Vol 13 No 11, November 2004
1009-1963/2004/13(11)/1830-05

Chinese Physics

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# A new energy source originating from extra dimensions*

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(Received 18 February 2004; revised manuscript received 7 June 2004)

In this work the Einstein gravitational field equations and the Lichnerowicz boundary formalism in the extra dimensions are used to build up our black hole model from 6-dimensional space–time. From the internal stress–energy tensor the solutions with energy levels and semiclassical space-quantization are obtained, which combines with only one metric condition outside the defect. We show a new type of energy source, which originates from extra dimensions. A part of the energy source of quasi-stellar object (QSO) maybe come from extra dimensions in that way. The theoretical arithmetic upper limit is identical to that of the output energy of QSO.

**Keywords:** space quantization, energy levels in 6-dimensions, new energy source
**PACC:** 1110, 0460, 9880

Brane worlds, the models in which our universe appears as some kind of domain wall or brane embedded in a highly-dimensional space–time, have gained considerable attention recently.[1] Gravity in this scenario is 'trapped' on the four-dimensional brane due to the geometry of the bulk space–time.[2,3] This model allows the presence of large or even infinite noncompact extra dimensions.[4]

Clearly, a very appealing feature of this idea is the fact that the defect itself arises naturally from a simple theoretical model of field (the Abelian Higgs model).[5]

It is now quite generally agreed that scenarios modelling our four-dimensional world as a topological defect in a higher dimensional space–time can be perfectly compatible with four-dimensional gravity, without need to compactify the extra dimensions.[5] In their two recent papers Randall and Sundrum[2] have shown that a scenario with a large or infinite fifth dimension in the presence of a brane can generate a theory of gravity which mimics purely four-dimensional gravity, both with respect to the classical gravitational potential and with respect to gravitational radiation.

In recent years, a series of topological defects and $p$-branes in space–time physics and gravity has been investigated extensively, but cannot be applied to the quantum geometry.[6−10]

The purpose of this paper is to show the separated energy levels and space quantization in the extra dimensions, and try to explore new physics in terms of the theory in 6D space–time.

Our 6D space–time is equipped with the metric of the form

$$\begin{aligned} \mathrm{d}s^2 =& \sigma(\rho)\left[f(r)\mathrm{d}t^2 - \frac{\mathrm{d}r^2}{f(r)} - r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2)\right] \\ & - \mathrm{d}\rho^2 - \rho^2\mathrm{d}\psi^2, \\ & 0 \le \rho < \gamma, \quad 0 \le \psi < 2\pi, \end{aligned} \tag{1}$$

with spherically symmetric defect (4-brane) localized at $\rho = 0$. In the model, $\gamma$ can be very large or infinite; the defect is modelled by a continuous matter distribution within a core of radius $\rho_0$, and vanishes outside the core, i.e. for $\rho > \rho_0$.

A specific model of so-called universal extra dimensions (UED) has recently been proposed by Appelquist *et al.*[11] In this model, the UED may be relatively large (i.e. $\gamma$ is finite in Eq.(1)), the evolution should be governed by Einstein field equations too.

After setting the four-dimensional cosmological constant $\Lambda_{\mathrm{phys}}$ to be zero, the Einstein equations become

$$R_B^A - \frac{1}{2}Rg_B^A = \frac{1}{M_6^4}\left(\Lambda + T_B^A\right), \tag{2}$$

where $M_6$ is the six-dimensional Planck scale, related

*Project supported by the Science and Technology Foundation of Guizhou Province of China (Grant No 2002-3026).
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to the four-dimensional Planck scale $M_{\rm p}$ by[5]

$$M_{\rm p}^2 = 2\pi M_6^4 \int_0^\gamma \sigma\rho {\rm d}\rho. \tag{3}$$

In UED cosmology, one has[12]

$$M_{\rm p}^2 = V_{(2)} M_6^4, \tag{4}$$

where $V_{(2)}$ is the volume of extra dimensions.

In order to guarantee the regularity of the geometry, the solution inside the core has to satisfy the boundary conditions at the origin:

$$\sigma\,|_{\rho=0} = A, \quad \sigma^*\,|_{\rho=0} = 0, \tag{5}$$

where $A > 0$ is an arbitrary constant, and a star denotes $\partial/\partial\rho$.

After setting the six-dimensional cosmological constants to be zero, the Einstein equations become

$$\frac{1}{r\sigma}\left(\frac{1}{r} - \frac{f}{r} - f'\right) - \frac{3}{2}\left(\frac{\sigma^{**}}{\sigma} + \frac{\sigma^*}{\rho\sigma}\right) = \frac{1}{M_6^4} T_0^0, \tag{6}$$

$$\frac{-1}{\sigma}\left(\frac{f'}{r} + \frac{f''}{2}\right) - \frac{3}{2}\left(\frac{\sigma^{**}}{\sigma} + \frac{\sigma^*}{\rho\sigma}\right) = \frac{1}{M_6^4} T_2^2, \tag{7}$$

$$\frac{1}{\sigma}\left(\frac{1}{r^2} - \frac{f}{r^2} - 2\frac{f'}{r} - \frac{f''}{2}\right) - \frac{3}{2}\frac{\sigma^{*2}}{\sigma^2} - 2\frac{\sigma^*}{\rho\sigma} = \frac{1}{M_6^4} T_4^4, \tag{8}$$

$$\frac{1}{\sigma}\left(\frac{1}{r^2} - \frac{f}{r^2} - 2\frac{f'}{r} - \frac{f''}{2}\right) - 2\frac{\sigma^{**}}{\sigma} - \frac{1}{2}\frac{\sigma^{*2}}{\sigma^2} = \frac{1}{M_6^4} T_5^5, \tag{9}$$

where the stress–energy tensor $T_0^0 = T_1^1$, $T_2^2 = T_3^3$, and a prime denotes $\partial/\partial r$. The metric outside the core is the solution of Eqs.(6), (7), (8) and (9) in the absence of sources

$$\sigma_{\rm out} = B, \quad f_{\rm out} = 1, \tag{10}$$

where $B > 0$ is an arbitrary constant. The solution inside the core has the form

$$\sigma_{\rm in} = A\sin\left(c\rho + \frac{\pi}{2}\right), \quad f_{\rm in} = f(r), \tag{11}$$

where $c > 0$, it is an arbitrary constant. The solution (11) satisfies the boundary conditions at the origin (Eq.(5)).

We now determine the boundary $\rho_0$ of the brane in the fifth dimensional $\rho$ direction. We simply require that at some boundary value of $\rho$, say $\rho_0$, the values of the internal form of $\sigma(\rho)$ and its first derivative match those of the external form given above. That is, we assume that the surface of the brane satisfies the Lichnerowicz boundary conditions, and that the coordinate system is admissible.[5,13]

Matching the interior and exterior forms of $\sigma(\rho)$ at $\rho_0$ gives the two relations

$$A\sin\left(c\rho_0 + \frac{\pi}{2}\right) = B, \tag{12}$$

$$\cos\left(c\rho_0 + \frac{\pi}{2}\right) = 0. \tag{13}$$

From Eq.(13) we have

$$c\rho_0 = n\pi, \quad n = 1, 2, 3, \ldots. \tag{14}$$

Equations (10), (12) and (14) give the two conditions

$$\sigma_{\rm out} = A, \quad n = 2, 4, 6, \ldots, \tag{15}$$

$$\sigma_{\rm out} = -A, \quad n = 1, 3, 5, \ldots. \tag{16}$$

In order to ensure the uniqueness of solution and obtain sensible ones,[13–16] we removed the solution (16), and now have the new sensible one

$$\sigma_{\rm out} = A, (n = 2, 4, 6, \ldots), \quad f_{\rm out} = 1, \tag{17}$$

$$\sigma_{\rm in} = A\sin\left(c\rho + \frac{\pi}{2}\right), \quad (\sigma\,|_{\rho=0} = A), \quad f_{\rm in} = f(r). \tag{18}$$

At the boundary

$$\rho_0 = \left(\frac{\pi}{c}\right) n, \quad n = 2, 4, 6, \ldots.$$

Since $c$ can be sufficient large, we let $\pi/c = L_{\rm P}$, then

$$\rho_0 = nL_{\rm P}, \quad n = 2, 4, 6, \ldots, \tag{19}$$

where $L_{\rm P}$ is the Planck length ($\sim 10^{-33}$cm).

For the Reissner–Nordstrom metric, $f(r)$ has the form

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} = \frac{1}{r^2}(r - r_+)(r - r_-), \tag{20}$$

and $r_\pm = M \pm \sqrt{M^2 - Q^2}$. The outer horizon is at $r = r_+$.

Now the internal stress–energy tensor must take the following form:

$$T_0^0 = M_6^4\left\{\frac{Q^2}{r^4}\frac{1}{A\sin\left(c\rho + \frac{\pi}{2}\right)} + \frac{3}{2}c\left[c - \frac{1}{\rho}{\rm ctg}\left(c\rho + \frac{\pi}{2}\right)\right]\right\}, \tag{21}$$

$$T_2^2 = M_6^4 \left\{ \frac{-Q^2}{r^4} \frac{1}{A \sin\left(c\rho + \frac{\pi}{2}\right)} + \frac{3}{2} c \left[ c - \frac{1}{\rho} \mathrm{ctg}\left(c\rho + \frac{\pi}{2}\right) \right] \right\}, \tag{22}$$

$$T_4^4 = M_6^4 \left\{ (-c) \left[ \frac{3c}{2} \mathrm{ctg}^2\left(c\rho + \frac{\pi}{2}\right) + \frac{2}{\rho} \mathrm{ctg}\left(c\rho + \frac{\pi}{2}\right) \right] \right\}, \tag{23}$$

$$T_5^5 = M_6^4 c^2 \left[ 2 - \frac{1}{2} \mathrm{ctg}^2\left(c\rho + \frac{\pi}{2}\right) \right], \tag{24}$$

where $T_0^0 = T_1^1$, $T_2^2 = T_3^3$; $T_B^A = 0$ for $A \neq B$.

We now consider the graded energy in the brane. Integrating Eq.(21)–(24) yields (more details in Appendix A,B,C)

$$\begin{aligned} E_0 =& \frac{3}{2} M_6^4 c \left\{ \left( \frac{2}{2m2\pi + \pi} - \frac{2}{\pi} \right) + \frac{4}{3} m2\pi + \frac{1}{135} \left[ \left( m2\pi + \frac{\pi}{2} \right)^3 - \left( \frac{\pi}{2} \right)^3 \right] \right. \\ & \left. + \frac{2}{4725} \left[ \left( m2\pi + \frac{\pi}{2} \right)^5 - \left( \frac{\pi}{2} \right)^5 \right] + \cdots + \frac{2^{2k} B_k}{(2k-1)(2k)!} \left[ \left( m2\pi + \frac{\pi}{2} \right)^{2k-1} - \left( \frac{\pi}{2} \right)^{2k-1} \right] + \cdots \right\}, \end{aligned} \tag{25}$$

$$\begin{aligned} E_4 =& M_6^4 c \left\{ \frac{3}{2} m2\pi + 2 \left[ \left( \frac{2}{2m2\pi + \pi} - \frac{2}{\pi} \right) + \frac{1}{3} m2\pi + \frac{1}{135} \left( \left( m2\pi + \frac{\pi}{2} \right)^3 - \left( \frac{\pi}{2} \right)^3 \right) \right] \right. \\ & \left. + 2 \left[ \frac{2}{4725} \left( \left( m2\pi + \frac{\pi}{2} \right)^5 - \left( \frac{\pi}{2} \right)^5 \right) + \cdots + \frac{2^{2k} B_k}{(2k-1)(2k)!} \left( \left( m2\pi + \frac{\pi}{2} \right)^{2k-1} - \left( \frac{\pi}{2} \right)^{2k-1} \right) + \cdots \right] \right\}, \end{aligned} \tag{26}$$

$$E_5 = M_6^4 c \left( \frac{5}{2} \right) m2\pi = 5 M_6^4 c m\pi, \tag{27}$$

where $E_0 = E_1 = E_2 = E_3$ and $m = 1, 2, 3, \ldots$. So the energies are separated. This means that we have obtained the quantized energy levels in the brane, i.e. $m = 1$ corresponds to the ground level of energy, $m = 2, 3, 4 \ldots$ correspond to the excited levels. If black holes are in the excited states, they may jump down to the ground level and release their excess energies. This is a new energy source, which comes from extra dimensions. A part of the energy source of a quasi-stellar object (QSO) maybe originates from extra dimensions as we have described above, the others from gravitational energy.

We now estimate the life-span of excited black holes that are in higher energy levels. From the uncertainty principle of energy–time

$$\Delta E \cdot \Delta t \geq \frac{\hbar}{2},$$

we obtain the lifetime

$$\Delta t \sim \left( \frac{\hbar}{2} \right) \left( \frac{1}{\Delta E} \right). \tag{28}$$

Let $\Delta E_{0,31} = E_0 \mid_{n=3} - E_0 \mid_{n=1}$, $\Delta E_{2,51} = E_2 \mid_{n=5} - E_2 \mid_{n=1}$ and so on. And since $\Delta E_{\alpha,11} = E_\alpha \mid_{n=1} - E_\alpha \mid_{n=1} = 0$, the lifetime $\Delta t_{\alpha,11} \to \infty$, $\alpha$=0, 1, 2, 3, 4, 5. So the black holes that are to be on the ground levels of energy are stable only, excited black holes are all unstable, as they jump to the ground levels they will release their excess energies in radiation. This is a new energy source originating from extra dimensions, and a new way to release energies like that. Astronomers have found abnormal fountain of black holes in recent years, it may be relevant to the new energy source that we described above, and the energy source of QSO may be relevant to it too.

Since Matthews and Sandage found the first QSO in the 1960s, a number of questions have not been solved:[17,18] 1) why the feature of radiation of QSO is non-thermal, 2) why the changes of their luminosity are nonperiodic, 3) how the central engines of QSO can be produced, 4) the output energy of their radiation can be $10^7$-fold that of the Milky Way, and about $10^{48} - 10^{53}$J, why it is so large.

Now we come back to Eq.(2), in which light-speed $C$=1, and gravitational constant $G$=1 too, so it can be in the form

$$R_B^A - \frac{1}{2} R g_B^A = \frac{8\pi G}{M_6^4 C^4} \left( \Lambda + T_B^A \right). \tag{29}$$

From Eqs.(4), (19), (25) and (29), we can estimate $\Delta E_{\alpha,\beta\delta} \approx V_{(2)}^{-1} \times 10^{69}$J ($\beta > \delta$, and they are both positive integers). Since the scale of active galactic nucleus of QSO is about 1pc=$3.0856 \times 10^{16}$m, if the extra volume $V_{(2)}$ has the same scale, we can come

to the meaningful result, i.e. $\Delta E_{\alpha,\beta\delta} \approx 10^{37}\mathrm{J}-10^{53}\mathrm{J}$, its upper limit is identical to that of output energy of QSO. So we believe that a part of QSO-energy comes from extra dimensions, and the others maybe originate from gravitational energy of black hole which is in the centre of QSO, and there is a violent and small region around the black hole.

In Refs.[19,20], quantum dynamics of self-gravitating membranes and membranes in flat space has been studied. In that case black holes can be regarded as quantum membranes.[21,22] One can describe them with a 'principal quantum number' $m$, and at least for a large $m$ one expects the area of a membrane $A = mA_0$, the quantum of area in this approach is $A_0 = \frac{1}{2}L_{Pl}^2$.

Braneworld cosmology has opened up a possible new phenomenology for the cosmology of the early universe.[21] In this work we have obtained a new way to quantized spaces of extra dimensions, and the separated and graded energy levels in 6-dimensions. From this and according to quantum mechanics principles a new energy source coming from extra dimensions has been found too. It may be relevant to the energy source of QSO and that of black hole fountains.

# Appendix A

Integrating Eq.(21) we have

$$E_0 = M_6^4 \left\{ \frac{1}{r^4}\frac{Q^2}{cA} \ln \mathrm{tg}\left(\frac{c\rho}{2}\right) + \frac{3}{2}c\left[c\rho + \frac{1}{c\rho} + \frac{c\rho}{3} + \frac{(c\rho)^3}{135} + \frac{2(c\rho)^5}{4725} + \cdots\right]\right\}\Big|_{c\rho=\frac{\pi}{2}}^{m2\pi+\frac{\pi}{2}},$$

where $m = 1, 2, 3, \ldots$ and

$$\int_0^{m2\pi/c} \frac{\mathrm{d}\rho}{\sin\left(c\rho+\frac{\pi}{2}\right)} = \frac{1}{c}\int_0^{m2\pi} \frac{\mathrm{d}x}{\sin\left(x+\frac{\pi}{2}\right)} = \frac{1}{c}\int_{\pi/2}^{m2\pi+\pi/2} \frac{\mathrm{d}y}{\sin y} = \frac{1}{c}\ln\left(\mathrm{tg}\,\frac{y}{2}\right)\Big|_{\frac{\pi}{2}}^{m2\pi+\frac{\pi}{2}} = 0,$$

where $x = c\rho, \quad y = c\rho + \pi/2,$

$$\ln\left(\mathrm{tg}\left(\frac{y}{2}\right)\right)\Big|_{y=\frac{\pi}{2}}^{m2\pi+\frac{\pi}{2}} = \ln\left(\mathrm{tg}\left(m\pi+\frac{\pi}{4}\right)\right) - \ln\mathrm{tg}\left(\frac{\pi}{4}\right) = 0.$$

So we have Eq.(25).

# Appendix B

Integrating Eq.(23) we have

$$\begin{aligned} E_4 =& M_6^4 c\left\{\frac{3c}{2}\left[\frac{\mathrm{ctg}(c\rho)}{c}+\rho\right]_{c\rho=\frac{\pi}{2}}^{m2\pi+\frac{\pi}{2}} + 2\left[\frac{1}{c\rho}+\frac{c\rho}{3}+\frac{(c\rho)^3}{135}+\frac{2(c\rho)^5}{4725}+\cdots+\frac{2^{2k}B_k(c\rho)^{2k-1}}{(2k-1)(2k)!}+\cdots\right]_{c\rho=\frac{\pi}{2}}^{m2\pi+\frac{\pi}{2}}\right\} \\ =& M_6^4 c\left\{\frac{3}{2}\left[c\rho\right]_{c\rho=\frac{\pi}{2}}^{m2\pi+\frac{\pi}{2}} + 2\left[\frac{1}{c\rho}+\frac{c\rho}{3}+\frac{(c\rho)^3}{135}+\frac{2(c\rho)^5}{4725}+\cdots+\frac{2^{2k}B_k(c\rho)^{2k-1}}{(2k-1)(2k)!}+\cdots\right]_{c\rho=\frac{\pi}{2}}^{m2\pi+\frac{\pi}{2}}\right\}. \end{aligned}$$

So we can obtain Eq.(26)

# Appendix C

**Table 1.** The values of Bernoullischen numbers $B_k$.

| k | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $B_k$ | $\frac{1}{6}$ | $\frac{1}{30}$ | $\frac{1}{42}$ | $\frac{1}{30}$ | $\frac{5}{66}$ | $\frac{691}{2730}$ | $\frac{7}{6}$ | $\frac{3617}{510}$ | $\frac{43867}{798}$ | $\frac{174611}{330}$ | $\frac{854513}{138}$ |

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