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Stochastic resonance of bias signal-modulated noise in a single-mode laser*

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Stochastic resonance (SR) for bias signal modulation is studied in a single-mode laser system. By investigating a gain-noise model driven by correlated pump noise and quantum noise, we find that, whether the correlation coefficient between both the noises is positive or negative, SR always appears in the dependence of signal-to-noise ratio (SNR) upon the noise correlation time and the frequency of the modulation signal. However, only when the correlation coefficient between both noises is negative can SR occur in the dependence of SNR upon the quantum noise intensity and pump noise intensity, while when the correlation coefficient between both noises is positive, it shows monotonically.

Keywords: bias signal-modulated noise, stochastic resonance, linear approximation method
PACC: 0540, 4260K

1. Introduction

Since Benzi *et al*[1] originally discovered the phenomenon of stochastic resonance (SR) in 1981, it has attracted extensive investigation.[2–6] In recent years, there have been considerable developments in the process of exploring the essence of SR. Compared to the concept of SR in the early stage, the new progresses in theories and experiments lie in the following three aspects. First, the restriction from the three necessary factors (nonlinearity system, noise, and periodic signal),[5,6] which were thought of as three essential ingredients for the onset of SR, was broken through because SR was observed without external signal or in linear systems.[7,8] Second, SR in the early stage was confined to the single-peak form, i.e. there was only one resonance peak in SR.[5,6] The multi-peak form SR, namely stochastic multiresonance, was detected first by Vilar and Rubi [9] in 1997. Now it is a focus in the investigations of SR.[10] Finally, in the early works, the dependence of signal-to-noise ratio (SNR) upon noise intensity was generally used to quantify the characteristic quantity of SR.[2,6,10,11] In 1998, Barzykin and Seki [12] proposed that, in the broad sense, SR can also be characterized by the dependence of SNR upon noise correlation time.

Due to fluctuations of spontaneous emission and population inversion, the output intensity of a laser varies randomly and continuously in a variety of forms, even when the laser operates in steady-state.[6,13] In 2002, using the linear approximation method, we investigated SR in a gain-noise model of a single-mode laser by studying an equation of the laser intensity with an input signal.[14] In the process of optical communications [15] a signal inevitably modulates noises in laser output intensity, resulting in signal-modulated noises, when the signal is modulated on the laser carrier wave. In 1992, Dykman *et al*[16] studied the phenomenon of stochastic resonance appearing in an asymmetric bistable model driven by a bias signal-modulated noise. They found that the bias signal-modulated noises can better describe and reveal SR characters of this kind of stochastic system, and their results were in agreement with experiments. Further, in 2002, Cao and Wu[17] studied SR characteristics in a linear system driven by a bias signal-modulated noise.

In order to apply SR to optical communications, it is very important to investigate this kind of bias signal-modulated noise for practical applications.

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Based on our previous works,[14,17] in this paper, we firstly introduce a bias signal-modulated noise into a single-mode laser system, and the characteristics of SR induced by the bias signal-modulated noise are studied in a gain-noise model driven by correlated pump noise and quantum noise. We calculate the laser intensity correlation function, power spectrum and SNR, and find that, not only does SR appear in the dependence of SNR upon the noise correlation time and the frequency of the modulation signal, but also there are both SR and monotony in the dependence of SNR upon the quantum noise intensity and pump noise intensity.

2. The intensity output power spectrum and SNR

We consider a gain-noise model of a single-mode laser with an input bias signal which is described by the following equation of intensity:[14]

$$\frac{\mathrm{d}I}{\mathrm{d}t} = -2KI + \frac{2\Gamma}{1+\beta I}I + D + \frac{2I}{1+\beta I} \times \left[1 + \frac{1}{2}A_0\cos(\Omega t)\right]\xi(t) + 2\sqrt{I}\eta(t), \quad (1)$$

where the pump noise $\xi(t)$ and the quantum noise $\eta(t)$ are correlated in the following form:

$$\langle\xi(t)\rangle = \langle\eta(t)\rangle = 0,$$
$$\langle\xi(t)\xi(t')\rangle = \frac{Q}{2\tau}\exp\left(-\frac{|t-t'|}{\tau}\right),$$
$$\langle\eta(t)\eta(t')\rangle = \frac{D}{2\tau}\exp\left(-\frac{|t-t'|}{\tau}\right),$$

$$<\xi(t)\eta(t')> = <\eta(t)\xi(t')> = \frac{\lambda\sqrt{DQ}}{2\tau}\exp\left(-\frac{|t-t'|}{\tau}\right), \quad (-1 \le \lambda \le 1). \quad (2)$$

In Eqs.(1) and (2), I is the laser intensity, K is the loss coefficient, $\beta = \dfrac{A}{\Gamma}$, A and Γ represent the self-saturation and gain coefficients, Q and D are the intensities of the pump and the quantum noise respectively, λ is the correlation coefficient between both noises, A_0 is the amplitude of a periodic signal, Ω is its frequency, and it is assumed that the self-correlation and cross-correlation time of the pump noise and quantum noise are of the same value τ. Since $A_0\cos(\Omega t)$ is the modulated signal, it is required that $A_0 \le 1$.

Let

$$I = I_0 + \varepsilon(t), \quad (3)$$

where $I_0 = (\Gamma - K)/\beta K$, is the deterministic steady-state intensity, and $\varepsilon(t)$ is the perturbation term. We linearize Eq.(1) around the deterministic steady-state intensity I_0, thus the linear equation of the laser intensity is found to be

$$\frac{\mathrm{d}\varepsilon(t)}{\mathrm{d}t} = -\gamma\varepsilon(t) + D + \frac{2I_0}{1+\beta I_0} \times \left[1 + \frac{1}{2}A_0\cos(\Omega t)\right]\xi(t) + 2\sqrt{I_0}\eta(t), \quad (4)$$

where the attenuation coefficient $\gamma = 2K(\Gamma - K)/\Gamma$. By virtue of Eq.(4), we have

$$\langle\varepsilon(t)\rangle = \frac{D}{\gamma}(1 - \mathrm{e}^{-\gamma t}). \quad (5)$$

The intensity correlation function is defined by

$$B(t) = \lim_{t'\to\infty}\overline{\langle I(t'+t)I(t')\rangle}, \quad (6)$$

where $\overline{\langle I(t'+t)I(t')\rangle} = \dfrac{\Omega}{2\pi}\displaystyle\int_{t'}^{t'+\frac{2\pi}{\Omega}}\langle I(t'+t)I(t')\rangle\,\mathrm{d}t'$. Integrating Eq.(4) and combining with Eqs.(3) and (6), we can have the analytical solution of $B(t)$ (only consider the case of $\gamma \ne \tau^{-1}$). Finally, in order to obtain the expression for SNR, we have to first work out the output power spectra of the signal and the noise. Thus, translate the intensity correlation function $B(t)$ into the power spectrum $S(\omega)$ by Fourier transform

$$S(\omega) = S_1(\omega) + S_2(\omega), \quad (7)$$

in which $S_1(\omega)$ and $S_2(\omega)$ are output power spectrum of the signal and the noise, respectively. The SNR is defined as the ratio of the output power of the signal P_s and the broadband noise output at $\omega = \Omega$. We have

$$\mathrm{SNR} = \frac{P_\mathrm{s}}{S_2(\omega = \Omega)}, \quad (8)$$

where $P_\mathrm{s} = \int_0^\infty S_1(\omega)\mathrm{d}\omega = \dfrac{\pi\Lambda^2 Q(\gamma^2 - 1/\tau^2)}{16\tau\kappa_+\kappa_-}$,

$$S_2(\omega = \Omega) = \frac{Q\Lambda^2A_0^2\gamma}{4\tau\kappa}\left(\frac{\gamma + 1/\tau}{4\kappa_+} - \frac{\gamma - 1/\tau}{2\kappa_-}\right) - \frac{Q\Lambda^2/2 + 2DI_0 + 2\Lambda\lambda\sqrt{DQI_0}}{\gamma\tau^2(\gamma^2 - 1/\tau^2)} + \frac{Q\Lambda^2/2 + 2DI_0 + 2\Lambda\lambda\sqrt{DQI_0}}{(1+\tau^2\Omega^2)(\gamma^2 - 1/\tau^2)},$$

with $\kappa = \gamma^2 + \Omega^2$, $\Lambda = 2I_0/(1+\beta I_0)$,

$$\kappa_+ = (\gamma + 1/\tau)^2 + \Omega^2, \quad \kappa_- = (\gamma - 1/\tau)^2 + \Omega^2.$$

3. Conclusion and discussion

According to the expression of SNR of Eq.(8), the curves of SNR as functions of the noise correlation time, the noise intensities, and the frequency of the modulation signal are plotted in Figs.1–4. From them, we can see that single-peak SR appears in the dependence of SNR upon the noise correlation time τ and the frequency of the modulation signal Ω. Moreover, there are two cases in which both the single-peak SR and monotony in the dependence of SNR upon the quantum noise intensity and pump noise intensity are shown.

(1) Resonance in the SNR vs τ curve

With the correlation coefficient between the noises λ as the parameter, the SNR vs τ curve is plotted in Fig.1. There is always a peak (i.e. stochastic resonance) in the SNR vs τ curve for the cases of $\lambda > 0$, $\lambda = 0$ and $\lambda < 0$. Moreover, when λ increases, the peak becomes lower, and the position of the peak moves to the left gradually.

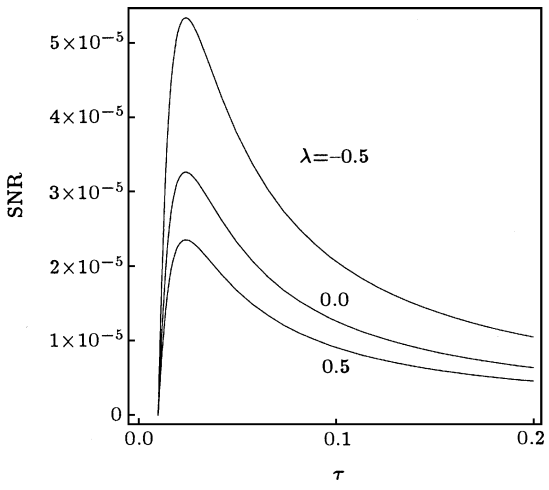


Fig.1. SNR versus noise correlation time τ for with Q=1.0, D=0.1, Ω=100, I_0=10.0, β=2.0, γ=0.5, A=0.6.

(2) Resonance and monotony in the SNR vs D and the SNR vs Q curve

Taking λ as the parameter, Figs.2 and 3 depict the curves of the dependence of SNR upon the intensities of the quantum noise D and the pump noise Q, respectively. In Fig.2, there are two cases in the SNR vs D curve. For the case of $\lambda < 0$, a resonance peak appears in the curve, and the peak becomes lower as $|\lambda|$ decreases. For the case of $\lambda \geq 0$, SNR decreases monotonically as D increases. There are also two cases in the SNR vs Q curve as shown in Fig.3. For the case of $\lambda < 0$, a resonance peak appears in the curve, and the peak becomes lower as $|\lambda|$ decreases. For the case of $\lambda \geq 0$, SNR increases slowly as Q increases.

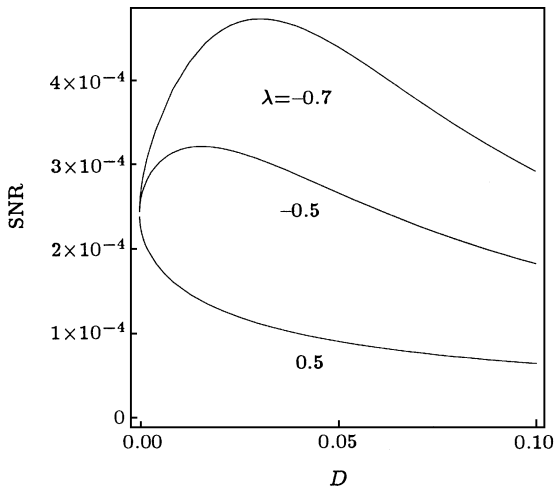


Fig.2. SNR versus quantum noise intensity D with Q=1.0, τ=0.2, Ω=100, I_0=1.0, β=3.0, γ=0.5, A=0.6.

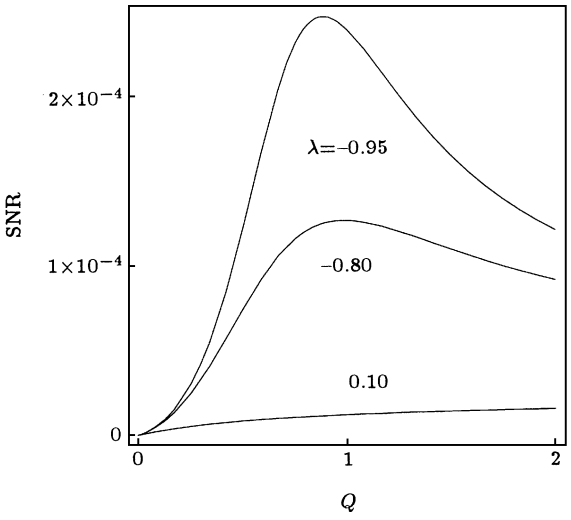


Fig.3. SNR versus pump noise intensity Q with D=0.05, τ=0.2, Ω=100, I_0=1.0, β=3.0, γ=0.5, A=0.6.

(3) Resonance in the SNR vs Ω curve

Choosing λ as the parameter, Fig.4 depicts the curve of the dependence of SNR upon the frequency of the signal Ω. A peak (i.e. stochastic resonance) always appears in the SNR vs Ω curve for the cases of $\lambda > 0$, $\lambda = 0$ and $\lambda < 0$. Furthermore, when λ increases, the peak becomes lower, and the position of the peak moves to the left gradually. That is, the laser system only responds to the signal whose frequency is within a certain range, so it can be used as a frequency scale selector or a frequency filter.

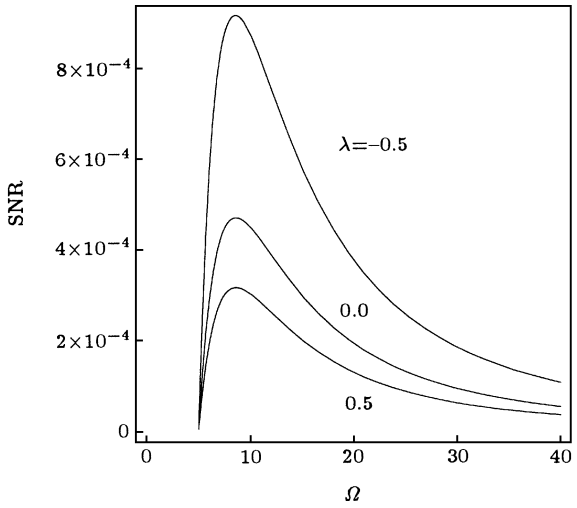


Fig.4. SNR versus frequency of the modulated signal Ω with Q=1.0, D=0.1, τ=0.2, I_0=1.0, β=3.0, γ=0.5, A=0.6.

Now, we draw a comparison between the bias signal-modulated noise and the non-modulated noise.[14] In the former, the noise and the signal were introduced multiplicatively into the single-mode laser system, while the noise and the signal were introduced additively in the latter. Hence, they represent different physical processes. Further, as viewed from the results: (1) The difference is that, in the former, SR occurs not only in the curve of dependence of SNR upon the intensities of pump noise and quantum noise, but also in the curve of the dependence of SNR upon the noise correlation time and the frequency of the modulation signal, while SR only occurs in the curve of dependence of SNR upon the intensities of pump noise and quantum noise in the latter. (2) The similarity is that, in the former, SR in the dependence of SNR upon the intensities of pump noise and quantum noise appears in the cases of $\lambda < 0$, and SR in the latter also appears in the cases of $\lambda < 0$.

It is emphasized that the conclusions drawn by the linear approximation method are correct and reliable only under the condition of the normalized intensity fluctuation (NIF) $B(0) \ll 1$.[18] Hence, we calculate the NIF and ensure $B(0) < 1 \times 10^{-2}$ throughout this paper. We find that, when the correlation coefficient between the pump noise and the quantum noise is negative, a minimum appears in the curve of the dependence of the NIF upon quantum and pump noise intensity. Moreover, at the minimum, the NIF can decrease further with increasing frequency of the modulation signal.

In summary, SR induced by the bias signal-modulated noise is studied in a gain-noise model of a single-mode laser system driven by correlated pump noise and quantum noise in this paper. According to the results, we know that SR appears not only in the dependence of SNR upon the noise correlation time, but also in the dependence of SNR upon the frequency of the modulation signal, the quantum noise intensity and pump noise intensity. Therefore, compared to the non-modulated noise, the bias signal-modulated noise has two advantages: (1) there are more ways to carry out SR; (2) the ranges of parameters satisfying the condition of the linear approximation method is wider, so SR can take place more easily. Consequently, it is more propitious to improve the quality of optical communications where SR is applied. It is believed that the results propose a new method and theoretic basis to design and optimize optical communication systems.

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