable, i.e. the time-step size is not dependent on the Courant condition, but rather on numerical errors. Subsequently, the perfectly matched layer (PML) absorbing media for ADI-FDTD method (it cannot corrupt the unconditional stability of this method) was proposed by Liu and Gedney[5] and Wang and Teixeira.[6] Nevertheless, the ADI-FDTD method, introduced by Zheng *et al*[1] and Namiki,[2−4] consumes more computer memory and computation resources than the conventional FDTD. This is because the ADI-FDTD solution marching from the nth time step to the $(n+1)$th time step is broken into two sub-updating procedures: the first updating procedure involves the advance from the nth time step to the $(n+1/2)$th time step, whereas in the second updating procedure, the fields are advanced from the $(n+1/2)$th time step to the $(n+1)$th time step.

In this paper, we propose a new ADI-FDTD method called the one-step ADI-FDTD method. The one-step ADI-FDTD solution marching from the nth time step to the $(n+1)$th time step is just one updating procedure, rather than two updating procedures. One significant advance over the conventional ADI-FDTD is that the one-step ADI-FDTD method consumes less computer memory and computation resources. Also, the one-step ADI-FDTD calculations remain unconditionally stable.

2. Derivation of the one-step ADI-FDTD method

In an isotropic loss-free medium, with the well-known Yee finite difference grid arrangement, The Maxwell curl equations can be written as in a one-dimensional case:

*Project supported by the National High Technology Research and Development Programme of China (Grant No 2002AA835040), and Key Laboratory of Defence Technology (Grant No 51483010301KG0102).

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$$\frac{E_x^{n+1}(k)-E_x^n(k)}{\Delta t}=-\frac{1}{\varepsilon}\left[\frac{\frac{1}{2}\left[H_y^{n+1}(k+1/2)+H_y^n(k+1/2)\right]-\frac{1}{2}\left[H_y^{n+1}(k-1/2)+H_y^n(k-1/2)\right]}{\Delta z}\right], \quad (1)$$

$$\frac{H_y^{n+1}(k+1/2)-H_y^n(k+1/2)}{\Delta t}=-\frac{1}{\mu}\left[\frac{\frac{1}{2}\left[E_x^{n+1}(k+1)+E_x^n(k+1)\right]-\frac{1}{2}\left[E_x^{n+1}(k)+E_x^n(k)\right]}{\Delta z}\right], \quad (2)$$

where ε and μ are the permittivity and permeability of the medium, respectively.

In Eqs.(1) and (2), the field components on the right-hand side are replaced with a set of unknown implicit values at the $(n+1)$th time step and a set of known explicit values at the previous nth time step. Equations (1) and (2) are extremely attractive because, compared with the conventional ADI-FDTD algorithm, the proposed FDTD solution marching from the nth time step to the $(n+1)$th time step is performed only in one updating procedure.

Equations (1) and (2) can be simplified as

$$\begin{aligned}E_x^{n+1}(k)=&E_x^n(k)-\frac{\Delta t}{2\varepsilon\Delta z}[H_y^{n+1}(k+1/2)\\&+H_y^n(k+1/2)-H_y^{n+1}(k-1/2)\\&-H_y^n(k-1/2)], \quad (3)\\H_y^{n+1}(k+1/2)=&H_y^n(k+1/2)\\&-\frac{\Delta t}{2\mu\Delta z}[E_x^{n+1}(k+1)+E_x^n(k+1)\\&-E_x^{n+1}(k)-E_x^n(k)]. \quad (4)\end{aligned}$$

By substituting the expression for H_y at the $(n+1)$th time step into Eq.(3), E_x value of $(n+1)$th time step can be obtained:

$$\begin{aligned}&E_x^{n+1}(k-1)-\left[2+\left(\frac{2\sqrt{\varepsilon\mu}\Delta z}{\Delta t}\right)^2\right]\\&\times E_x^{n+1}(k)+E_x^{n+1}(k+1)\\&=-E_x^n(k-1)+\left[2-\left(\frac{2\sqrt{\varepsilon\mu}\Delta z}{\Delta t}\right)^2\right]\\&\times E_x^n(k)-E_x^n(k+1)\\&+\frac{4\mu\Delta z}{\Delta t}\left[H_y^n(k+1/2)-H_y^n(k-1/2)\right]. \quad (5)\end{aligned}$$

The implicit equation (5) can be written in tri-diagonal matrix form. It can be efficiently solved. Thereafter, Eq.(4) can be calculated directly using the E_x value at the $(n+1)$th time step.

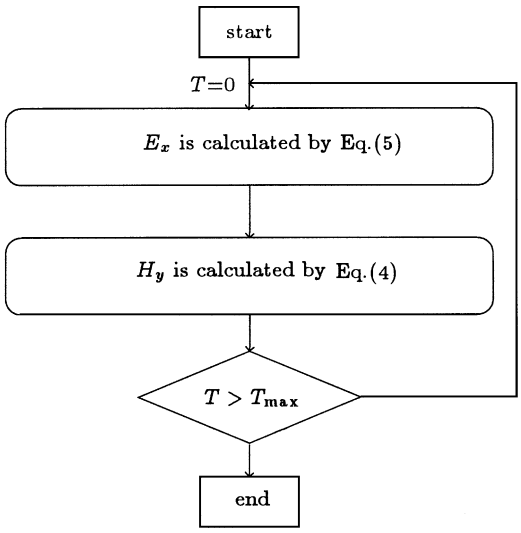


Fig.1. Flowchart of the one-step ADI-FDTD method.

The basic flowchart of the one-step ADI-FDTD method is shown in Fig.1.

Note that the above one-step ADI-FDTD method is different from the conventional ADI-FDTD method. In the conventional ADI-FDTD method, the FDTD solution marching from the nth time step to the $(n+1)$th time step is performed in two sub-updating procedures. In the proposed method, however, the FDTD solution for one discrete time step is performed in one updating procedure only. Consequently, the proposed method consumes less computer memory and computation resources than the conventional ADI-FDTD.

The one-step ADI-FDTD method can be extended to 2-D and 3-D cases. The implicit equation of field components can be written in quintuple–diagonal matrix form in a 2-D case. It will be presented in our future publications.

3. Analysis of stability

In order to prove theoretically the numerical stability, the fields are written as follows:

$$E_x=\psi_{\mathrm{A}}\xi\exp(\mathrm{j}kz), \quad (6a)$$

$$H_y=\psi_{\mathrm{B}}\xi\exp(\mathrm{j}kz), \quad (6b)$$

where k is the wavenumber, and ξ is the growth factor

$$\xi = \frac{E_x^{n+1}}{E_x^n} = \frac{H_y^{n+1}}{H_y^n} = \exp(\mathrm{j}\omega\Delta t). \tag{7}$$

Substituting Eqs.(6) into Eqs.(1) and (2), we obtain

$$\psi_\mathrm{A}\xi = \psi_\mathrm{A} - \mathrm{j}\left(\frac{\Delta t}{\varepsilon\Delta z}\right)\sin(k\Delta z/2)[\psi_\mathrm{B}\xi + \psi_\mathrm{B}], \tag{8a}$$

$$\psi_\mathrm{B}\xi = \psi_\mathrm{B} - \mathrm{j}\left(\frac{\Delta t}{\mu\Delta z}\right)\sin(k\Delta z/2)[\psi_\mathrm{A}\xi + \psi_\mathrm{A}]. \tag{8b}$$

By eliminating ψ_A and ψ_B of Eqs.(8), we obtain

$$p\xi^2 + q\xi + p = 0, \tag{9}$$

where

$$p = 1 + \left(\frac{\Delta t}{\sqrt{\varepsilon\mu}\Delta z}\right)^2 \sin^2\left(\frac{k\Delta z}{2}\right), \tag{10a}$$

$$q = 2\left[\left(\frac{\Delta t}{\sqrt{\varepsilon\mu}\Delta z}\right)^2 \sin^2\left(\frac{k\Delta z}{2}\right) - 1\right]. \tag{10b}$$

The growth factor ξ is $(4p^2 - q^2 > 0)$

$$\xi = \frac{-q \pm \mathrm{j}\sqrt{4p^2 - q^2}}{2p}. \tag{11}$$

The absolute value of growth factor ξ is

$$|\xi| = 1. \tag{12}$$

At first glance, it is seen that the proposed scheme is theoretically unconditionally stable without the constraint of the Courant stability condition.

4. Numerical example

To validate the aforementioned one-step ADI-FDTD method for one-dimensional EM problems, consider the following problem: the plane electromagnetic wave propagates in a free space.

4.1. Numerical verification of the stability

To verify numerically that the proposed one-step ADI-FDTD method is unconditional stable, simulations with the conventional FDTD and the proposed one-step ADI-FDTD were run with a time step, Δt, exceeding the time step limit.

Figure 2 shows a comparison of the conventional FDTD[7,8] and proposed one-step ADI-FDTD results for the time-domain normalized electric fields. The computational domain is 15cm long. The incident wave used in the simulation is a plane wave whose frequency is 20GHz. For conventional FDTD parameters, the spatial discretization, Δx, used in the simulations is 75μm, and the time step Δt_c=0.3ps (the maximum stable time step $\Delta t_{\mathrm{c\ max}} = \Delta x/c$=0.25ps). For the one-step ADI-FDTD method, the time step is 100 times of $\Delta t_{\mathrm{c\ max}}$. As can be seen in Fig.1, the conventional FDTD quickly becomes unstable while the one-step ADI-FDTD is still stable just as the conventional ADI-FDTD is.

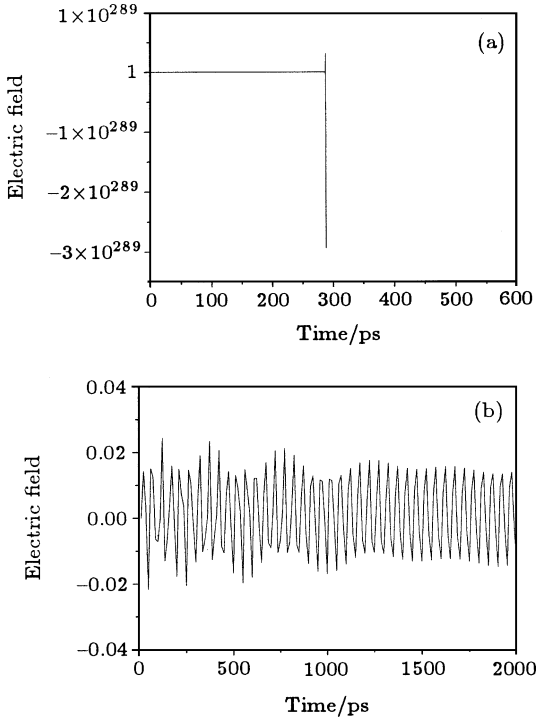


Fig.2. Time-domain electric fields with the conventional FDTD and the proposed one-step ADI-FDTD. (a) The conventional FDTD solutions that becomes unstable with Δt_c=0.3ps. (b) The proposed one-step ADI-FDTD solutions that is stable with Δt_c=25ps.

4.2. Accuracy versus time step

Figures 3(a) and 3(b) show a comparison of the conventional FDTD[7] and proposed one-step ADI-FDTD results for the normalized electric field. In Fig.2(a), the time step of conventional FDTD Δt_c=0.125ps, the time step of the proposed one-step ADI-FDTD $\Delta t_\mathrm{one} = 5\Delta t_\mathrm{c}$. In Fig.3(b), the time step of conventional FDTD Δt_c=0.125ps, the time step of the proposed one-step ADI-FDTD $\Delta t_\mathrm{one} = 10\Delta t_\mathrm{c}$. Note that the one-step ADI-FDTD method picks up the same sinusoidal behaviour as the conventional FDTD method. But the error of results is dependent on the time step size.

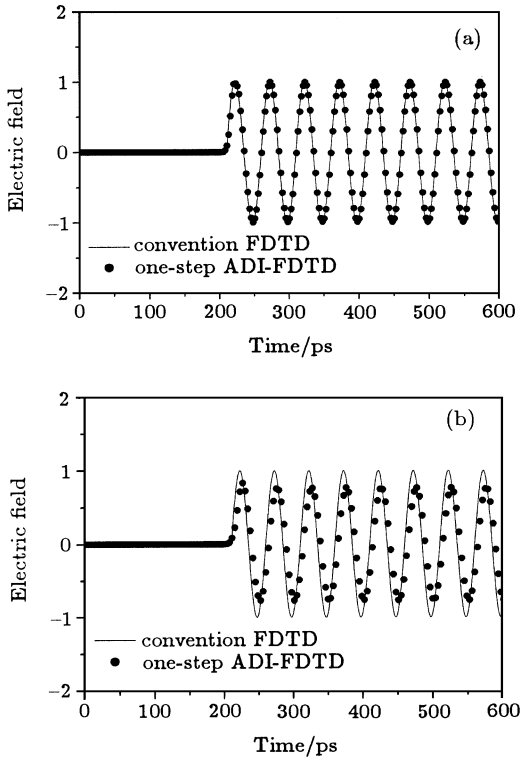


Fig.3. A comparison of the conventional FDTD and proposed one-step ADI-FDTD for time-domain electric fields. (a) Δt_c=0.125ps, $\Delta t_{one} = 5\Delta t_c$. (b) Δt_c=0.125ps, $\Delta t_{one} = 10\Delta t_c$.

4.3. Computation memory and CPU time saving

For the comparison purpose, both the conventional FDTD[7,8] and one-step ADI-FDTD were used to simulate the electromagnetic wave propagation again. In this case, the time step of conventional FDTD Δt_c=0.125ps, the time step of the proposed one-step ADI-FDTD $\Delta t_{one} = 5\Delta t_c$=0.625ps. The reason for such time step selections is that they will provide similar accuracy. The results of computing in a P4 PC computer (1.7GHz) are shown in Table 1. In this case, the CPU time is reduced to 24% when the time step size is set 5 times as large as the conventional FDTD method. Meanwhile, the computer memory is the same as for the conventional FDTD method.

Table 1. The comparison of the results for two methods.

	Δt	Steps	CPU time	Memory
FDTD	0.125ps	10000	8.9s	4k
One-step ADI-FDTD	0.625ps	2000	2.1s	3.9k

5. Conclusion

In this article, we propose a new ADI-FDTD method called the one-step ADI-FDTD method. In the proposed method, the FDTD solution for one discrete time step is performed in only one updating procedure. Consequently, the proposed method consumes less computer memory and computation resources than the conventional ADI-FDTD. Subsequently, its numerical stability is studied analytically and numerically.

For the one-step ADI-FDTD, the implicit equation of the field component can be written in tridiagonal matrix form in the one-dimensional case. As a return, it can be efficiently solved. And the one-step ADI-FDTD method can be extended to 2-D and 3-D cases. The implicit equation of field components can be written in quintuple–diagonal matrix form in a 2-D case. This will be presented in our future publications.

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