Vol 13 No 11, November 2004
1009-1963/2004/13(11)/1835-08

Chinese Physics

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Triaxial interaction effect in the structure of $^{148-158}$Sm*

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(Received 25 February 2004; revised manuscript received 29 June 2004)

The structures of $^{148-158}$Sm are studied in the frame of the interacting boson model. Attention is given to the triaxial interaction, and it is found that the triaxial interaction plays an important role in the detailed structure of this isotopic chain. The potential surfaces of these nuclei are also studied and the shapes of them are analysed. And it is found that though the triaxial interaction plays an important role in these nuclei, there are no stable triaxial shapes in the low-lying structures of these nuclei.

Keywords: Mixed Symmetry States, $^{148-158}$Sm isotopes, interacting boson model
PACC: 2110F, 2110R, 2160E, 2760

1. Introduction

The interacting boson model (IBM) [1–3] has been successfully used in the description of nuclear collective excitation of medium and heavy nuclei. Different from the geometrical picture,[4] it has an inherent group structure. In the first version (IBM-1), only pairs with angular momentum L=0 (called s boson) and L=2 (called d boson) are included. Despite its simplicity, it can describe the properties of many nuclei. With the work of Ginocchio and Kirson [5] and, simultaneously, Dieperink *et al*[6] the relation between the IBM and the geometrical models of nuclear collective motion has been established; and in the large N-limit, where N is the number of bosons, the result of IBM approaches the geometrical model result. Over the past few years, the structure of medium and heavy nuclei have received intensive attention.[7–12] The isospin effects were studied in Refs.[13,14]. The structures of odd–odd nuclei were studied in Refs.[15,16]. Nuclear deformation is important in determining the structure of nuclei, and it can also increase the stability in superheavy nucleus.[17,18] Octuple deformation has a significant influence on the structure of heavy nucleus.[19] Triaxial deformation plays an important role in the staggering of the γ-band.

To calculate the classical limit of the Hamiltonian of the IBM,[6] one obtains a potential energy surface in the space spanned by β and γ bands. It became clear that even the most general Hamiltonian of the model could not form a stable, triaxial shape for the nucleus.[6] Heyde *et al*[20] studied this discrepancy and gave the settlement. They introduced cubic terms into the Hamiltonian of the IBM, so that one could indeed obtain stable, triaxially shaped nucleus near the $O(6)$ limit. Their calculated data fit to the experimental data better than the IBM calculations without these terms. Recently, some other nuclei are studied by using the IBM Hamiltonian with cubic terms and satisfactory results are obtained.[21,22] In the work of Scholten *et al*,[23] the Sm isotopes were well described by IBM in the transition from $U(5)$ to $SU(3)$. The Sm isotopes from A=148 to A=158 have a neutron number varying from 86 to 96 respectively, covering the range from the typical nucleus of an enharmonic vibrator (namely $U(5)$ symmetry in the IBM) at neutron number 86 to the typical nucleus of a rigid axial rotor ($SU(3)$ symmetry in the IBM) at neutron number 92. In this paper, we incorporate the cubic terms in the Hamiltonian of the IBM, compare the low-lying

*Project supported by the National Natural Science Foundation of China (Grant Nos 10047001, 10325521 and 1047113), the State Key Development Programme for Basic Research of China (Grant No G200077400), the SRFDP Programme of Education Ministry of China.
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spectra and $E2$ transition rates of Sm isotopes between the original IBM without cubic terms and the IBM with cubic terms. In addition, we also calculate the classical limit of the Hamiltonians and obtain the potential energy surfaces. We will show the effect of cubic terms on the nucleus in the transition from $U(5)$ symmetry to $SU(3)$ symmetry. The results obtained are in agreement with those of Ref.[20].

2. Model

The IBM model we use here is IBM-1. The Hamiltonian is of the form

$$\begin{aligned} H_{\rm sd} =&\varepsilon_{\rm d} n_{\rm d} + a_0 P^+ \cdot P + a_1 L \cdot L \\ &+ a_2 Q \cdot Q + a_3 T_3 \cdot T_3 + a_4 T_4 \cdot T_4, \end{aligned} \tag{1}$$

where $n_{\rm d} = (d^+ \cdot \tilde{d})$ is the d-boson number operator, and

$$P = \frac{1}{2}(\tilde{d} \cdot \tilde{d} - s \cdot s) \tag{2}$$

is the pairing operator,

$$\hat{L} = \sqrt{10}[d^+ \tilde{d}]^{(1)}, \tag{3}$$

$$Q = [d^+ \tilde{s} + s^+ \tilde{d}]^{(2)} + \chi [d^+ \tilde{d}]^{(2)}, \tag{4}$$

$$T_l = [d^+ \tilde{d}]^{(l)} \qquad (l = 3, 4), \tag{5}$$

are the angular momentum, the quadruple moment, the octuple and hexadecuple operators respectively.

The electric quadruple transition operator in the IBM has the form

$$T(E2) = \alpha [d^+ \tilde{s} + s^+ \tilde{d}]^{(2)} + \beta [d^+ \tilde{d}]^{(2)}. \tag{6}$$

In this context, because the Sm isotopes transit from $U(5)$ to $SU(3)$ symmetry, we let a_0=0 (namely without $P^+ \cdot P$ term), default $\beta = \chi \cdot \alpha$ in Eq.(6), with $\chi = \dfrac{\sqrt{7}}{2}$ as in the consistent $Q \cdot Q$ formalism.[24] This convention is not an essential requirement of the model,[25,26] but in most cases it gives a good approximation.

When we introduce the cubic terms, the Hamiltonian changes to

$$H = H_{\rm sd} + H_{\rm cub}, \tag{7}$$

in which $H_{\rm cub}$ contains the cubic terms

$$H_{\rm cub} = \sum_L \theta_L [d^+ d^+ d^+]^{(L)} \cdot [\tilde{d}\tilde{d}\tilde{d}]^{(L)}. \tag{8}$$

The classic limit of the IBM Hamiltonian can be obtained through the IBM coherent intrinsic state (boson condensate) introduced in Refs.[5] and [6],

$$\begin{aligned} |N;\beta,\gamma\rangle =&[N!(1+\beta)^2]^{-1/2}\{s^+ + \beta[\cos\gamma d_0^+ \\ &+ 1/\sqrt{2}\sin\gamma(d_{+2}^+ + d_{-2}^+)]\}^N |0\rangle. \end{aligned} \tag{9}$$

Then we can calculate the potential energy function from Eq.(9)[6]

$$E(N;\beta,\gamma) = \langle N,\beta,\gamma|\ H|\ N,\beta,\gamma\rangle. \tag{10}$$

Starting from the Hamiltonian of Eqs.(1) and (7), the corresponding potential energy surfaces have been given by many authors, and they are given in Eqs.(11) and (12) respectively in Section 4. It was concluded that only the L=3 term can induce triaxial shapes since it forms a term involving $\cos^2 3\gamma$,[20,27] so we only add the L=3 term to the IBM Hamiltonian, and choose θ_3=0.03 for all the Sm isotopes.

Table 1. Parameters for the Sm isotopes.

Nucleus	ε/MeV	a_1/MeV	a_2/MeV	a_3/MeV	a_4/MeV	α/MeV
^{148}Sm	0.580	0.001	–0.004	0.001	0.003	0.130
^{150}Sm	0.470	0.001	–0.0075	0.001	0.006	0.240
^{152}Sm	0.390	0.003	–0.0159	0.010	0.005	0.228
^{154}Sm	0.300	0.0044	–0.019	0.010	0.010	0.254
^{156}Sm	0.170	0.0044	–0.020	0.010	0.030	
^{156}Sm	0.140	0.0044	–0.021	0.010	0.030	

We determine the parameters, which are given in Table 1, by fitting to the experimental results. From these parameters, we can see that the values of $\varepsilon_{\rm d}$ decreases as the absolute values of a_2 increase when the neutron number increases.

3. Energy spectra and $E2$ transition rate

In Figs.1–6, we give the comparisons between the calculations using a Hamiltonian without the cubic term (denoted by IBM) and a Hamiltonian with a L=3 cubic term(denoted by CUB) for $^{148-168}$Sm, to-

gether with the experimental (denoted by EXP) energy levels. From Figs.1–6, we can see that the IBM data fit the experimental data very well, especially in the low-energy levels. The level spectra show the transition from $U(5)$ to $SU(3)$ symmetry. The theoretical and experimental energy levels of ^{148}Sm are shown in Fig.1. From Fig.1, we see a good agreement between the experimental and theoretical results in the ground-state band (g.s.band). The levels space equally in g.s.band, which indicates the vibrational nature of ^{148}Sm. Figure 1 also includes the β'-band and γ'-band, where 0_3^+(β'-band head) is calculated to be at 1.931MeV, corresponding to 1.922MeV in experiment. The 4_4^+ member of the same band has a value 2.657MeV(2.713MeV) in the IBM calculation (in experiment). The ratio $E_{4_1^+}/E_{2_1^+}$ equals 2.03 according to the IBM calculation, compared to the experimental data 2.14. This is a typical character of the vibrational nature. Figure 2 shows the comparison between the experimental and the calculated energy levels of ^{150}Sm nucleus. In general, there is a good agreement between experimental and the IBM results for low-lying states. The experimental results of γ'-band are also plotted, and the IBM levels fit them very well. For instance, 2_5^+(γ'-band head) is 1.820MeV which is very close to 1.794MeV obtained in the experiment; 3_2^+ state is at 2.212MeV and 2.044MeV which are the calculated and experimental values respectively, and the energy ratios $E_{4_1^+}/E_{2_1^+}$ obtained theoretically and experimentally equal 2.08 and 2.31 respectively. The agreements between the experimental and theoretical results for β-band and γ-band in Figs.1 and 2 are not very good, however they may be improved by using the IBM-2, and the properties of ^{150}Sm are studied in detail in Ref.[28]. In Fig.3, we see that the spectrum of ^{152}Sm becomes rotational. The level spacing changes approximately in proportion to $L(L+1)$, and the γ-band levels space regularly. The fit of the IBM calculation to the data is very good in all the five bands. The ratios $E_{4_1^+}/E_{2_1^+}$ obtained are 3.14 and 3 in the IBM calculation and the experimental data respectively, indicating that the spectrum is rotational. In Figs.4–6, the spectra of $Sm^{154-158}$ are shown, and the theoretical calculations all agree well with the data. The spectra of these nuclei have the same characters as those of ^{152}Sm, indicating that they also have the rotational properties. These are supported by the energy ratios $E_{4_1^+}/E_{2_1^+}$ in these nuclei, which are (3.32, 3.26), (3.35, 3.29), (3.32, 3.29) for $Sm^{154-158}$ respectively, where the first number in parentheses is the IBM calculation and the second number is the experimental value.

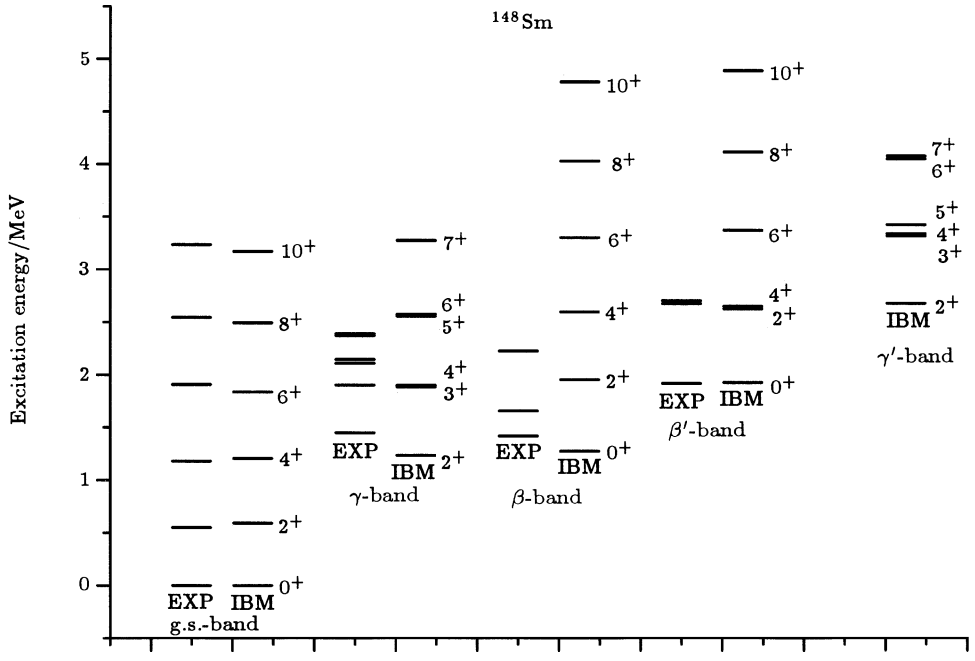


Fig.1. ^{148}Sm spectrum.

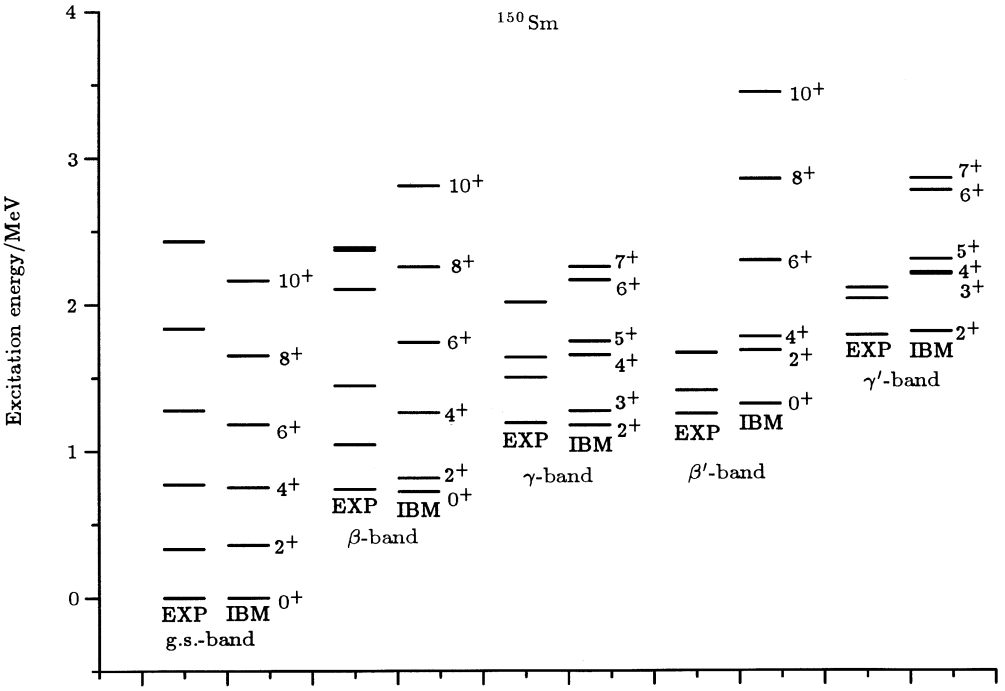


Fig.2. ^{150}Sm spectrum.

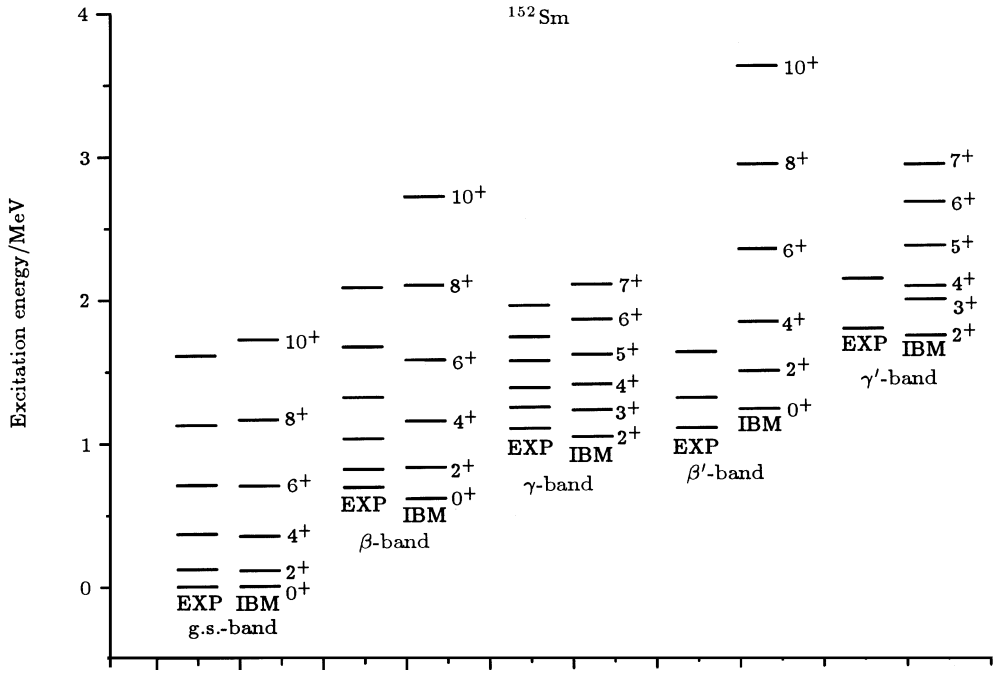


Fig.3. ^{152}Sm spectrum.

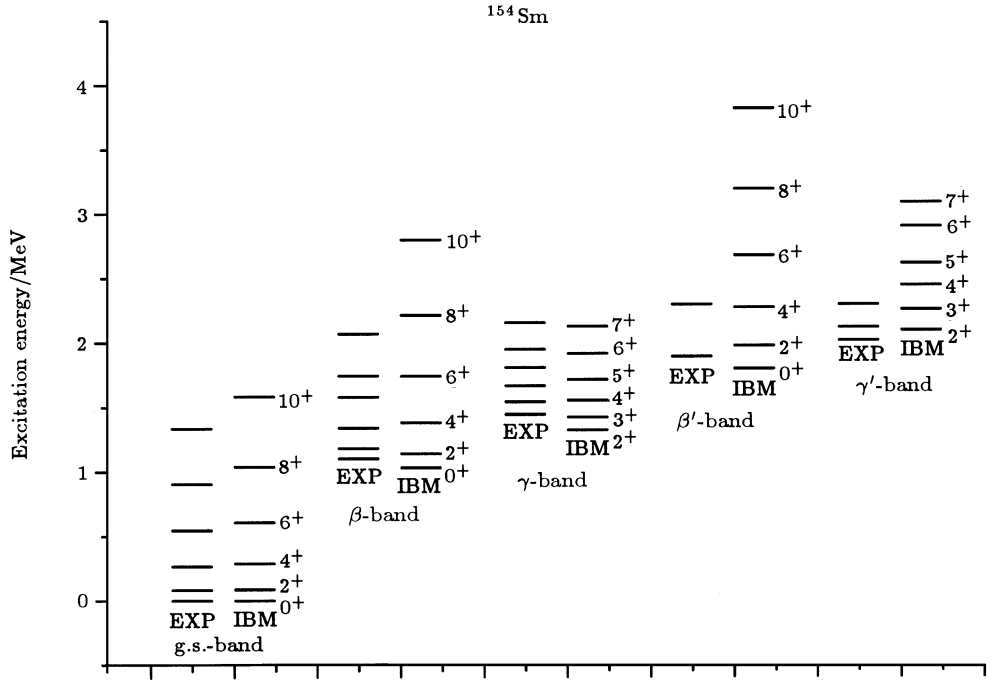


Fig.4. ^{154}Sm spectrum.

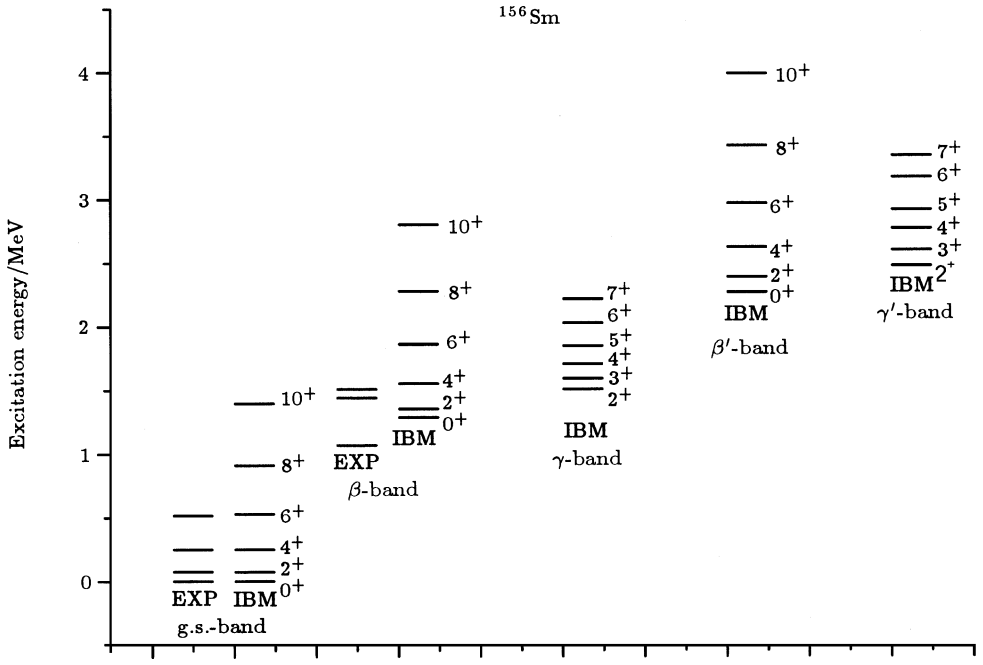


Fig.5. ^{156}Sm spectrum.

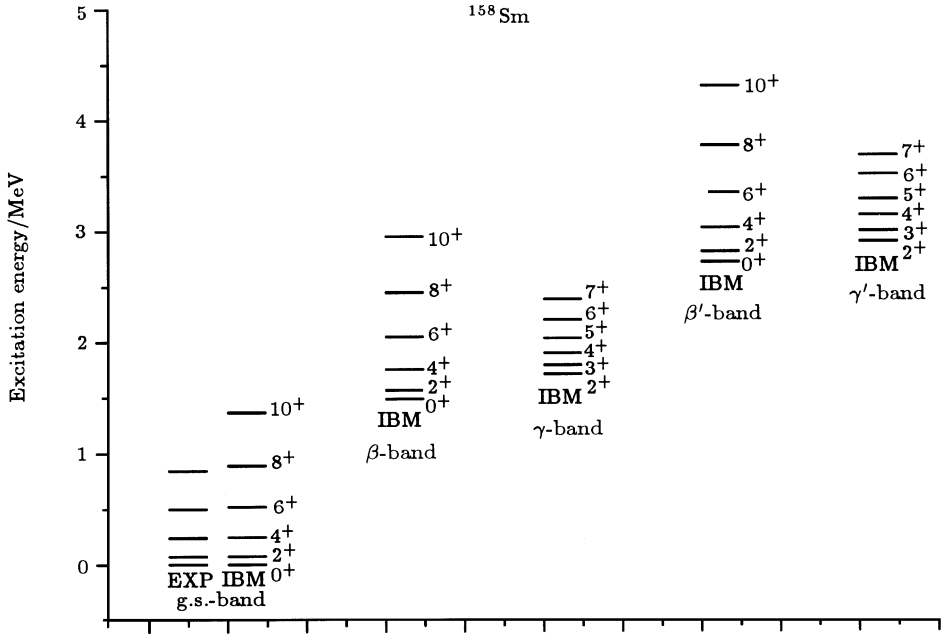


Fig.6. ^{158}Sm spectrum.

We have also calculated the $B(E2)$ values using the Hamiltonian of IBM-1. The operator of $T(E2)$ is given in Eq.(6), and the values of α are given in Table 1. From Table 2, one can find a good agreement between IBM and experimental results for transitions within the ground state band levels for $^{148-154}$Sm. Moreover, this table includes some transitions from the β-band levels to ground state band levels and γ-band levels to ground state band levels. The discrepancy between theory and experiment in some transition is somehow a little bigger, for instance, $B(E2, 2_3^+ \to 0_1^+)$ in ^{152}Sm and ^{154}Sm. We think that this is because some intruder components may come into play.

Table 2. Comparisons of B(E2) (in units of e^2b^2) for $^{148-158}$Sm in CUB calculation with IBM-1, and with experimental values.[23,29]

	^{148}Sm		^{150}Sm	
$J_i \to J_f/e^2b^2$	IBM	EXP	IBM	EXP
$2_1^+ \to 0_1^+$	0.146	0.151	0.23	0.255
$4_1^+ \to 2_1^+$	0.245	0.25	0.60	0.53
$6_1^+ \to 4_1^+$	0.304		0.2747	
$6_1^+ \to 4_3^+$	0.0038		0.288	
$0_2^+ \to 2_1^+$	0.212		0.7488	
$0_2^+ \to 2_3^+$	0.697		2.9952	
$2_2^+ \to 0_1^+$	0.0013	0.0079	0.3917	0.0038
$2_2^+ \to 2_1^+$	0.2651	0.035	0.00	
$2_3^+ \to 0_1^+$	0.00		0.0576	0.0096
$2_3^+ \to 4_1^+$	0.0895		0.2715	0.043
$3_3^+ \to 2_4^+$	0.001		0.2469	
$4_3^+ \to 4_1^+$	0.0004		0.1371	

	^{152}Sm		^{154}Sm	
$J_i \to J_f/e^2b^2$	IBM	EXP	IBM	EXP
$2_1^+ \to 0_1^+$	0.707	0.694	0.855	0.853
$4_1^+ \to 2_1^+$	0.985	1.007	1.197	1.196
$6_1^+ \to 4_1^+$	1.047		1.271	
$6_1^+ \to 4_3^+$	0.061		0.073	
$0_2^+ \to 2_1^+$	0.0016		0.119	
$0_2^+ \to 2_3^+$	0.00		0.095	
$2_2^+ \to 0_1^+$	0.0489	0.0044	0.0263	0.0046
$2_2^+ \to 2_1^+$	0.0279	0.026	0.0161	
$2_3^+ \to 0_1^+$	0.2538	0.017	0.4053	0.016
$2_3^+ \to 4_1^+$	0.0368	0.0038	0.048	0.0033
$3_3^+ \to 2_4^+$	0.034		0.197	
$4_3^+ \to 4_1^+$	0.551		0.816	

4. Potential energy surface

We study the shapes of these nuclei using the classic limit of the Hamiltonians in Eq.(10). To this end, we calculate the potential energy surfaces. For the IBM Hamiltonian with cubic term given in Eq.(1), the potential energy formulation is [5,6]

$$E(\beta,\gamma)=\frac{N\varepsilon_d\beta^2}{1+\beta^2}+a_2\left[\frac{N}{1+\beta^2}\left(5+\frac{11}{4}\beta^2\right)+\frac{N(N-1)}{(1+\beta^2)^2}\times\left[\frac{\beta^2}{2}+2\sqrt{2}\beta^3\cos(3\gamma)+4\beta^2\right]\right]+\frac{a_1 6N\beta^2}{1+\beta^2}+\frac{a_0N(N-1)}{4}\left[\frac{1-\beta^2}{1+\beta^2}\right]^2+\frac{7a_3N\beta^2}{5(1+\beta^2)}+a_4\left[\frac{9N\beta^2}{5(1+\beta^2)}+\frac{18N(N-1)\beta^4}{35(1+\beta^2)^2}\right]. \quad (11)$$

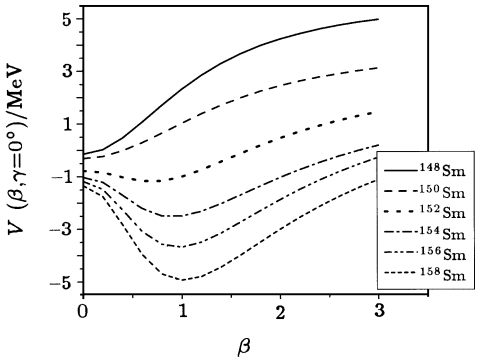
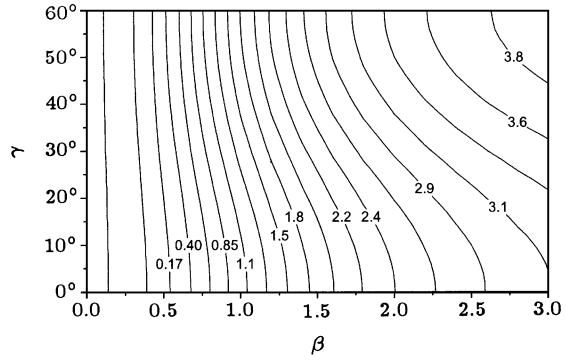


Fig.7. Potential energy surfaces $V(\beta, \gamma = 0°)$ for $Sm^{148-158}$, the Hamiltonian used is of the IBM-1.

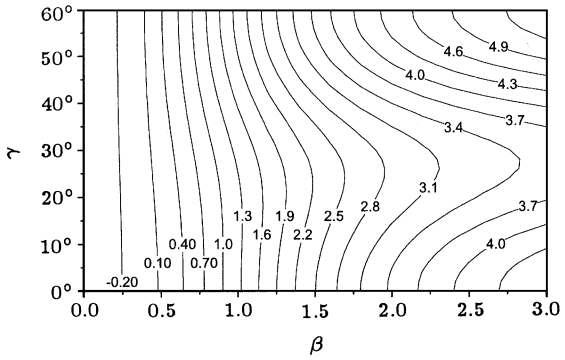
Figure 7 plots the potential energy surfaces $V(\beta, 0)$ for $^{148-158}$Sm, from which we can see that the minimum of the curve for ^{148}Sm is at the origin of the coordinate, indicating that the equilibrium shape of ^{148}Sm is spherical. Similarly, the minimum of the curve of ^{150}Sm is also at the origin and displays the same nature, but the slope becomes small and shows that the spherical equilibrium shape is not so stable as ^{148}Sm. From ^{152}Sm onward, we find that there are minima as β approaches 1. These properties indicate that the deformation of those nuclei has occurred.

When the cubic term is added to the Hamiltonian, the potential energy formulation becomes [5,6,20,27]

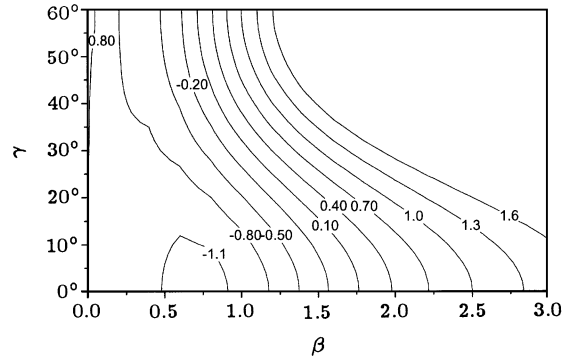
$$\begin{aligned} E(\beta,\gamma) = & \frac{N\varepsilon_{\mathrm{d}}\beta^2}{1+\beta^2} + a_2\left[\frac{N}{1+\beta^2}\left(5+\frac{11}{4}\beta^2\right)+\frac{N(N-1)}{(1+\beta^2)^2}\right. \\ & \left.\times\left[\frac{\beta^2}{2}+2\sqrt{2}\beta^3\cos(3\gamma)+4\beta^2\right]\right] + \frac{a_1 6N\beta^2}{1+\beta^2} \\ & + \frac{a_0 N(N-1)}{4}\left[\frac{1-\beta^2}{1+\beta^2}\right]^2 + \frac{7a_3 N\beta^2}{5(1+\beta^2)} \\ & + a_4\left[\frac{9N\beta^2}{5(1+\beta^2)}+\frac{18N(N-1)\beta^4}{35(1+\beta^2)^2}\right] \\ & + \frac{\theta_3 N(N-1)\beta^6}{7(1+\beta^2)^3}(-1+\cos^2 3\gamma). \end{aligned} \tag{12}$$



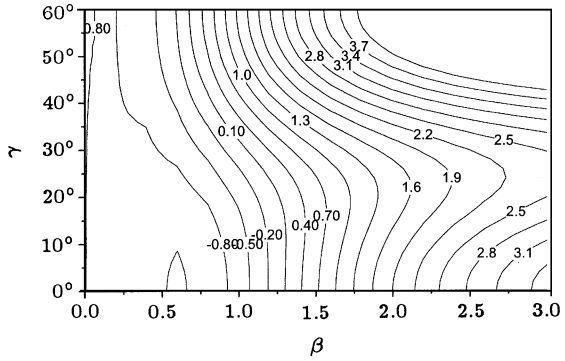
(a) V (β,γ) for ^{150}Sm with Hamitonian H_{sd}



(b) V (β,γ) for ^{150}Sm with Hamitonian $H_{\mathrm{sd}}+H_{\mathrm{cub}}$



(c) V (β,γ) for ^{152}Sm with Hamitonian H_{sd}



(d) V (β,γ) for ^{152}Sm with Hamitonian $H_{\mathrm{sd}}+H_{\mathrm{cub}}$

Fig.8. Comparison of the potential energy surfaces $V(\beta, \gamma)$ for ^{150}Sm and ^{152}Sm between Hamiltonians H_{sd} and $H_{\mathrm{sd}} + H_{\mathrm{cub}}$.

Figures 8(a) and 8(b) compare the potential energy surfaces calculated using the Hamiltonians with and without the cubic term for ^{150}Sm respectively, and Figs.8(c) and 8(d) are those for ^{152}Sm. From these figures, we see that although we do not obtain a stable triaxial shape which exists in some $O(6)$ nucleus,[20] the cubic term produces a local minimum at γ=30° when we fix the deformation parameter β. This indi-

cates a tendency toward triaxial deformation at higher energies. We note that the local extremum is at about $\beta = 1$ and is shallow, namely the effect of the cubic term is weak. From the above discussion, we obtain that when we simulate the high energy levels of a nucleus, we should consider much of the cubic terms.

5. Conclusion

In conclusion, we have studied the low-lying spectra and $E2$ transition rates of Sm isotopes in the frame of IBM-1. The simulation with IBM-1 of Sm isotopes proves in many aspects that the Sm isotope chain changes from $U(5)$ to $SU(3)$ symmetry. By incorporating the cubic and quadruple terms in the Hamiltonian of the IBM-1, we have studied the effect of the cubic and quadruple terms on the low-lying spectra of Sm isotopes. In addition, we also calculate the classic limit of the Hamiltonian to obtain potential energy surfaces which is dependent on β and γ. Comparing the result obtained by using IBM-1 plus cubic and quadruple terms with that by using the IBM-1 and the experimental result, we have shown that it is instructive to introduce the cubic and quadruple terms into the IBM, thus giving a new extension and modification to the IBM in some aspects.

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