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# Teleportation of $N$-particle entangled $W$ state via entanglement swapping*

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A scheme for teleporting an unknown $N$-particle entangled $W$ state is proposed via entanglement swapping. In this scheme, $N$ maximally entangled particle pairs are used as quantum channel. As a special case, the teleportation of an unknown four-particle entangled $W$ state is studied.

**Keywords:** teleportation, $N$-particle entangled $W$ state, entanglement swapping, unitary transformation

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Quantum entanglement is a key ingredient of quantum techniques for information processing. The entangled states of two or more particles not only provide the possibilities to test quantum mechanics against local-hidden variable theory,[1,2] but also have practical applications in quantum information processing, such as quantum cryptography,[3] quantum dense coding,[4] quantum teleportation,[5] and quantum computers.[6] Recently, there has been much interest in the investigation of multiparticle entangled states. In Ref.[7], it is found that truly tripartite pure state entanglement of three qubits is either equivalent to the maximally entangled Greenberger–Horne–Zeilinger (GHZ) state[8] or to the entangled $W$ state.[7] The entanglement of the $W$ state is maximally robust under disposal of any one of the three qubits.[7] Guo and Zhang[9] presented an experimental scheme of preparing the tripartite $W$ state via cavity quantum electrodynamics. A natural generalization of the $W$ state to $N$-particle, called $N$-particle entangled $W$ state, is discussed.[10−12] More recently, Zou *et al*[13] have shown a scheme to produce an entangled four-photon $W$ state by using linear optical elements.

Quantum teleportation was originally proposed by Bennett *et al.*[5] In their scheme, an unknown quantum state is transferred without material transportation from a sender (Alice) to a receiver (Bob) via a quantum channel with the help of some classical information, and such a quantum channel is represented by a maximally entangled Bell state. Teleportation has been demonstrated with the polarization photon[14] and a single coherent mode of field[15] in optical experiments. Several schemes for teleportation of a two-particle entangled state have been presented.[16−21] Recently, quantum teleportations of a three-particle entangled GHZ state[8] and a three-particle entangled $W$ state[7] have been investigated.[22−26] In this paper, we propose a scheme to teleport an unknown $N$-particle entangled $W$ state via entanglement swapping. In this scheme, $N$ maximally entangled particle pairs are used as quantum channels. As a special case, a scheme of teleporting an unknown four-particle entangled $W$ state is investigated.

Suppose Alice has an unknown $N$-particle entangled $W$ state

$$\begin{aligned}|W_N\rangle &= \sum_{k=1}^{N} x_k \left|0_{(N-1)}, 1\right\rangle_{12\ldots N} \\ &= x_1 \left|000\ldots1\right\rangle_{12\ldots N} + x_2 \left|000\ldots10\right\rangle_{12\ldots N} + \cdots \\ &\quad + x_{N-1} \left|010\ldots0\right\rangle_{12\ldots N} + x_N \left|100\ldots0\right\rangle_{12\ldots N}, \end{aligned} \tag{1}$$

where

$$|x_1|^2 + |x_2|^2 + \cdots + |x_N|^2 = 1, \tag{2}$$

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and $\{|000\ldots1\rangle_{12\ldots N}, |000\ldots10\rangle_{12\ldots N}, \ldots, |100\ldots0\rangle_{12\ldots N}\}$ is the basis of an $N$-dimensional Hilbert space. To teleport the state $|W_N\rangle$ to the receiver Bob, Alice needs to set up $N$ entangled pairs (assuming $N$ independent sources, each emits a pair of particles in a beam-entangled state and, by chance, these emissions are nearly simultaneous[27]) between Bob and herself, which are located in the following maximally entangled states:

$$|\psi\rangle_{k'k''} = \frac{1}{\sqrt{2}}\left(|00\rangle_{k'k''} + |11\rangle_{k'k''}\right), \tag{3}$$

where $k' = 1', 2', \ldots, N'$ and $k'' = 1'', 2'', \ldots, N''$. Alice uses these as quantum channels between Bob and herself. We assume the particles $1, 2, \ldots, N$, $1'', 2'', \ldots, N''$ belong to Alice while particles $1', 2', \ldots, N'$ belong to Bob. The state of the system composed of $N$ particles and $N$ entangled pairs reads

$$\begin{aligned}|\Psi\rangle &= |W_N\rangle \otimes |\psi\rangle_{1'1''} \otimes |\psi\rangle_{2'2''} \otimes \cdots \otimes |\psi\rangle_{N'N''} \\ &= |W_N\rangle \otimes \prod_{k',k''=1}^{N',N''} |\psi\rangle_{k'k''}\,. \end{aligned} \tag{4}$$

In order to realize teleportation, Alice performs Bell measurements on particles $(1, 1'')$, $(2, 2'')$, $\ldots$, $(N, N'')$, separately. For two two-state particles $(k, k'')$, the corresponding Bell basis state are actually $N$ maximally entangled states which form a complete orthonormal basis in the $N$-dimensional space. They can be expressed as

$$|\Phi^{\pm}\rangle_{kk''} = \frac{1}{\sqrt{2}}\left(|00\rangle \pm |11\rangle\right)_{kk''}, \tag{5}$$

$$|\Psi^{\pm}\rangle_{kk''} = \frac{1}{\sqrt{2}}\left(|01\rangle \pm |10\rangle\right)_{kk''}. \tag{6}$$

where $kk''$ represents $11'', 22'', \ldots, NN''$. After the $N$ measurements, all the probable $2^{2N}$ kinds of outcomes are

$$\prod_{k,k''=1}^{N,N''} {}_{k,k''}\langle\varphi|\,\Psi\rangle = 2^{-N/2}\sum_{k=1}^{N}\delta_k x_k \prod_{i=1}^{N}|q_i\rangle, \tag{7}$$

where the $|\varphi\rangle_{k,k''}$ is the Bell state of particles $(k, k'')$, $\prod_{k,k''=1}^{N,N''} {}_{k,k''}\langle\varphi|$ is one of the $2^{2N}$ kinds of combination of Bell states $|\Phi^{\pm}\rangle_{kk''}$ and $|\Psi^{\pm}\rangle_{kk''}$, $\delta_k$ denotes the signum of coefficient $x_k$, $\prod_{i=1}^{N}|q_i\rangle = |q_1 q_2 \ldots q_N\rangle_{12\ldots N}$ is a basis state, and $q_i \in \{0, 1\}$ is dependent on the Bell-state measurement results.

After operations, Alice informs the receiver Bob of the measured results via a classical channel. Bob can reconstruct the original state only by performing a unitary transformation on his particles $1', 2', \ldots, N'$. For instance, assume that for the cases when Alice measures particles $(k, k'')$ in the Bell states $|\Phi^-\rangle_{kk''}$, $|\Psi^+\rangle_{kk''}$, $|\Phi^+\rangle_{kk''}$ and $|\Psi^-\rangle_{kk''}$, Bob may perform the corresponding unitary transformation $(\sigma_z)_{k'}$ $(\sigma_x)_{k'}$, $I$, $(\sigma_x)_{k'}$ and $(\sigma_z)_{k'}$ on the particle $k'$, respectively (where $\sigma_x$ and $\sigma_z$ are the Pauli operators, and $I$ is an identity operator). It is evident that Bob must operate relevant unitary transformation against different measurement results of Alice. After all the operations stated above, Alice successfully sends the unknown state (1) to Bob.

As a special case, we investigate a scheme for teleporting an unknown four-particle entangled $W$ state via entanglement swapping. We suppose that the sender Alice wants to transmit an unknown four-particle entangled $W$ state to receiver Bob,

$$\begin{aligned}|\psi\rangle_{1234} =& (x_0|0001\rangle + x_1|0010\rangle \\ &+ x_2|0100\rangle + x_3|1000\rangle)_{1234}, \end{aligned} \tag{8}$$

where

$$|x_0|^2 + |x_1|^2 + |x_2|^2 + |x_3|^2 = 1. \tag{9}$$

To teleport the state $|\psi\rangle_{1234}$ to receiver Bob, Alice needs to set up four distant entangled pairs (particles $(1', 1'')$, $(2', 2'')$, $(3', 3'')$ and $(4', 4'')$) between herself and Bob, which are produced by four independent sources (each emits a pair of particles and, by chance, these emissions are nearly simultaneous). These entangled pairs are located in the following maximally entangled states, respectively:

$$|\psi\rangle_{1'1''} = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)_{1'1''}, \tag{10}$$

$$|\psi\rangle_{2'2''} = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)_{2'2''}, \tag{11}$$

$$|\psi\rangle_{3'3''} = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)_{3'3''}, \tag{12}$$

$$|\psi\rangle_{4'4''} = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)_{4'4''}. \tag{13}$$

We assure that particles 1, 2, 3, 4, $1''$, $2''$, $3''$ and $4''$ belong to Alice while particles $1'$, $2'$, $3'$ and $4'$ belong to Bob. Therefore, the initial state of the whole system composed of particles (1, 2, 3, 4) and four entangled pairs $(1', 1'')$, $(2', 2'')$, $(3', 3'')$ and $(4', 4'')$ is

$$|\Psi\rangle = |\psi\rangle_{1234} \otimes |\psi\rangle_{1'1''} \otimes |\psi\rangle_{2'2''} \otimes |\psi\rangle_{3'3''} \otimes |\psi\rangle_{4'4''}. \tag{14}$$

In order to realize teleportation, Alice performs Bell-state measurements on particles $(1,1'')$, $(2,2'')$, $(3,3'')$ and $(4,4'')$, separately. After the four measurements, all the possible results are

$$
\begin{aligned}
&{}_{44''}\langle\Phi^{\pm}|_{33''}\langle\Phi^{\pm}|_{22''}\langle\Phi^{\pm}|_{11''}\langle\Phi^{\pm}|\Psi\rangle \\
=&\frac{1}{16}(\pm x_1|0001\rangle \pm x_2|0010\rangle \\
&\pm x_3|0100\rangle \pm x_4|1000\rangle)_{1'2'3'4'}, \qquad (15)
\end{aligned}
$$

$$
\begin{aligned}
&{}_{44''}\langle\Psi^{\pm}|_{33'}\langle\Phi^{\pm}|_{22''}\langle\Phi^{\pm}|_{11''}\langle\Phi^{\pm}|\Psi\rangle \\
=&\frac{1}{16}(\pm x_1|0000\rangle \pm x_2|0011\rangle \\
&\pm x_3|0101\rangle \pm x_4|1001\rangle)_{1'2'3'4'}, \qquad (16)
\end{aligned}
$$

$$
\begin{aligned}
&{}_{44''}\langle\Phi^{\pm}|_{33''}\langle\Psi^{\pm}|_{22''}\langle\Phi^{\pm}|_{11''}\langle\Phi^{\pm}|\Psi\rangle \\
=&\frac{1}{16}(\pm x_1|0011\rangle \pm x_2|0000\rangle \\
&\pm x_3|0110\rangle \pm x_4|1010\rangle)_{1'2'3'4'}, \qquad (17)
\end{aligned}
$$

$$
\begin{aligned}
&{}_{44''}\langle\Psi^{\pm}|_{33''}\langle\Psi^{\pm}|_{22''}\langle\Phi^{\pm}|_{11''}\langle\Phi^{\pm}|\Psi\rangle \\
=&\frac{1}{16}(\pm x_1|0010\rangle \pm x_2|0001\rangle \\
&\pm x_3|0111\rangle \pm x_4|1011\rangle)_{1'2'3'4'}, \qquad (18)
\end{aligned}
$$

$$
\begin{aligned}
&{}_{44''}\langle\Phi^{\pm}|_{33''}\langle\Phi^{\pm}|_{22''}\langle\Psi^{\pm}|_{11''}\langle\Phi^{\pm}|\Psi\rangle \\
=&\frac{1}{16}(\pm x_1|0101\rangle \pm x_2|0110\rangle \\
&\pm x_3|0000\rangle \pm x_4|1100\rangle)_{1'2'3'4'}, \qquad (19)
\end{aligned}
$$

$$
\begin{aligned}
&{}_{44''}\langle\Psi^{\pm}|_{33''}\langle\Phi^{\pm}|_{22''}\langle\Psi^{\pm}|_{11''}\langle\Phi^{\pm}|\Psi\rangle \\
=&\frac{1}{16}(\pm x_1|0100\rangle \pm x_2|0111\rangle \\
&\pm x_3|0001\rangle \pm x_4|1101\rangle)_{1'2'3'4'}, \qquad (20)
\end{aligned}
$$

$$
\begin{aligned}
&{}_{44''}\langle\Phi^{\pm}|_{33''}\langle\Psi^{\pm}|_{22''}\langle\Psi^{\pm}|_{11''}\langle\Phi^{\pm}|\Psi\rangle \\
=&\frac{1}{16}(\pm x_1|0111\rangle \pm x_2|0100\rangle \\
&\pm x_3|0010\rangle \pm x_4|1110\rangle)_{1'2'3'4'}, \qquad (21)
\end{aligned}
$$

$$
\begin{aligned}
&{}_{44''}\langle\Psi^{\pm}|_{33''}\langle\Psi^{\pm}|_{22''}\langle\Psi^{\pm}|_{11''}\langle\Phi^{\pm}|\Psi\rangle \\
=&\frac{1}{16}(\pm x_1|0110\rangle \pm x_2|0101\rangle \\
&\pm x_3|0011\rangle \pm x_4|1111\rangle)_{1'2'3'4'}, \qquad (22)
\end{aligned}
$$

$$
\begin{aligned}
&{}_{44''}\langle\Phi^{\pm}|_{33''}\langle\Phi^{\pm}|_{22''}\langle\Phi^{\pm}|_{11''}\langle\Psi^{\pm}|\Psi\rangle \\
=&\frac{1}{16}(\pm x_1|1001\rangle \pm x_2|1010\rangle \\
&\pm x_3|1100\rangle \pm x_4|0000\rangle)_{1'2'3'4'}, \qquad (23)
\end{aligned}
$$

$$
\begin{aligned}
&{}_{44''}\langle\Psi^{\pm}|_{33''}\langle\Phi^{\pm}|_{22''}\langle\Phi^{\pm}|_{11''}\langle\Psi^{\pm}|\Psi\rangle \\
=&\frac{1}{16}(\pm x_1|1000\rangle \pm x_2|1011\rangle \\
&\pm x_3|1101\rangle \pm x_4|0001\rangle)_{1'2'3'4'}, \qquad (24)
\end{aligned}
$$

$$
\begin{aligned}
&{}_{44''}\langle\Phi^{\pm}|_{33''}\langle\Psi^{\pm}|_{22''}\langle\Phi^{\pm}|_{11''}\langle\Psi^{\pm}|\Psi\rangle \\
=&\frac{1}{16}(\pm x_1|1011\rangle \pm x_2|1000\rangle \\
&\pm x_3|1110\rangle \pm x_4|0010\rangle)_{1'2'3'4'}, \qquad (25)
\end{aligned}
$$

$$
\begin{aligned}
&{}_{44''}\langle\Psi^{\pm}|_{33''}\langle\Psi^{\pm}|_{22''}\langle\Phi^{\pm}|_{11''}\langle\Psi^{\pm}|\Psi\rangle \\
=&\frac{1}{16}(\pm x_1|1010\rangle \pm x_2|1001\rangle \\
&\pm x_3|1111\rangle \pm x_4|0011\rangle)_{1'2'3'4'}, \qquad (26)
\end{aligned}
$$

$$
\begin{aligned}
&{}_{44''}\langle\Phi^{\pm}|_{33''}\langle\Phi^{\pm}|_{22''}\langle\Psi^{\pm}|_{11''}\langle\Psi^{\pm}|\Psi\rangle \\
=&\frac{1}{16}(\pm x_1|1101\rangle \pm x_2|1110\rangle \\
&\pm x_3|1000\rangle \pm x_4|0100\rangle)_{1'2'3'4'}, \qquad (27)
\end{aligned}
$$

$$
\begin{aligned}
&{}_{44''}\langle\Psi^{\pm}|_{33''}\langle\Phi^{\pm}|_{22''}\langle\Psi^{\pm}|_{11''}\langle\Psi^{\pm}|\Psi\rangle \\
=&\frac{1}{16}(\pm x_1|1100\rangle \pm x_2|1111\rangle \\
&\pm x_3|1001\rangle \pm x_4|0101\rangle)_{1'2'3'4'}, \qquad (28)
\end{aligned}
$$

$$
\begin{aligned}
&{}_{44''}\langle\Phi^{\pm}|_{33''}\langle\Psi^{\pm}|_{22''}\langle\Psi^{\pm}|_{11''}\langle\Psi^{\pm}|\Psi\rangle \\
=&\frac{1}{16}(\pm x_1|1111\rangle \pm x_2|1100\rangle \\
&\pm x_3|1010\rangle \pm x_4|0110\rangle)_{1'2'3'4'}, \qquad (29)
\end{aligned}
$$

$$
\begin{aligned}
&{}_{44''}\langle\Psi^{\pm}|_{33''}\langle\Psi^{\pm}|_{22''}\langle\Psi^{\pm}|_{11''}\langle\Psi^{\pm}|\Psi\rangle \\
=&\frac{1}{16}(\pm x_1|1110\rangle \pm x_2|1101\rangle \\
&\pm x_3|1011\rangle \pm x_4|0111\rangle)_{1'2'3'4'}, \qquad (30)
\end{aligned}
$$

where $|\Phi^{\pm}\rangle_{kk''}$ and $|\Psi^{\pm}\rangle_{kk''}$ are the Bell states of particles $k$ and $k''$,

$$|\Phi^{\pm}\rangle_{kk''} = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)_{kk''}, \qquad (31)$$

$$|\Psi^{\pm}\rangle_{kk''} = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)_{kk''}, \qquad (32)$$

where $kk'' = 11'$, $22''$, $33''$ and $44''$ respectively. After operations, Alice informs the receiver Bob of her measured results via a classical channel. According to the information, Bob can perform some unitary operations on particles $1'$, $2'$, $3'$ and $4'$ to reconstruct the original state. For instance, if Alice makes Bell measurements $|\Phi^{-}\rangle_{11''}$, $|\Phi^{+}\rangle_{22''}$, $|\Psi^{-}\rangle_{33''}$ and $|\Phi^{+}\rangle_{44''}$,

the state of particles $1'$, $2'$, $3'$ and $4'$ collapses into the following state:

$$\begin{aligned}&{}_{44''}\langle\Phi^+|_{33''}\langle\Psi^-|_{22''}\langle\Phi^+|_{11''}\langle\Phi^-|\,\Psi\rangle\\&=\frac{1}{16}(x_1|0011\rangle-x_2|0000\rangle\\&\quad+x_3|0110\rangle-x_4|1010\rangle)_{1'2'3'4'}.\end{aligned}\tag{33}$$

That is, after four separate measuring, the entanglement among particles 1, 2, 3 and 4 disappears, the new entanglement among particles $1'$, $2'$, $3'$ and $4'$ is set up, and the entanglement swapping happens. According to the measurement result Eq.(33) by Alice, Bob operates a unitary transformation $U_1$

$$U_1=U(\sigma_z)U(\sigma_x),$$

$$U(\sigma_x)=(\sigma_x)_{3'},U(\sigma_z)=(\sigma_z)_{3'}(\sigma_z)_{1'}\tag{34}$$

on Eq.(33), and thus reconstructs entangled state (8). Therefore, Alice successfully sends the unknown state (9) to Bob. It is evident that Bob must operate a relevant unitary transformation against different measurement results of Alice. Tables 1 and 2 provide all the different measurement results of Alice and the relevant unitary transformations $U(\sigma_x)$ and $U(\sigma_z)$ of Bob, respectively.

**Table 1.** Unitary transformations $U(\sigma_x)$ corresponding to the states of particles $1'$, $2'$, $3'$ and $4'$.

| measurement results of Alice | unitary transformations of Bob |
|---|---|
| ${}_{44''}\langle\Phi^\pm|_{33''}\langle\Phi^\pm|_{22''}\langle\Phi^\pm|_{11''}\langle\Phi^\pm|\,\Psi\rangle$ | $I$ |
| ${}_{44''}\langle\Psi^\pm|_{33''}\langle\Phi^\pm|_{22''}\langle\Phi^\pm|_{11''}\langle\Phi^\pm|\,\Psi\rangle$ | $(\sigma_x)_{4'}$ |
| ${}_{44''}\langle\Phi^\pm|_{33''}\langle\Psi^\pm|_{22''}\langle\Phi^\pm|_{11''}\langle\Phi^\pm|\,\Psi\rangle$ | $(\sigma_x)_{3'}$ |
| ${}_{44''}\langle\Psi^\pm|_{33''}\langle\Psi^\pm|_{22''}\langle\Phi^\pm|_{11''}\langle\Phi^\pm|\,\Psi\rangle$ | $(\sigma_x)_{3'}(\sigma_x)_{4'}$ |
| ${}_{44''}\langle\Phi^\pm|_{33''}\langle\Phi^\pm|_{22''}\langle\Psi^\pm|_{11''}\langle\Phi^\pm|\,\Psi\rangle$ | $(\sigma_x)_{2'}$ |
| ${}_{44''}\langle\Psi^\pm|_{33''}\langle\Phi^\pm|_{22''}\langle\Psi^\pm|_{11''}\langle\Phi^\pm|\,\Psi\rangle$ | $(\sigma_x)_{2'}(\sigma_x)_{4'}$ |
| ${}_{44''}\langle\Phi^\pm|_{33''}\langle\Psi^\pm|_{22''}\langle\Psi^\pm|_{11''}\langle\Phi^\pm|\,\Psi\rangle$ | $(\sigma_x)_{2'}(\sigma_x)_{3'}$ |
| ${}_{44''}\langle\Psi^\pm|_{33''}\langle\Psi^\pm|_{22''}\langle\Psi^\pm|_{11''}\langle\Phi^\pm|\,\Psi\rangle$ | $(\sigma_x)_{2'}(\sigma_x)_{3'}(\sigma_x)_{4'}$ |
| ${}_{44''}\langle\Phi^\pm|_{33''}\langle\Phi^\pm|_{22''}\langle\Phi^\pm|_{11''}\langle\Psi^\pm|\,\Psi\rangle$ | $(\sigma_x)_{1'}$ |
| ${}_{44''}\langle\Psi^\pm|_{33''}\langle\Phi^\pm|_{22''}\langle\Phi^\pm|_{11''}\langle\Psi^\pm|\,\Psi\rangle$ | $(\sigma_x)_{1'}(\sigma_x)_{4'}$ |
| ${}_{44''}\langle\Phi^\pm|_{33''}\langle\Psi^\pm|_{22''}\langle\Phi^\pm|_{11''}\langle\Psi^\pm|\,\Psi\rangle$ | $(\sigma_x)_{1'}(\sigma_x)_{3'}$ |
| ${}_{44''}\langle\Psi^\pm|_{33''}\langle\Psi^\pm|_{22''}\langle\Phi^\pm|_{11''}\langle\Psi^\pm|\,\Psi\rangle$ | $(\sigma_x)_{1'}(\sigma_x)_{3'}(\sigma_x)_{4'}$ |
| ${}_{44''}\langle\Phi^\pm|_{33''}\langle\Phi^\pm|_{22''}\langle\Psi^\pm|_{11''}\langle\Psi^\pm|\,\Psi\rangle$ | $(\sigma_x)_{1'}(\sigma_x)_{2'}$ |
| ${}_{44''}\langle\Psi^\pm|_{33''}\langle\Phi^\pm|_{22''}\langle\Psi^\pm|_{11''}\langle\Psi^\pm|\,\Psi\rangle$ | $(\sigma_x)_{1'}(\sigma_x)_{2'}(\sigma_x)_{4'}$ |
| ${}_{44''}\langle\Phi^\pm|_{33''}\langle\Psi^\pm|_{22''}\langle\Psi^\pm|_{11''}\langle\Psi^\pm|\,\Psi\rangle$ | $(\sigma_x)_{1'}(\sigma_x)_{2'}(\sigma_x)_{3'}$ |
| ${}_{44''}\langle\Psi^\pm|_{33''}\langle\Psi^\pm|_{22''}\langle\Psi^\pm|_{11''}\langle\Psi^\pm|\,\Psi\rangle$ | $(\sigma_x)_{1'}(\sigma_x)_{2'}(\sigma_x)_{3'}(\sigma_x)_{4'}$ |

**Table 2.** Unitary transformations $U(\sigma_z)$ corresponding to the states of particles $1'$, $2'$, $3'$ and $4'$.

| signs of $x_i(i=1,2,3,4)$ in states of particles $1',2',3'$ and $4'$ | | | | unitary transformations of Bob |
|---|---|---|---|---|
| $x_1$ | $x_2$ | $x_3$ | $x_4$ | |
| + | + | + | + | $I$ |
| − | + | + | + | $(\sigma_z)_{4'}$ |
| + | − | + | + | $(\sigma_z)_{3'}$ |
| − | − | + | + | $(\sigma_z)_{4'}(\sigma_z)_{3'}$ |
| + | + | − | + | $(\sigma_z)_{2'}$ |
| − | + | − | + | $(\sigma_z)_{4'}(\sigma_z)_{2'}$ |
| + | − | − | + | $(\sigma_z)_{3'}(\sigma_z)_{2'}$ |
| − | − | − | + | $(\sigma_z)_{4'}(\sigma_z)_{3'}(\sigma_z)_{2'}$ |
| + | + | + | − | $(\sigma_z)_{1'}$ |
| − | + | + | − | $(\sigma_z)_{4'}(\sigma_z)_{1'}$ |
| + | − | + | − | $(\sigma_z)_{3'}(\sigma_z)_{1'}$ |
| − | − | + | − | $(\sigma_z)_{4'}(\sigma_z)_{3'}(\sigma_z)_{1'}$ |
| + | + | − | − | $(\sigma_z)_{2'}(\sigma_z)_{1'}$ |
| − | + | − | − | $(\sigma_z)_{4'}(\sigma_z)_{2'}(\sigma_z)_{1'}$ |
| + | − | − | − | $(\sigma_z)_{3'}(\sigma_z)_{2'}(\sigma_z)_{1'}$ |
| − | − | − | − | $(\sigma_z)_{4'}(\sigma_z)_{3'}(\sigma_z)_{2'}(\sigma_z)_{1'}$ |

In conclusion, we have presented a scheme for teleporting an unknown $N$-particle entangled $W$ state via entanglement swapping. In the present scheme, the quantum channel is composed of $N$ maximally entangled particle pairs shared by sender (Alice) and receiver (Bob). As a special case, we have investigated a protocol of teleportation of an unknown four-particle entangled $W$ state in detail. It is shown that, after a sender operates the Bell-state measurement, the original $N$-particle ($N \geq 4$) entangled $W$ state can be reconstructed when the receiver performs a corresponding measurement with unitary transformation.

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