Vol 13 No 11, November 2004
1009-1963/2004/13(11)/1843-05

Chinese Physics

© 2004 Chin. Phys. Soc.
and IOP Publishing Ltd

Description of nucleus ^{44}Ti in new symmetry limit in IBM-3*

Zhang Jin-Fu(张进富)[a)b)†] and Bai Hong-Bo(白洪波)[a)b)]

[a)] *Department of Physics, Chifeng College, Chifeng* 024001, *China*

[b)] *Department of Physics, Tsinghua University, Beijing* 100084, *China*

(Received 18 January 2004; revised manuscript received 3 June 2004)

Analysis of the spectra and $E2$ transition rates shows that the empirical scheme of ^{44}Ti is in good agreement with the predictions of the $[U_{\rm d}(5) \otimes U_{T_{\rm d}}(3)] \oplus U_{T_{\rm s}}(3)$ symmetry limit.

Keywords: spectra, transition rate, symmetry limit, IBM-3
PACC: 2160F, 2320

1. Introduction

The interacting boson model (IBM) of nuclei introduced by Arima and Iachello,[1] is phenomenologically successful in describing the spectra of medium heavy nuclei[2−4] and heavy nuclei. This model treats pairs of valence nucleons (particles/holes) as bosons with angular momentum l=0 (s bosons) or l=2 (d bosons). In the original version of the interacting boson model, IBM-1, no distinction is made between neutron bosons and proton bosons. IBM-1 has been extended to IBM-2 by distinguishing neutron bosons from proton bosons,[1] and proved to be a first approximation for IBM-2.

IBM-2 is effective for nuclear states of valence protons and valence neutrons filling the different single particle orbits. However, in light nuclei, the valence protons and valence neutrons are in the same single particle orbit; therefore the isospin effect has to be included. To this end, IBM-3[5] and IBM-4[6] were proposed. IBM-4 describes the nuclei in the sd shell where the nucleon couple is in the LS scheme.[6,7] In the pf shells, the nucleon couple is in the jj scheme, and IBM-3 has to be used here. As has been shown,[5−7] IBM-3 is introduced for light nuclei. But there are some heavy nuclei with protons and neutrons filling the same single particle orbit, for example, the proton rich nuclei with $N \sim Z \sim 40$, and the lighter isotopes of tellurium, xenon, barium. As far as the nuclei with $N \sim Z \sim 40$ are concerned,[8] IBM-3 may be used. Furthermore, in radioactive ion beams, this method may be used to describe the case of many nuclei.

Although new radioactive beam facilities[9] and relative experiments[10,11] mainly search for the new regions of nuclei far from the stability line, prior to data on such exotic nuclei becoming available, it is necessary to have a model applicable to such nuclei, for example the proton-rich nuclei with $N \sim Z$, in order to obtain some guideline for new experiments. Some information on such nuclei with $A \geq 40$ region suggests that their collective behaviours are different from those of nuclei with $N \gg Z$. Such a new type of collective motion for $N \sim Z$ nuclei was studied by Ginocchio and Leviatan.[12,13]

The properties of ^{44}Ti nucleus are most challenging. First, the properties of radionuclide ^{44}Ti are of great interest in astrophysics. In most radionuclides, the half-life[14] of ^{44}Ti is about the same as the mean time between supernovae events in our Galaxy. From the decay chain ^{44}Ti–^{44}Sc–^{44}Ca, an abundance of ^{44}Ca in solar matter is the decay product of ^{44}Ti. This is a very important clue in understanding the origin of

*Project supported in part by National Natural Science Foundation of China (Grant Nos 10047001 and 10265001), the State Education Commission Foundation for Excellent Young Teachers of China Education Ministry, the Fok Ying Tung Education Foundation, the State Key Development Programme for Basic Research of China (Grant No G200077400), and the Key Scientific Research Fund of Inner Mongolian Educational Bureau (Grant No ZD01038).

†E-mail: zjf@cfxy.cn

the solar system. On the other hand, apart from nucleus ^{48}Cr, the nucleus ^{44}Ti is the heaviest even–even nucleus with T=0 for which the electromagnetic properties are measured. It is thus possible to make some theoretical calculations by using models such as the shell model,[15] the α-cluster model,[16] the algebraic model,[17] the pair plus quadrupole force model[18] and the cranked Skyrme–Hartree–Fock method,[19] etc.

Recently, Kota[20,21] proposed a class of new symmetry limit $[U_{\rm d}(5)\otimes U_{T_{\rm d}}(3)]\oplus U_{T_{\rm s}}(3)$ of the IBM-3 model with good s and d boson isospins, which is applicable to proton-rich nuclei with $N\sim Z$. In order to treat it, complicated numerical calculations have to be made. Therefore, it is clearly desirable to find an empirical example for the nucleus with the new symmetry limit $[U_{\rm d}(5)\otimes U_{T_{\rm d}}(3)]\oplus U_{T_{\rm s}}(3)$. In this paper we analyse the data for nucleus ^{44}Ti using the new symmetry limit and find that nucleus ^{44}Ti establishes some new symmetry limit characteristics.

In this paper a new symmetry limit $[U_{\rm d}(5)\otimes U_{T_{\rm d}}(3)]\oplus U_{T_{\rm s}}(3)$, described in more detail in Refs.[20] and [21], is reviewed and the data for nucleus ^{44}Ti are analysed. The extent to which the nucleus ^{44}Ti can be described by the new symmetry limit is also investigated. In addition, we attempt to apply the new symmetry limit to other nuclei with $N\sim Z$.

2. $[U_{\rm d}(5)\otimes U_{T_{\rm d}}(3)]\oplus U_{T_{\rm s}}(3)$ symmetry limit

As in IBM-1, the IBM-3 model includes s and d bosons with no intrinsic spin but with isospin permitted to be carried. To take into account the isospin conservation in the framework of boson models, besides the proton–proton and neutron–neutron pairs (π, ν bosons), a proton–neutron pair (δ bosons) is also introduced. The building blocks of IBM-3 are s^+_ν, s^+_δ, s^+_π, d^+_ν, d^+_δ and d^+_π. The three s bosons and three d bosons form an isospin T=1 triplet ($T_{\rm z}$=1, 0, –1 correspond to pp, pn and nn pairs, respectively). With the three $T_{\rm z}$ values, IBM-3 has $U(18)$ group as its dynamical symmetry group. Dynamical symmetries are important, because (1) they provide a useful mathematical basis for numerical calculation and (2) they provide us with a typical spectrum in analytical form with clear physical interpretation. We can look for experimental candidates for them, which are useful in guiding numerical studies.

In general, the dynamical symmetric group chain must contain $O_L(3)$ and $SU_T(2)$, because the angular momentum and the isospin are good quantum numbers. There are quite a few group chains satisfying these requirements.[22,23] One class of physically significant group chain for the $[U_{\rm d}(5)\otimes U_{T_{\rm d}}(3)]\oplus U_{T_{\rm s}}(3)$ limit[20,21] is

$$\begin{array}{c} U_{\rm sd}(18)\\ \{N\}\end{array}\supset\left\{\begin{array}{ccccc} U_{\rm d}(15) & \supset & U_{\rm d}(5) & \supset & O_{\rm d}(5) \supset O_{\rm d}(3)\\ \{n_{\rm d}\} & & \{f_1f_2f_3\} & & [v_1v_2]\quad L\end{array}\right]\otimes\left[\begin{array}{ccc} U_{T_{\rm d}}(3) & \supset & O_{T_{\rm d}}(3)\\ \{f_1f_2f_3\} & & T_{\rm d}\end{array}\right]\Bigg\}$$
$$\oplus\left\{\begin{array}{ccc} U_{T_{\rm s}}(3) & \supset & O_{T_{\rm s}}(3)\\ \{n_{\rm s}\} & & T_{\rm s}\end{array}\right\}\supset\begin{array}{ccc} O_{\rm d}(3) & \otimes & O_T(3)\\ L & & T\end{array},$$

where the labels below the groups are the corresponding irreducible representations (irreps). The states corresponding to $n_{\rm d}$ of $U_{\rm d}(15)$ are totally symmetric and therefore the $U_{\rm d}(5)$ and the $U_{T_{\rm d}}(3)$ have the same irreps. The reduction from $U_{\rm d}(15)$ to $U_{\rm d}(5)$ (or $U_{T_{\rm d}}(3)$) is

$$[n_{\rm d}]=\sum_{\substack{f_1\geq f_2\geq f_3\geq 0\\ f_1+f_2+f_3=n_{\rm d}}}\otimes[f_1f_2f_3]. \qquad (1)$$

The reduction of $U_{\rm d}(5)\to O_{\rm d}(5)\to O_{\rm d}(3)$ follows from the results of the $SU_{\rm sdg}(5)$ limit of sdgIBM.[24] The $U_{T_{\rm d}}(3)\to O_{T_{\rm d}}(3)$ follows directly from Elliott's rule,[25]

$$[f_1f_2f_3]\equiv(\lambda=f_1-f_2,\ \mu=f_2-f_3). \qquad (2)$$

For the $U_{T_{\rm d}}(3)$ irreps (λ,μ),

$$K=\min(\lambda,\mu),\ \min(\lambda,\mu)-2,\cdots\cdots,1\text{ or }0. \qquad (3)$$

For $K\neq 0$,

$$T_{\rm d}=K,\ K+1,\ K+2,\cdots\cdots,K+\max(\lambda,\mu). \qquad (4)$$

For K=0,

$$T_{\rm d}=\max(\lambda,\mu),\max(\lambda,\mu)-2,\cdots\cdots,1\text{ or }0. \qquad (5)$$

Similarly the reduction $[n_{\rm s}]\to[T_{\rm s}]$ has been carried out. Because $[n_{\rm s}]\equiv(\lambda=n_{\rm s},\ \mu=0)$, $T_{\rm s}=n_{\rm s}$, $n_{\rm s}$ –2, $n_{\rm s}$ –4, $\cdots\cdots$, 1 or 0. We list the results of the reduction for T=0 in Table 1.

Table 1. State labels for $[U_{\rm d}(5)\otimes U_{T_{\rm d}}(3)]\oplus U_{T_{\rm s}}(3)$limit with T=0 (for our purpose, the results with $n_{\rm d}\leq 5$ are given).

$U_{\rm d}(15)$	$U_{\rm d}(5)$	$O_{\rm d}(5)$	$O_{\rm d}(3)$	$O_{T_{\rm d}}(3)$	$O_{T_{\rm s}}(3)$
$n_{\rm d}$	$[f_1f_2f_3]$	$[v_1v_2]$	L	$T_{\rm d}$	$T_{\rm s}$
0	[000]	[00]	0	0	0
1	[100]	[10]	2	1	1
2	[200]	[20]	2,4	0,2	0,2
		[00]	0	0,2	0,2
3	[300]	[30]	0,3,4,6	1,3	1,3
		[10]	2	1,3	1,3
	[210]	[21]	1,2,3,4,5	1	1
		[10]	2	1	1
4	[400]	[40]	3,4,5,6,8	0,2,4	0,2,4
		[20]	2,4	0,2,4	0,2,4
		[00]	0	0,2,4	0,2,4
	[310]	[31]	1,2,3,4,5,6,7	2	2
		[11]	1,3	2	2
		[20]	2,4	2	2
	[220]	[22]	0,2,3,4,6	0,2	0,2
		[20]	2,4	0,2	0,2
		[00]	0	0,2	0,2
5	[500]	[50]	2,4,5,6,7,8,10	1	1
		[41]	1,2,3^2,4^2,5^2,6^2,7^2,8,9	1	1

Wavefunctions in the symmetry limit are characterized by $|N;n_{\rm d}[f_1f_2f_3](v_1v_2)L;(T_{\rm d}T_{\rm s})T\rangle$. With the generators of subgroup, Casimir operator and their eigenvalues, the energy formula was obtained:[20,21]

$$\begin{aligned}E =&E_0+\varepsilon_{\rm d}n_{\rm d}+\alpha n_{\rm d}(n_{\rm d}+14)\\&+\beta[f_1(f_1+4)+f_2(f_2+2)+f_3^2]\\&+\gamma[v_1(v_1+3)+v_2(v_2+1)]\\&+\gamma_1L(L+1)+\gamma_2T_{\rm d}(T_{\rm d}+1)\\&+\gamma_3T_{\rm s}(T_{\rm s}+1)+\gamma_4T(T+1),\end{aligned}\qquad(6)$$

where E_0, $\varepsilon_{\rm d}$, α, β, γ, γ_1, γ_2, γ_3 and γ_4 are free parameters.

Another important aspect of a dynamical symmetry relates to its transition properties. The general one-boson E2 operator in IBM-3 consists of isovector and isoscalar parts. For the even–even nuclei with T=0, only the isoscalar part contributes to B(E2); hence, the E2 operation is $T^{\rm E2}=e_{\rm eff}({\rm d}^+\tilde{\rm s}+{\rm s}^+\tilde{\rm d})^{2,0}_{\mu,0}$, where $e_{\rm eff}$ is effective charge. The E2 transition properties have also been obtained in Refs.[20] and [21], we will use these results in our investigation.

3. Results and discussions

The comparisons between the calculated and the measured energy levels[26,27] for ^{44}Ti are shown in Fig.1. Figure 1 (left) shows some detailed predictions of the $[U_{\rm d}(5)\otimes U_{T_{\rm d}}(3)]\oplus U_{T_{\rm s}}(3)$ limit with appropriate energy scales for comparison with the measured energy levels of ^{44}Ti. In order to modify the energy levels of the collective states, the model space is extended from the N=16 shell[28–30] to the N=28 shell. The parameters in energy formula are determined by a best fit to experimental levels. It is found that $\varepsilon_{\rm d}$=1.1(MeV), γ=β=–0.006(MeV), γ_1=0.025(MeV), $E_0=\alpha=\gamma_2=\gamma_3=\gamma_4$=0. In general, they are in good agreement with the measurements, especially for the ground-state band levels and the backbanding at the 10^+_1 level. The quantum number $n_{\rm d}$ of $U(15)$ is similar to a phonon number. Thus the nuclei with T=0 have more eigenstates than the quadrupole vibrator because of the presence of additional monopole and quadrupole isospin quantum numbers. From Fig.1, it is found that 4^+_1, 2^+_1 and 0^+_2 states with $T_{\rm s}$=0 belong to $n_{\rm d}$=2, and are reproduced satisfactorily by the symmetry limit. The states with $n_{\rm d}$=3 belong to $T_{\rm s}$=1. The calculation predicts an excited 5^+_1 state at 3.9Mev. It is noticed that the calculated level for the second excited 0^+ state is much lower than the measured. We thus tempt to assume this 0^+ state belongs to $n_{\rm d}$=4. This remains to be further studied. The calculated levels with $n_{\rm d}$=4 are in agreement with the data overall. But the agreement for 6^+_2 and 8^+_2 states is poor. The origin of the discrepancy can be

understood by considering the strong mixture of the 6_1^+ state and the 8_1^+ state, respectively. Good agreement with the data is also obtained for those states with n_d=5. The calculation also predicts an excited 7_1^+ state at 6.4Mev.

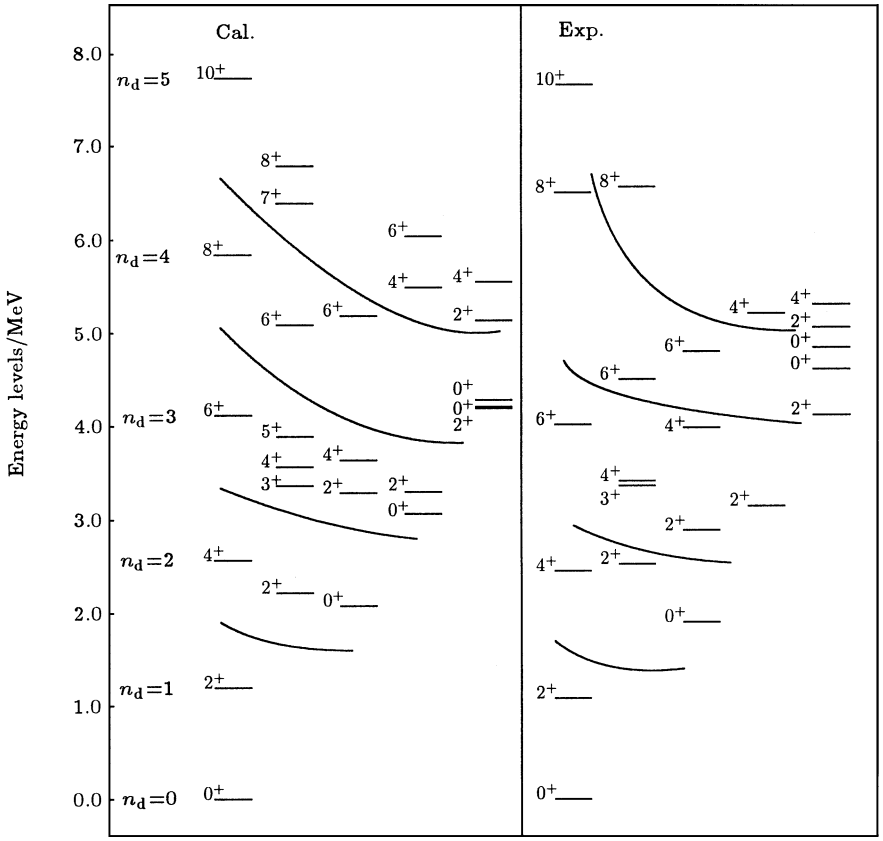


Fig.1. Comparison of energies calculated in the symmetry limit in IBM-3 with experimental excitation energies of nucleus ^{44}Ti. The parameters are given in the text.

The low-lying spectrum of ^{44}Ti indicates a phonon structure overall. The ratio of the energy of two-phonon state 4_1^+ to that of one-phonon state 2_1^+ is 2.27, in between the spherical vibrator or the $U(5)$ limit (2.0) and the deformed quadrupole vibrator or the $O(6)$ limit(2.5).[1] Utilizing the relation[31] $\beta_2 = 4\pi\sqrt{B(\mathrm{E2}: 0_1^+ \to 2_1^+)/(3eZR_0^2)}$ with $R_0 = 1.2A^{1/3}$, one estimated the deformation parameter β_2 of nucleus ^{44}Ti, which is equal to 0.26. The large deformation and a low value of the energy ratio $R = E(4_1^+)/E(2_1^+)$ are features specific to the nucleus ^{44}Ti also. As has been known,[32] many rare earth nuclei with $\beta_2 \approx 0.3$ indicate the rotational characteristics with $R \approx 3.3$.

In Table 2, we give the B(E2) values for nucleus ^{44}Ti and compare with the data.[18,26,27] It is found that the calculations are in good agreement with the data. The observed $B(\mathrm{E2}: J \to J-2)$ values in yrast-state band are reproduced satisfactorily by this symmetry limit. In addition, the differences between the calculated B(E2) values and the corresponding experimental data are found in Table 2. In the $[U_d(5) \otimes U_{T_d}(3)] \oplus U_{T_s}(3)$ symmetry limit, the B(E2) values would vanish by the selection rule Δn_d= 1. As shown previously, the nuclei with T=0 have more eigenstates than the quadrupole vibrator except for n_d=1: for example, the two-phonon 0^+, 2^+, 4^+ triplet occurs twice, one with T_s=0 and another with T_s=2, which will probably mix. The breaking of the symmetry limit will, however, destroy this selection rule. A non-negligible deviation in the $2_2^+ \to 0_2^+$ decay is observed. The origin of the discrepancy $2_2^+ \to 0_2^+$ can be understood by considering the strong interference effects between the vibrational and the intruder (rotational) contributions to the transition rate, or 0_2^+ as core–excited state, which is a general phenomenon of shape coexistence. The vibration–rotation interaction has to be included.

Table 2. Comparison of the experimental B(E2) values for ^{44}Ti with the calculated in the $[U_{\rm d}(5) \otimes U_{T_{\rm d}}(3)] \oplus U_{T_{\rm s}}(3)$ symmetry limit. The effective charge $e_{\rm eff}$=0.1(eb) is employed.

$L_{\rm i}^{\pi}$	$L_{\rm f}^{\pi}$	B(E2) (e^2fm^4)	
		Exp.	Cal.
2_1^+	0_1^+	120 ± 36	200
4_1^+	2_1^+	277 ± 55	156
2_2^+	2_1^+	66 ± 12	156
0_2^+	2_1^+		156
2_2^+	0_1^+	1.4 ± 0.37	0
2_2^+	0_2^+	222 ± 55	0
6_1^+	4_1^+	157 ± 22	222
4_2^+	2_2^+	203 ± 92	116
4_2^+	2_1^+	25 ± 5.5	0
8_1^+	6_1^+	>18	220
10_1^+	8_1^+	138 ± 28	156
4_3^+	2_3^+	258 ± 129	0
4_3^+	2_1^+	4.2 ± 1.9	0
2_4^+	0_1^+	1.3 ± 0.65	0

4. Conclusion

We have given a detailed study of the energy levels and E2 transition rates for the nucleus ^{44}Ti in the $[U_{\rm d}(5) \otimes U_{T_{\rm d}}(3)] \oplus U_{T_{\rm s}}(3)$ symmetry limit. A good agreement with the data is obtained for both the spectra and the E2 transition rates. It is shown that ^{44}Ti is close to this symmetry limit. Meanwhile, we find that the heavier deformed nuclei with $A \sim 40$ exhibit much stronger quadrupole collectivity. We should note that discrepancy between the calculated results and the experimental data is observed. The dynamical symmetry limit is usually not perfect in its nature. The coefficients depend strongly on the nucleon number and the isospin.[33] It would be interesting to carry out detailed numerical calculations in the $A \sim 40$ nuclei.

Acknowledgments

The authors are greatly indebted to Professor G. L. Long for his continuing interest in this work and also for his many suggestions.

References

[1] Iachello F and Arima A 1987 *The Interacting Boson Model* (Cambridge: Cambridge University Press)
[2] Liu F Y, Zhang J F and Pak J Y 2001*Chin. Phys.* **10** 194
[3] Shi Z Y, Liu Y and Sang J P 2000 *Chin. Phys.* **9** 9
[4] Shi Z Y, Zao X Z and Tong H 2003 *Chin. Phys.* **12** 732
[5] Elliott J P and White A P 1980 *Phys. Lett.* B **97** 169
[6] Elliott J P and Evans J A 1981 *Phys. Lett.* B **101** 216
[7] Han Q Z, Sun H Z and Li G H 1987*Phys. Rev.* C **35** 786
[8] Sugita M 1997 *Phys. Lett.* B **394** 235
[9] Tanihata I 1996 *J. Phys.* G **22** 157
[10] Ma Y G *et al* 2000 *Acta Phys. Sin.* **49** 654 (in Chinese)
[11] Li W F, Zhang F S and Chen L W 2001 *Acta Phys. Sin.* **50** 1040 (in Chinese)
[12] Ginocchio Joseph N and Leviatan Amiram 1994 *Phys. Rev. Lett.* **73** 1903
[13] Ginocchio Joseph N 1996 *Phys. Rev. Lett.* **77** 28
[14] Hashimoto T, Nakai K and Wakasaya Y *et al* 2001 *Nucl. Phys.* A **686** 591
[15] O'Leary C D, Bentley M A, Brown B A *et al* 2000 *Phys. Rev.* C **61** 064314
[16] Hasegawa M, Tazaki S and Okamoto R 1995 *Nucl. Phys.* A **592** 45
[17] Garcia-Ramos J E and Van Isacker P 1999 *Ann. Phys.* **274** 45
[18] Hasegawa M, Kaneko K and Tazaki S 2000 *Nucl. Phys.* A **674** 411
[19] Inakura T, Mizutori S, Yamagami M *et al* 2002 *Nucl. Phys.* A **710** 261
[20] Kota V K B 1997 *Phys. Lett.* B **399** 185
[21] Kota V K B 1998 *Ann. Phys.* **265** 101
[22] Long G L 1994 *Chin. J. Phys.* **16** 331
[23] Long G L, Gan C Y and Shen T Y 1997 *Commun. Theor. Phys.* **27** 317
[24] Kota V K B, De Meyer H, Vander Jeugt J *et al* 1987 *J. Math. Phys.* **28** 1644
[25] Elliott J P 1958 *Proc. R. Soc.* (London) A **245** 128
[26] Cameron J A and Singh B 1999 *Nucl. Data Sheets* **88** 299
[27] Chu S Y, Nordberg H, Firestone R B *et al* 1997 *Isotopes Explorer* 2.00 Lawrence Berkeley National Laboratory, Physics Department, Lund University.
[28] Ozawa A *et al* 2000 *Phys. Rev. Lett.* **84** 5493
[29] Otsuka T *et al* 2001 *Phys. Rev. Lett.* **87** 082502
[30] Kanungo Rituparna, Tanihata I and Ozawa A 2002 *Phys. Lett.* B **528** 58
[31] Raman S *et al* 1987 *At. Data Nucl. Data Tables* **36** 1
[32] Bohr A and Mottelson B R 1975 *Nuclear Structure* vol 2 (New York: Benjamin)
[33] Evans J A, Long G L and Elliott J P 1993 *Nucl. Phys.* A **561** 201