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Entanglement and disentanglement from cascade beam-splitter for squeezed vacuum state inputs*

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Properties of the entanglement at the outputs of a cascade beam-splitter are investigated for two single-mode squeezed vacuum state inputs. It is shown that the entanglement depends on the squeezing amplitudes of the input states and the reflection coefficients of the cascade beam-splitter, and the composite phase shift Δ between the cascade beam-splitter and the input fields has a great effect on the entanglement. In particular, the properties of the entanglement of a cascade beam-splitter differ from those of only one beam-splitter. A further method for manipulating entanglement by adjusting the parameters of the cascade beam-splitter and the input fields is proposed.

Keywords: entanglement, disentanglement, cascade beam-splitter, squeezed vacuum states
PACC: 4280H, 4250

1. Introduction

Quantum entanglement, as one of the most striking features of the quantum theory, has become a new focus of activity in quantum physics because of immense theoretical and experimental progress in the foundation of quantum mechanics and the new field of quantum information science. In particular, creating and manipulating of entangled states are essential for quantum information applications, such as quantum computation,[1−4] quantum teleportation,[5−7] quantum dense coding,[8−10] etc. On the other hand, disentanglement of an entangled quantum state, which is the process that transforms an entangled state into an unentangled state, has played an important role in quantifying the quantum entanglement.[11] Hence, quantum entangled states have been viewed as a useful information 'resource' for quantum information processing.[12−17]

An entangled state can be generated from unentangled states by a beam-splitter (BS). That means a BS has the ability to entangle a quantum state. Properties of the entanglement of a BS for various input states including the discrete variable and the continuous variable have been studied extensively .[18−25] From these works, one can see that nonclassicality of the input fields is a necessary condition for entangling the output state with the help of a BS.[26] In the laboratory, coherent states and squeezed states are two familiar practically existing states. It is well known that no entanglement is produced for coherent states due to their classicality. Therefore it is important to know the properties of the entanglement when squeezed states are used as the inputs. Recently, more attention has been paid to continuous variable quantum information processing.[25−32] For instance, the output entanglement quantity is studied in Ref.[26] for a given squeezed vacuum state (SVS) and an explicit formula expressing the output state in the form of the two-mode SVS has been obtained. In particular, Paris[27] studied entanglement of the output state from a Mach–Zehnder interferometer for SVS inputs. However, the results in Refs.[26,27] are only limited to specific cases. Here BS is limited to a 50:50 symmetrical BS (where the symmetry implies that an input experiences a phase shift of $\pi/2$ upon reflection relative to transmission), and the SVS is confined to the

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real squeezing parameters, and so on. Hence, it is natural to investigate the properties of the entanglement under the more general conditions. In this paper, we shall explore the properties of the entanglement and disentanglement of a cascade beam-splitter (CBS) for two single-mode SVS inputs with arbitrary input parameters by introducing a more general formula of the entanglement entropy. Especially, we have the interest to know how to maximize (minimize) entanglement through adjusting these parameters in the whole system. Here CBS consists of the first BS (BS1) and the second BS (BS2) in cascade configuration, and the parameters of BS1 and BS2 are arbitrary. In particular, CBS possesses properties of the entanglement which differ from those of only one BS, and would be reduced to a Mach–Zehnder interferometer when both BS1 and BS2 are restricted to 50:50 BSs.

2. Properties of entanglement and disentanglement of CBS

Let us analyse the action of a CBS on two single-mode SVSs briefly before we discuss the entanglement and disentanglement. Each of the two lossless BSs combined in the CBS can be described by a unitary transformation operator[18]

$$\hat{B}=\exp\left[\theta(\hat{a}^{+}\hat{b}\mathrm{e}^{\mathrm{i}\phi}-\hat{a}\hat{b}^{+}\mathrm{e}^{-\mathrm{i}\phi})\right], \tag{1}$$

where the transmission and the reflection coefficients of the BS are determined by a single angular parameter θ (i.e. $t=\cos\theta$ and $r=\sin\theta$), and the phase shift ϕ between the reflected and transmitted fields is given by the BS. The BS operator given by Eq.(1) provides the well-known transformation relations

$$\hat{B}^{+}\begin{pmatrix}\hat{a}\\ \hat{b}\end{pmatrix}\hat{B}=\begin{pmatrix}t & r\mathrm{e}^{\mathrm{i}\phi}\\ -r\mathrm{e}^{-\mathrm{i}\phi} & t\end{pmatrix}\begin{pmatrix}\hat{a}\\ \hat{b}\end{pmatrix}, \tag{2}$$

where we have used the operator identity

$$\begin{aligned}&\exp(\hat{A})\hat{B}\exp(-\hat{A})\\ =&\hat{B}+[\hat{A},\hat{B}]+\frac{1}{2!}\left[\hat{A},[\hat{A},\hat{B}]\right]\\ &+\frac{1}{3!}\left[\hat{A},\left[\hat{A},[\hat{A},\hat{B}]\right]\right]+\ldots,\end{aligned} \tag{3}$$

together with the fact that $\hat{B}^{+}=\hat{B}^{-1}$ owing to the unitarity of the BS operator.

It is well known that the SVSs are generated by the action of the single-mode squeezing operators

$$\begin{aligned}\hat{S}_{\mathrm{a}}(\xi_{\mathrm{a}})&=\exp\left(-\frac{1}{2}\xi_{\mathrm{a}}\hat{a}^{+2}+\frac{1}{2}\xi_{\mathrm{a}}^{*}\hat{a}^{2}\right),\\ \hat{S}_{\mathrm{b}}(\xi_{\mathrm{b}})&=\exp\left(-\frac{1}{2}\xi_{\mathrm{b}}\hat{b}^{+2}+\frac{1}{2}\xi_{\mathrm{b}}^{*}\hat{b}^{2}\right)\end{aligned} \tag{4}$$

on the two-mode vacuum state $|00\rangle$, respectively. Here the squeezing parameters $\xi_{a(b)}=\zeta_{a(b)}\exp(\mathrm{i}\gamma_{a(b)})$ are any complex numbers with the modulus $\zeta_{a(b)}$ and the argument $\gamma_{a(b)}$, and a and b denote the propagation directions of the two input single modes, respectively. Then the density operator of the joint input state can be written as

$$\hat{\rho}_{\mathrm{in}}=\hat{S}_{\mathrm{a}}(\xi_{\mathrm{a}})\hat{S}_{\mathrm{b}}(\xi_{\mathrm{b}})|00\rangle\langle 00|\hat{S}_{\mathrm{b}}^{+}(\xi_{\mathrm{b}})\hat{S}_{\mathrm{a}}^{+}(\xi_{\mathrm{a}}). \tag{5}$$

Using Eq.(4) and the theorem Eq.(3), we have the following properties:

$$\begin{aligned}\hat{S}_{\mathrm{a}}^{+}(\xi_{\mathrm{a}})\begin{pmatrix}\hat{a}\\ \hat{a}^{+}\end{pmatrix}\hat{S}_{\mathrm{a}}(\xi_{\mathrm{a}})&=\begin{pmatrix}\cosh\zeta_{\mathrm{a}} & -\mathrm{e}^{\mathrm{i}\gamma_{\mathrm{a}}}\sinh\zeta_{\mathrm{a}}\\ -\mathrm{e}^{-\mathrm{i}\gamma_{\mathrm{a}}}\sinh\zeta_{\mathrm{a}} & \cosh\zeta_{\mathrm{a}}\end{pmatrix}\begin{pmatrix}\hat{a}\\ \hat{a}^{+}\end{pmatrix},\\ \hat{S}_{\mathrm{b}}^{+}(\xi_{\mathrm{b}})\begin{pmatrix}\hat{b}\\ \hat{b}^{+}\end{pmatrix}\hat{S}_{\mathrm{b}}(\xi_{\mathrm{b}})&=\begin{pmatrix}\cosh\zeta_{\mathrm{b}} & -\mathrm{e}^{\mathrm{i}\gamma_{\mathrm{b}}}\sinh\zeta_{\mathrm{b}}\\ -\mathrm{e}^{-\mathrm{i}\gamma_{\mathrm{b}}}\sinh\zeta_{\mathrm{b}} & \cosh\zeta_{\mathrm{b}}\end{pmatrix}\begin{pmatrix}\hat{b}\\ \hat{b}^{+}\end{pmatrix}.\end{aligned} \tag{6}$$

With the help of Eqs.(2) and (6), it is simple to represent the joint input state $\hat{\rho}_{\mathrm{in}}$ by its Wigner characteristic function (WCF) given by

$$\begin{aligned}\chi_{\mathrm{in}}^{(w)}(\alpha,\beta)=&\mathrm{Tr}[\hat{\rho}_{\mathrm{in}}\hat{D}(\alpha,\beta)]\\ =&\exp\Big[-\frac{1}{2}(|\alpha\cosh\zeta_{\mathrm{a}}+\alpha^{*}\mathrm{e}^{\mathrm{i}\gamma_{\mathrm{a}}}\sinh\zeta_{\mathrm{a}}|^{2}\\ &+|\beta\cosh\zeta_{\mathrm{b}}+\beta^{*}\mathrm{e}^{\mathrm{i}\gamma_{\mathrm{b}}}\sinh\zeta_{\mathrm{b}}|^{2})\Big],\end{aligned} \tag{7}$$

where we have denoted the Glauber displacement operator as $\hat{D}(\alpha,\beta)=\exp(\alpha\hat{a}^{+}-\alpha^{*}\hat{a}+\beta\hat{b}^{+}-\beta^{*}\hat{b})$ and used the relation $\hat{S}_{\mathrm{a}}^{+}(\xi)\hat{D}(\alpha)\hat{S}_{\mathrm{a}}(\xi)=\hat{D}(\alpha\cosh\zeta_{\mathrm{a}}+\alpha^{*}\mathrm{e}^{\mathrm{i}\gamma_{\mathrm{a}}}\sinh\zeta_{\mathrm{a}})$.

Now let two independent single-mode SVSs be incident on a CBS simultaneously. For convenience, we suppose that the reflection coefficient and the internal phase shift of BS2 are equal to that of BS1, respectively, (i.e. $r_2=r_1=r$ and $\phi_{BS2}=\phi_{BS1}=\phi$). After transformation of BS1, the WCF is given by

$$\begin{aligned}\chi^{(1)}_{\rm out}(\alpha,\beta) =&{\rm Tr}[\hat{\rho}^{(1)}_{\rm out}\hat{D}(\alpha,\beta)]\\ =&\exp\Big\{-\frac{1}{2}[|(t\alpha - r{\rm e}^{{\rm i}\phi}\beta)\cosh\zeta_{\rm a}\\ &+(t{\rm e}^{{\rm i}\gamma_{\rm a}}\alpha^* - r{\rm e}^{-{\rm i}(\phi-\gamma_{\rm a})}\beta^*)\sinh\zeta_{\rm a}|^2\\ &+|(r{\rm e}^{-{\rm i}\phi}\alpha + t\beta)\cosh\zeta_{\rm b}\\ &+(r{\rm e}^{{\rm i}(\phi+\gamma_{\rm b})}\alpha^* + t{\rm e}^{{\rm i}\gamma_{\rm b}}\beta^*)\sinh\zeta_{\rm b}|^2]\Big\}, \quad (8)\end{aligned}$$

where $\hat{\rho}^{(1)}_{\rm out} = \hat{B}_1\hat{\rho}_{\rm in}\hat{B}_1^+$. Equation (8) is obtained by using the relation $\hat{B}_1^+\hat{D}_{\rm a}(\alpha)\hat{D}_{\rm b}(\beta)\hat{B}_1 = \hat{D}_{\rm a}(t\alpha - r{\rm e}^{{\rm i}\phi}\beta)\hat{D}_{\rm b}(r{\rm e}^{-{\rm i}\phi}\alpha + t\beta)$. One knows that the action of a unitary transformation on a state can lead to a simple modification of the WCF so that we can easily find the WCF for transformed state if we know that of the original state.[33] In our scheme the output state from BS1 is directly used as the input state of BS2. Then, the 'original state' with the density operator $\hat{\rho}^{(1)}_{\rm out}$ transforms into the resulting state with density operator $\hat{\rho}^{(2)}_{\rm out} = \hat{B}_2\hat{\rho}^{(1)}_{\rm out}\hat{B}_2^+$ by BS2. Hence, the modified WCF for $\hat{\rho}^{(2)}_{\rm out}$ can be written as

$$\begin{aligned}\chi^{(2)}_{\rm out}(\alpha,\beta) =&\exp\Big\{-\frac{1}{2}[|(t^2-r^2)(\alpha\cosh\zeta_{\rm a}\\ &+\alpha^*{\rm e}^{{\rm i}\gamma_{\rm a}}\sinh\zeta_{\rm a})\\ &-2rt(\beta{\rm e}^{{\rm i}\phi}\cosh\zeta_{\rm a} + \beta^*{\rm e}^{-{\rm i}(\phi-\gamma_{\rm a})}\sinh\zeta_{\rm a})|^2\\ &+|2rt(\alpha{\rm e}^{-{\rm i}\phi}\cosh\zeta_{\rm b} + \alpha^*{\rm e}^{({\rm i}\phi+\gamma_{\rm b})}\sinh\zeta_{\rm b})\\ &+(t^2-r^2)(\beta\cosh\zeta_{\rm b} + \beta^*{\rm e}^{{\rm i}\gamma_{\rm b}}\sinh\zeta_{\rm b})|^2]\Big\}. \quad (9)\end{aligned}$$

Because continuous variable entanglement generated by BSs experimentally leads to Gaussian state which can be transformed through an appropriate local linear unitary Bogoliubov operator (LLUBO) into the standard form given by Eqs.(6), (8), and (10) in Ref.[34], we can calculate the WCF given by

$$\begin{aligned}\chi^{(2)}_a(\alpha) =&\chi^{(2)}_{\rm out}(\alpha,\beta=0)\\ =&\exp\Big[-\frac{1}{2}(\alpha_{\rm R},\alpha_{\rm I})M(\alpha_{\rm R},\alpha_{\rm I})^{\rm T}\Big] \quad (10)\end{aligned}$$

in the output mode a of the CBS with the help of Eq.(9). Here $\alpha = \alpha_{\rm R} + i\alpha_{\rm I}$ and M is the 2×2 correlation matrix given by

$$M = \begin{pmatrix} m_{11} & m_{12}\\ m_{21} & m_{22}\end{pmatrix}. \quad (11)$$

After a little algebra, we obtain the correlation matrix Eq.(11) with its elements given by

$$\begin{aligned}m_{11} =&(t^2-r^2)^2[\cosh(2\zeta_{\rm a}) + \cos\gamma_{\rm a}\sinh(2\zeta_{\rm a})]\\ &+4r^2t^2[\cosh(2\zeta_{\rm b}) + \cos(2\phi+\gamma_{\rm b})\sinh(2\zeta_{\rm b})],\\ m_{22} =&(t^2-r^2)^2[\cosh(2\zeta_{\rm a}) - \cos\gamma_{\rm a}\sinh(2\zeta_{\rm a})]\\ &+4r^2t^2[\cosh(2\zeta_{\rm b}) - \cos(2\phi+\gamma_{\rm b})\sinh(2\zeta_{\rm b})],\\ m_{12} =&m_{21} = (t^2-r^2)^2\sin\gamma_{\rm a}\sinh(2\zeta_{\rm a})\\ &+4r^2t^2\sin(2\phi+\gamma_{\rm b})\sinh(2\zeta_{\rm b}). \quad (12)\end{aligned}$$

From Eqs.(12) it can be seen that the relative phase ϕ of each BS has an effect on the rotation of the squeezing angle for the input fields. In order to evaluate the entanglement, we note that any unitary transformation $\hat{U}$ acting on the single mode state $\hat{\rho}^{(2)}_{\rm a}$ does not change the value of the entanglement entropy[35] (i.e. $E(\hat{\rho}'_{\rm a} = \hat{U}\hat{\rho}^{(2)}_{\rm a}\hat{U}^+) = E(\hat{\rho}^{(2)}_{\rm a})$). Using this property, we can transform the output mode $\hat{\rho}^{(2)}_{\rm a}$ of the CBS through LLUBO into the entropy-equivalent state $\hat{\rho}'_{\rm a}$ whose WCF can be written as the standard form[34]

$$\tilde{\chi}_{\rm a}(\alpha) = \exp\Big[-\frac{1}{2}(\alpha_{\rm R},\alpha_{\rm I})\begin{pmatrix}\lambda & 0\\ 0 & \lambda\end{pmatrix}(\alpha_{\rm R},\alpha_{\rm I})^{\rm T}\Big], \quad (13)$$

and

$$\lambda = \sqrt{\det(M)} = \sqrt{m_{11}m_{22} - m_{12}^2}. \quad (14)$$

Clearly, the state defined by the WCF of Eq.(13) is a thermal state of form

$$\hat{\rho}'_{\rm a} = (1-{\rm e}^{-\eta}){\rm e}^{-\eta\hat{a}^+\hat{a}}, \quad (15)$$

with its Wigner characteristic function given by

$$\chi_{\rm th}(\alpha) = \exp\left[-\frac{1}{2}(\alpha_{\rm R},\alpha_{\rm I})\begin{pmatrix}\dfrac{1+{\rm e}^{-\eta}}{1-{\rm e}^{-\eta}} & 0\\ 0 & \dfrac{1+{\rm e}^{-\eta}}{1-{\rm e}^{-\eta}}\end{pmatrix}(\alpha_{\rm R},\alpha_{\rm I})^{\rm T}\right]. \quad (16)$$

Comparing Eq.(13) with Eq.(16), we find the parameter η satisfying

$${\rm e}^{-\eta} = \frac{\lambda-1}{\lambda+1}. \quad (17)$$

In order to discuss entanglement and disentanglement of the CBS, the entanglement entropy provides the essentially unique measure of entanglement for bi-

partite pure quantum state,[36] which is given by

$$E(\hat{\rho}_{\rm a}^{(2)}) = -\,{\rm Tr}(\hat{\rho}_{\rm a}^{(2)}\ln\hat{\rho}_{\rm a}^{(2)}) = -\ln\left(\frac{2}{\lambda+1}\right) - \frac{\lambda-1}{2}\ln\left(\frac{\lambda-1}{\lambda+1}\right), \quad (18)$$

where we have used the fact that the trace value does not change under any unitary transformation. From Eq.(18) we find that the entanglement entropy is completely determined by λ in Eq.(14). We note that the value of λ is related to the composite phase shift Δ between the CBS and the input fields, it is given by

$$\Delta = 2\phi + \gamma_{\rm b} - \gamma_{\rm a}, \quad (19)$$

where the values of Δ, which is independent of R, $\zeta_{\rm a}$, and $\zeta_{\rm b}$, are practically detectable and controllable in BS experiment. Using Eqs.(12), (14), and (18), we can explore the properties of the entanglement of the CBS. The numerical results for Eq.(18) are shown in Figs.1–3 which represent the properties of the entanglement for various choices of the BS parameters, as well as the squeezing parameters of the two single-mode SVSs. Figure 1 exhibits the entanglement entropy of the output state from the CBS against the reflection coefficient R and the squeezing amplitude $\zeta_{\rm b}$ for the given value of other parameters. Figure 2 displays that the influence of BS parameters on the entanglement entropy for the given squeezing parameters. Figure 3 gives the entanglement entropy as a function of the squeezing parameters of the input fields for the given values of BS parameters.

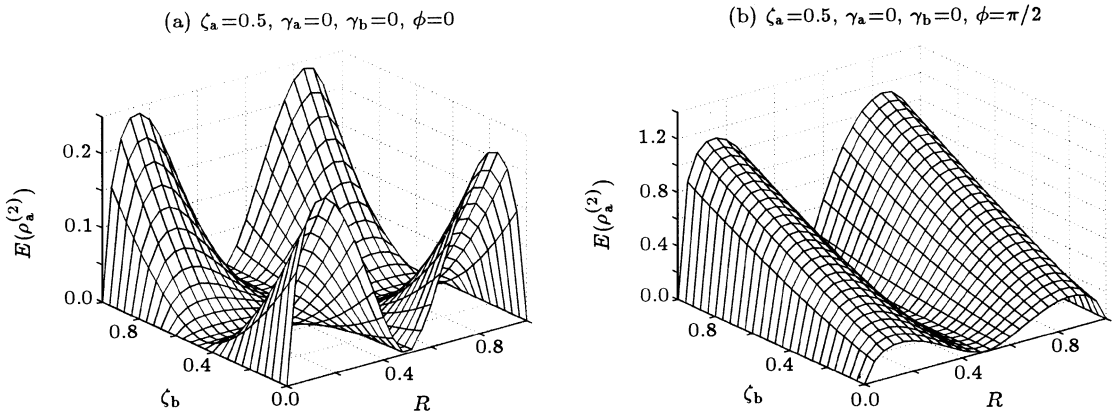


Fig.1. The entanglement entropy of the output state from the CBS against reflection coefficient R ($R \equiv r^2$) and the squeezing amplitude $\zeta_{\rm b}$ for $\zeta_{\rm a} = 0.5$, $\gamma_{\rm a} = \gamma_{\rm b} = 0$ and the different values of (a) $\phi = 0$, (b) $\phi = \pi/2$.

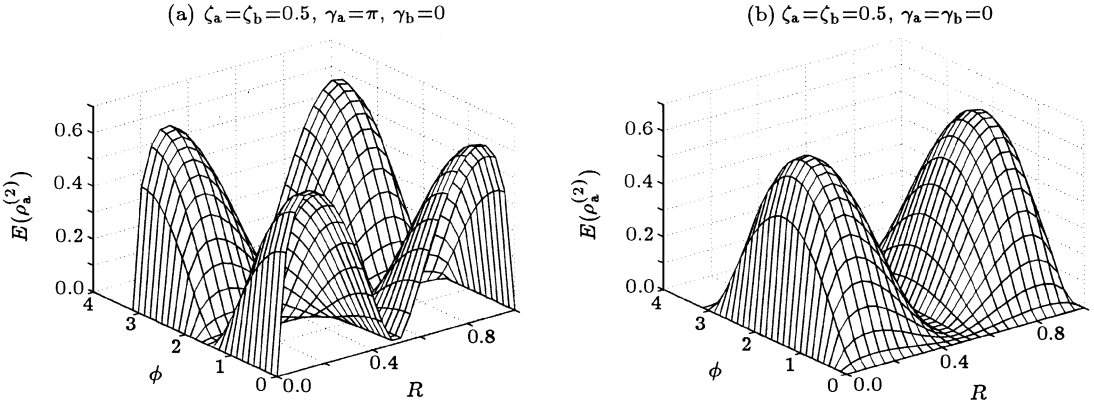


Fig.2. The entanglement entropy of the output state from the CBS versus the BS parameters R and ϕ for the given squeezing parameters $\zeta_{\rm a} = \zeta_{\rm b} = 0.5$ and $\gamma_{\rm b} = 0$, and for the different squeezing phases (a) $\gamma_{\rm a} = \pi$, (b) $\gamma_{\rm a} = 0$.

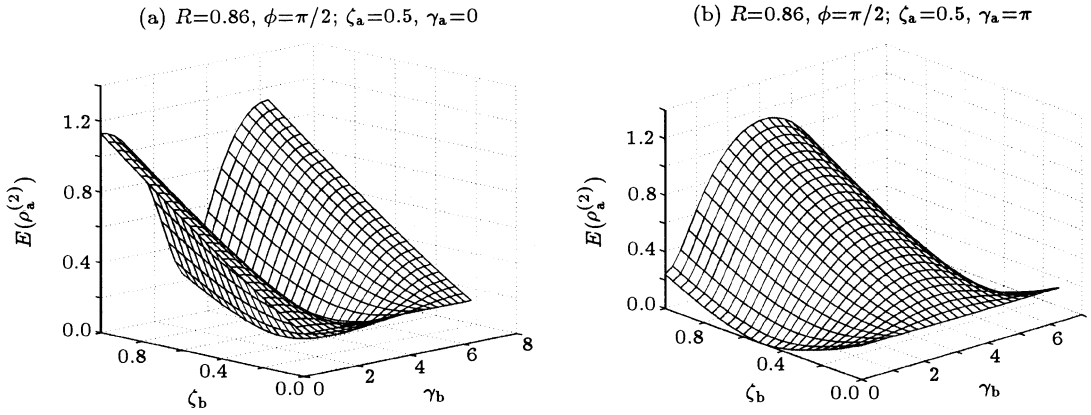


Fig.3. The entanglement entropy of the output state from the CBS as a function of the squeezing parameters of the input field in mode b for the given parameters of the input field in mode a and the CBS. The values of the various parameters are $\zeta_a = 0.5$, $\phi = \pi/2$, and $R = 0.86$. Different plots correspond to different values of the squeezing phase of (a) $\gamma_a = 0$ and (b) $\gamma_a = \pi$.

From Figs.1(a), 2(a), and 3(a) we find that only if $\Delta = 0$ (regardless of the values of ϕ, γ_a, and γ_b), the output modes of the CBS completely lose entanglement owing to destructive interference between reflected and transmitted fields when the two SVS inputs have the same squeezing amplitudes (where $\zeta_a = \zeta_b = 0.5$) without respect to the BS reflection coefficient. The same result can be obtained for the 50:50 BSs without regard for squeezing amplitudes due to the most basic interferometry from the CBS. In contrast to the cases of only one BS displayed in Fig.3 in Ref.[26], it can be seen that for the two 50:50 BSs combined in the CBS, BS1 actually acts as an entangler for the input SVSs, while BS2 is a disentangler of BS1. In other words, when the CBS is composed of two 50:50 BSs, it cannot produce entanglement at its output for the input quantum states in this case. More generally, when reflection coefficient R deviates from the value of $R = 0.5$, both the properties of the entanglement of BS1 and those of the disentanglement of BS2 would be destroyed. These show that the properties of the entanglement for a CBS differ from those for only one BS and a BS can be used not only as an entangler but also as a disentangler.

Figures 1(b) and 3(b) show that only when $\Delta = \pi$ (regardless of the values of ϕ, γ_a, and γ_b), the degree of entanglement is an increasing function of the squeezing amplitude ζ_b for the given squeezing amplitude ζ_a and reflection coefficient R. From Figs.1(b) and 2(b) we can see that the entanglement entropy is split into two symmetric peaks centred at $(R, \Delta) = (0.14, \pi)$ and $(0.86, \pi)$, respectively, with respect to the symmetric axis $R = 0.5$. This implies that the maximally entangled state at the output of the CBS can been obtained only when $\Delta = \pi$ and $R = 0.14$ or $R = 0.86$ for larger squeezing amplitudes. This is to say that we can always use a CBS as an entangler by varying the values of the parameters both of the CBS and the input fields properly.

Comparing Figs.1(a) and 1(b), we find that for the given reflection coefficient R and the squeezing amplitudes $\zeta_{a(b)}$, the entanglement is minimized when $\Delta = 0$ (as shown in Fig.1(a)), while it is maximized when $\Delta = \pi$ (as indicated in Fig.1(b)). Hence, the entanglement of the output fields of the CBS depends not only on the squeezing amplitudes and the reflection coefficient, but also on the values of Δ. It is obvious that the Δ has a great effect on the entanglement.

3. Conclusion

In summary, we have studied the properties of the entanglement and the disentanglement of the output state from the CBS for the two SVS inputs with arbitrary parameters of both the CBS (R and ϕ) and the two input SVSs (ξ_a and ξ_b). We have found that the degree of entanglement of the output state from the CBS depends on the reflection coefficient R and the squeezing amplitudes ζ_a and ζ_b, and the composite phase shift Δ is also an important factor that affects entanglement. For the two 50:50 BSs, BS1 usually acts as an entangler where maximally entangled state can be obtained when $\Delta = \pi$ and the squeezing amplitudes are larger, while BS2 is employed to disentangle the maximally entangled state from BS1. This is different from the case of only one BS. Here the CBS cannot generate quantum entanglement at its output ports. Only if $\Delta = \pi$, maximal entanglement at the output

of the CBS can be obtained for the larger squeezing amplitudes when $R = 0.14$ or 0.86. Hence, our results provide a method for manipulating entanglement by adjusting the parameters of the CBS and the input fields. It is important for quantum information processing.

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