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Effect of three-body interaction on hot asymmetric nuclear matter*

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The properties of hot asymmetric nuclear matter are studied in the framework of the finite temperature Brueckner–Hartree–Fock theory that is extended to include the contribution of microscopic three-body forces. We give the variation of the critical temperature with the asymmetry parameter and show the effect brought by this three-body repulsive potential on the value of the critical asymmetry of the phase transition for asymmetric nuclear matter. Owing to the additional repulsion provided by three-body forces, this value decreases. In addition, the domain of mechanical instability for hot nuclear matter is also indicated, which gradually shrinks with increasing asymmetry and temperature.

Keywords: nuclear matter, three-body force, asymmetry parameter, mechanical instability
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For a long time it has been believed that, due to the Van der Waals nature of the nucleon–nucleon interaction, nuclear matter is likely to exhibit a liquid–gas phase transition.[1] It is a common interesting phenomenon observed or expected in various fields: melting and boiling of condensed matters and atomic clusters,[2–4] decay of hot nuclei,[5,6] transition to quark gluon plasma in high energy nuclear collisions and so on. In heavy-ion collisions, part of the hot nuclear matter can be also produced. During these reactions, liquid–gas phase transitions may occur depending on the temperature and densities involved.[7,8] In addition, hot nuclear matter is also of particular interest for problems of astrophysics. Under astrophysical conditions, mainly at the later stage of the supernovae collapse, the temperature dependence of the nuclear equation of state has some important applications, even if the temperature in the process is probably below a critical temperature. So the investigation of the equation of state for strongly interacting, hot and dense nuclear matter provides a fundamental starting point in heavy-ion physics and astrophysics.[9,10] Thus various models, such as the semi-empirical Skyrme forces method,[11] a chiral hadronic model[12] and some other microscopic theories, are all used to study the properties of hot nuclear matter.

However, most microscopic approaches to nuclear matter are limited to its ground-state properties. This is understandable, since all non-relativistic calculations encounter difficulties in reliable reproduction of the saturation properties of nuclear matter in its ground state. It is therefore assumed that three-body forces must play an important role, for these forces lead to a density-dependent contribution to the effective nucleon–nucleon potential (NN potential). The addition of these microscopic three-body contributions in calculations confirmed these expectations.[13] Therefore we have recently embarked on a study on the temperature behaviour of nuclear matter by using a kind of microscopic approach, in which it is necessary to add the three-body forces contribution to the nucleon–nucleon interaction. As we know, microscopic finite temperature calculations are relatively rare. The most elaborate study so far is the variational calculation made by Friedman and Pandharipande[14] on hot nuclear and neutron mat-

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ter. There exist a few non-relativistic finite temperature Brueckner calculations.[15,16] In this article we extend the finite temperature Brueckner–Hartree–Fock (FTBHF) theory by introducing the microscopic three-body force (TBF) contributions. With respect to the temperature behaviour of nuclear matter, most calculations,[14,17] including the recent phenomenological Thomas–Fermi and Hartree–Fock calculations based on Skyrme interactions,[11] agree in their description of nuclear matter as a Van der Waals liquid, with the existence of a liquid–gas phase equilibrium below a critical temperature $T_{\rm C}$ of some 15MeV–20MeV. Using the extended FTBHF theory including TBF we have found the critical temperature of the liquid–gas phase transition for symmetric nuclear matter to be $T_{\rm C} \approx 13$MeV, which is similar to the above results. But in recent years, with the rapid advance in radioactive beam physics, exotic beams regions of large charge asymmetry could be formed in heavy-ion collisions. Therefore we mainly discuss the thermal properties of the asymmetric nuclear matter with the isospin degree of freedom taken into consideration, which is important for an understanding of nuclei and nuclear interaction.

Asymmetric nuclear matter is a two-component system consisting of Z protons and N neutrons, interacting via the strong nuclear forces (the Coulomb interaction between protons is not considered) within a volume V. The thermodynamic limit is

$$A = Z + N \to \infty, \quad V \to \infty,$$

$$\rho \equiv \frac{A}{V} \to \text{ finite.}$$

The neutron number density $\rho_{\rm n}$ and the proton number density $\rho_{\rm p}$ are related to the total number density $\rho = \rho_{\rm n} + \rho_{\rm p}$, the isospin asymmetry parameter is $\beta = \dfrac{\rho_{\rm n} - \rho_{\rm p}}{\rho}$ by definition, from which we can give $\rho_{\rm n} = \dfrac{1}{2}(1+\beta)\rho$ and $\rho_{\rm p} = \dfrac{1}{2}(1-\beta)\rho$. The Fermi momentum is $k_{\rm F}^{\tau} = \left[3\pi^2\rho_\tau\right]^{1/3}$ with $\tau = p$ or $\tau = n$.

The basic ingredient of the Brueckner–Bethe–Goldstone approach is the reaction matrix G that satisfies the generalized Bethe–Goldstone equation

$$\begin{aligned} &G_{\tau,\tau'}(\rho,T,\omega,\beta) \\ =&v + v\sum_{k_a k_b} \frac{|k_a k_b\rangle\, Q_{\tau,\tau'}\, \langle k_a k_b|}{\omega - e(k_a) - e(k_b)} G_{\tau,\tau'}(\rho,T,\omega,\beta), \end{aligned} \quad (1)$$

where v is the bare nucleon–nucleon (NN) interaction, ω is the starting energy. Here, we consider nuclear matter with no spin excess, therefore in Eq.(1) and in the following equations we drop the spin subscripts to simplify the formulation. $Q_{\tau,\tau'}$ is the finite temperature Pauli operator

$$\begin{aligned} Q_{\tau,\tau'} =&Q_{\tau,\tau'}(k_a,k_b,\rho,T,\beta) \\ =&[1 - f_\tau(k_a,T,k_{\rm F}^{\tau}(\rho,\beta))] \\ &\times \left[1 - f_{\tau'}(k_b,T,k_{\rm F}^{\tau'}(\rho,\beta))\right]. \end{aligned} \quad (2)$$

The Fermi distribution can be expressed as

$$f_\tau(k,T,k_{\rm F}^{\tau}(\rho,\beta)) = \left[1 + \exp(\frac{e_\tau(k) - \mu_\tau}{T})\right]^{-1}, \quad (3)$$

where $\mu_\tau = \mu_\tau(k,T,\rho)$ is the chemical potential. For a given density value ρ and temperature T, we can calculate the chemical potential μ_τ from the following implicit equation self-consistently by iteration:

$$\begin{aligned} \rho_\tau =&\frac{1}{V}\sum_k f_\tau(k,T,\rho,\beta) \\ =&\frac{1}{V}\sum_k \left[1 + \exp\left(\frac{e_\tau(k) - \mu_\tau}{T}\right)\right]^{-1}, \end{aligned} \quad (4)$$

where

$$\begin{aligned} e_\tau(k) =&e_\tau(k,T,\rho,\beta) \\ =&\frac{\hbar^2}{2m}k^2 + U_\tau(k,T,\rho,\beta) \end{aligned} \quad (5)$$

is the single particle energy, and $U_\tau(k,T,\rho,\beta)$ is the two hole-line contribution to the BBG expansion using continuous choice[18] for single particle potential, namely

$$U_\tau(k,T,\rho,\beta) = \frac{1}{2}\sum_{\tau'}\sum_{k'} f_\tau(k,T,k_{\rm F}^{\tau}) f_{\tau'}(k',T,k_{\rm F}^{\tau'})\, \langle kk'|\, G_{\tau,\tau'}(T,\rho,e(k)+e(k'),\beta)\, |kk'\rangle_A\,, \quad (6)$$

where the subscript A denotes the anti-symmetrization of the matrix elements.

The microscopic three-body force adopted in the present calculation is constructed from the meson-exchange current approach as described in Ref.[19]. In the one boson exchange (OBE) approximation, four important mesons π, ρ, σ and ω are considered,[20] in which ω meson provides strong vector coupling to nuclei, ρ meson presents strong tensor coupling, and σ scalar meson offers medium-range attraction in the

NN interaction. A detailed description at zero temperature is given by Ref.[19]. Here we directly show the contribution of the 2σ-exchange TBF at a finite temperature, and this contribution indicates the effective interaction of the nuclear at positions $\boldsymbol{r_1}$ and $\boldsymbol{r_2}$. The function $\eta(\boldsymbol{r})$ is an average over spin and momenta in the Fermi sea of a defect function.[19] With $g(\boldsymbol{r}) = 1-\eta(\boldsymbol{r})$, the three-body force contribution can be written as

$$\begin{aligned} & V_3^{\tau,2\sigma}(T,\rho,\beta) \\ = & -\frac{g_{\sigma N}^4 m_\sigma^2}{4M^3(4\pi)^2}\frac{1}{V}\sum_k f_\tau(k,T,\rho)\int \mathrm{d}^3\boldsymbol{r}_3 \\ & \times\left[\int \mathrm{d}^3\boldsymbol{r}_3' \mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{r}_3'} g(|\boldsymbol{r}_3'-\boldsymbol{r}_1|)g(|\boldsymbol{r}_3'-\boldsymbol{r}_2|)\right. \\ & \times Z_\sigma(|\boldsymbol{r}_3'-\boldsymbol{r}_1|)Z_\sigma(|\boldsymbol{r}_3'-\boldsymbol{r}_2|) \\ & \left.\times \nabla_{r_3'}^2\delta^{(3)}(|\boldsymbol{r}_3'-\boldsymbol{r}_3|)\right] \\ & \times g(|\boldsymbol{r}_3-\boldsymbol{r}_1|)g(|\boldsymbol{r}_3-\boldsymbol{r}_2|)\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{r}_3}. \end{aligned} \tag{7}$$

Here the function $Z_\sigma(\boldsymbol{r})$ is given by $Z_\sigma(\boldsymbol{r}) = \frac{4\pi}{m_\sigma}\int \frac{\mathrm{d}^3\boldsymbol{q}}{(2\pi)^3}\frac{\mathrm{e}^{\mathrm{i}\vec{q}\cdot\vec{r}}F_\sigma^2(q^2)}{(q^2+m_\sigma^2)}$, in which $F_\sigma(q^2)$ is the form factor. Similarly, the other contributions from $\sigma\omega$ and 2ω exchange can also be extended. Because of the limited space of the article we cannot indicate each contribution, but the results calculated include the exchange effects of all the above mesons. At a finite temperature the three-body force contribution $V_3^\tau(T,\rho,\beta)$ may be expressed as follows:

$$\begin{aligned} V_3^\tau(T,\rho,\beta) = & V_3^{\tau,2\sigma}(T,\rho,\beta) \\ & + V_3^{\tau,2\omega}(T,\rho,\beta) \\ & + V_3^{\tau,(\sigma,\omega)}(T,\rho,\beta) \\ & + V_3^{\tau,(\pi,\rho)}(T,\rho,\beta). \end{aligned} \tag{8}$$

Equations (1), (5), (6), (7), (8) can be solved self-consistently for each density, by which we introduce the microscopic three-body force contributions into the FTBHF theory. The bare potential adopted as an input parameter in Eq.(1) is the Argonne V_{18}.[21] The equation needs five iterations to reach a satisfactory convergence. So the total energy obtained is $E(T,\rho,\beta) = E_{\mathrm{kin}}(T,\rho,\beta) + E_{\mathrm{pot}}(T,\rho,\beta)$, where

$$E_{\mathrm{kin}}(T,\rho,\beta) = \sum_\tau\sum_k f_\tau(k,T,\rho,\beta)\frac{\hbar^2}{2m}k^2$$

is the total kinetic energy, and the total potential energy is

$$E_{\mathrm{pot}}(T,\rho,\beta) = \frac{1}{2}\sum_{\tau,\tau'}\sum_{k,k'} f_\tau(k,T,\rho,\beta)f_{\tau'}(k',T,\rho,\beta)\left\langle kk'\right| G_{\tau,\tau'}(T,\rho,e(k)+e(k'))\left|kk'\right\rangle_A .$$

The total entropy S can be evaluated in the approximation of a non-interacting Fermi gas of quasiparticle in the mean field $U_\tau(k,T,\rho,\beta)$, and using the standard thermodynamic relation the free energy F can be given by

$$F = E - TS. \tag{9}$$

As shown in Ref.[8], we can straightforwardly express the mechanical stability criterion as follows:

$$\rho^2\left(\frac{\partial^2 F}{\partial\rho^2}\right)_{T,\beta} = \rho\left(\frac{\partial P}{\partial\rho}\right)_{T,\beta} > 0, \tag{10}$$

where P refers to the pressure. The inequality is the familiar requirement that the isothermal compressibility $K = 9\left(\frac{\partial P}{\partial\rho}\right)$ is positive, which is indicative of mechanical instability.

From the extended FTBHF approximation with three-body force contribution we have obtained a critical temperature $T_{\mathrm{C}} \approx 13$MeV for the phase transition of the symmetric nuclear matter; here we mainly see the properties of the phase transition for the asymmetric nuclear matter with the three-body interaction taken into account.

As we know, pure neutron matter is unbound, and therefore the corresponding finite temperature equation of state (FTEOS) cannot present a liquid–gas phase transition. If such a phase transition is indeed displayed by a symmetric nuclear matter, it must be interesting to ascertain the trend of the liquid–gas transition from symmetric matter to neutron matter. In hot nuclear matter the role played by the energy is replaced by the free energy F, which can be seen in formula (9). A calculated F is shown in Fig.1 for several values of the temperature T and the asymmetry parameter β. The minimum of free energy corresponds to an equilibrium density at a finite temperature, i.e. to a vanishing pressure. With increasing temperature, the equilibrium point disappears at a given asymmetry. As long as the asymmetry is high enough ($\beta \approx 0.75$ in the figure), the stability will be lost at any temperature. Making use of the FTBHF theory that does not include the contribution of three-body interaction, Lombardo and Zuo[22] gave the critical asymmetry parameter of the liquid–gas phase transition for asymmetric nuclear matter, which is $\beta = 0.8$. This indicates that the addition of

the TBF lowers the critical temperature at a higher β, i.e. the nuclear matter becomes easier to be gasified with the increase of asymmetry parameter as a result of the introduction of three-body repulsive potential. Therefore for pure neutron matter, the system only remains in a gas phase at zero temperature.

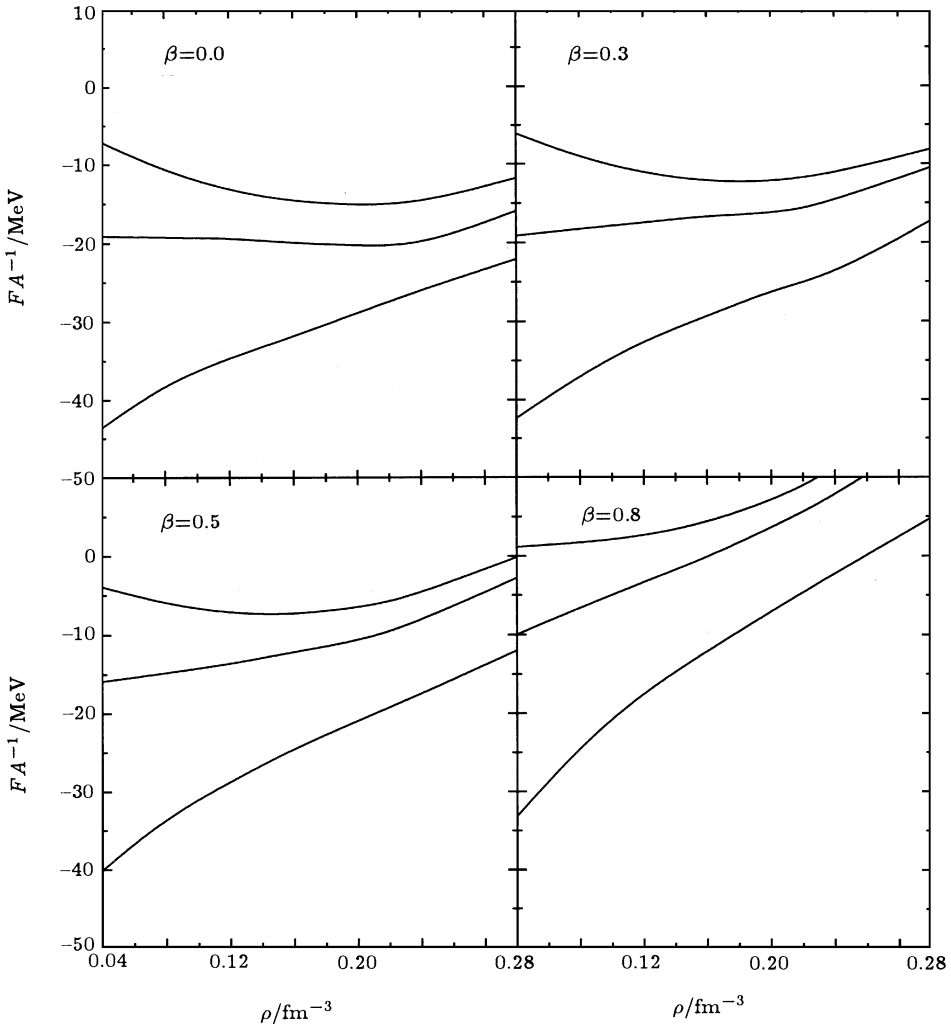


Fig.1. Free energy is expressed as a function of density for four different values of the asymmetry parameter. Three isothermal curves at a fixed asymmetry parameter refer to temperatures T=0,10, 20 MeV from top to bottom.

To be more specific, Fig.2 shows the pressure as a function of density at a fixed temperature but with different asymmetry parameter, i.e. with different proton fractions. For any fixed density at the fixed temperature, it can be observed from the figure that the pressure is not constant, or rather, it increases with asymmetry parameter. This clearly indicates that for asymmetric nuclear matter the pressure cannot remain constant but increases monotonically during liquid–gas phase transition. For a high β, and in particular, for the pure neutron matter (β=1), the pressure increases monotonically, so the matter is stable at all densities. In contrast, for $\beta < 0.0$, 0.2, 0.4, 0.6, the compressibility becomes negative, indicating a mechanical instability.

In order to clearly illustrate the effect brought by three-body interaction on the phase transition of the asymmetric nuclear matter, in Fig.3 we plot the domain of the mechanical instability in an asymmetry-density plane for the fixed temperature with and without three-body forces. From this figure we can see that with TBF taken into account the bound of the region becomes a little smaller, which further confirms the result shown in Fig.1. But compared with Fig.1, a slight difference can be found between the two values of the critical asymmetry parameters for phase transition. Here this value is about $\beta \approx 0.7$. Hence this shows that the mechanical instability domain turns

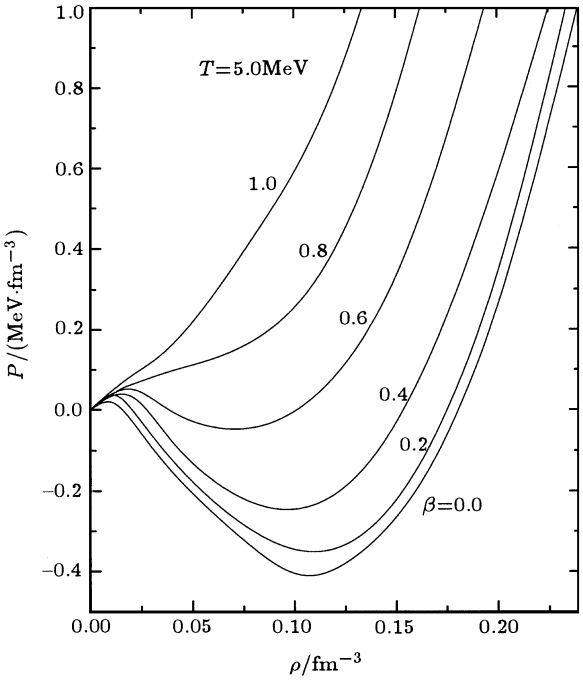


Fig.2. Pressure as a function of density for different asymmetry parameters at a fixed temperature T=5.0 MeV.

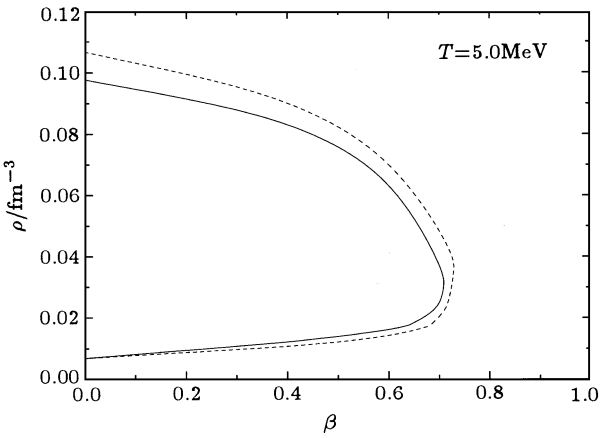


Fig.3. Regions of mechanical instability at a fixed temperature T=5.0 MeV. The solid line is for the presence of three-body interaction and dashed line for the absence of this interaction.

smaller with the increase of the temperature in the system. However, no matter whether three-body interaction is added or not, when the asymmetry increases, the two unstable domains may gradually shrink respectively.

In summary, in this paper we have introduced the contribution of microscopic three-body forces into the finite temperature Brueckner–Hartree–Fock theory and studied the thermal properties of asymmetric nuclear matter. We have found that with three-body forces introduced into nucleon–nucleon interaction, nuclear matter behaves like a liquid–gas phase transition. For asymmetric nuclear matter, we have investigated the variation of the critical temperature with the asymmetry parameter, and observed that $T_{\rm C}$ decreases with the increase of β. At a fixed temperature and density the pressure of the system increases with the increase of asymmetry parameter, which indicates that in the course of the phase transition pressure cannot remain constant for asymmetric nuclear matter. The value of the critical asymmetry is about 0.75 below which asymmetric nuclear matter can produce phase transition at a certain temperature. But for pure neutron matter, even at zero temperature the system remains only in gas phase. Owing to the additional repulsive potential provided by three-body forces, this value decreases from 0.8 to 0.75. In addition, the domain of mechanical instability for hot nuclear matter also gradually shrinks with increasing asymmetry and temperature.

The microscopic three-body interaction serves as a stronger repulsion at a high density,[19] hence we can better apply the extended FTBHF model including TBF to astrophysics. We will investigate the structure and properties of astrophysical objects, particularly of the proton–neutron star that is born following the gravitational collapse of the core of a massive star during a supernova explosion (type-II) and evolves to a cold and deleptonized neutron star, basically through neutrino emission. On the basis of the variation of pressure with energy density, the Tolman–Oppenheimer–Volkoff (TOV) equations[23,24] are solved so that we can discuss mass and radius of a star at a fixed temperature. This will be the subject of our next work.

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