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Quantum fluctuation of mesoscopic distributed parameter circuits*

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Under the Born–von-Karmann periodic boundary condition, we propose a quantization scheme for a non-dissipative distributed parameter circuit (a uniform transmission line). Quantum fluctuations of charge and current in the vacuum state are investigated by adopting the canonical transformation and unitary transformation method. Our results indicate that in distributed parameter circuits, quantum fluctuations, which also have distributed properties, are related to both the circuit parameters and the positions and the mode of signals.

Keywords: mesoscopic distributed parameter circuit, quantization, quantum fluctuation
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1. Introduction

The electronic and magnetic properties of a mesoscopic system have attracted much attention in the light of several experimental observations. In a typical mesoscopic system of nanometre size, the effective distance between inelastic collisions at low temperatures can exceed the sample dimension. In such a situation, the electrons maintain the coherence of the single-particle wave function across the entire sample. Recently many phenomena have been predicted and observed in mesoscopic systems. These phenomena include the normal-state Aharanov–Bohm resistance oscillations and persistent currents in metallic rings pierced by a magnetic field, Coulomb blockade effect in microtunnel junctions.[1−5]

Recently, the quantum effect in a mesoscopic circuit has been widely studied, and some interesting results have been obtained.[6−16] In the previous work, some investigators took the circuits as lumped parameter circuits. However, when the length of circuits and circuit elements is comparable to the work wavelength of signals, the circuits must be considered as distributed parameter circuits.[17,18] In this paper, we study the quantum effects of a uniform non-dissipative mesoscopic distributed parameter circuit. We first present the quantization of distributed parameter circuits, then study the quantum fluctuations of the charge and current of the circuits in the vacuum state. Our results show that the quantum fluctuations have distributed properties.

2. Quantum scheme

Now, we study a uniform non-dissipative mesoscopic distributed parameter circuit (see Fig.1). The classical kinetic equation of a distributed parameter circuit is

$$L\ddot{q}_n = \frac{1}{C}(\, q_{n+1} + q_{n-1} - 2q_n), \tag{1}$$

where L and C are inductance and capacitance of a unit length in a transmission line (per cell), respectively. q_n is the charge that is regarded as a coordinate of the system. The classical Lagrangian of this system is

$$\Xi = \sum_{n=1}^{N} \frac{1}{2} L\dot{q}_n^2 - \sum_{n=1}^{N-1} \frac{1}{2C}(q_n - q_{n+1})^2 - \frac{1}{2C}(q_N - q_1)^2. \tag{2}$$

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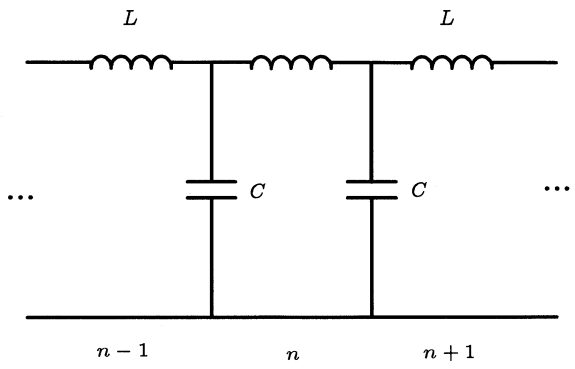


Fig.1. A lumped equivalent circuit of the uniform transmission line.

The last term in the above formula represents the Born–von-Karmann periodic boundary condition. From this Lagrangian, we obtain the conjugate momentum

$$p_n = \frac{\partial \Xi}{\partial \dot{q}_n} = L\dot{q}_n.$$

Then, the Hamiltonian of the system is

$$\begin{aligned} H &= \sum_{n=1}^{N} p_n \dot{q}_n - \Xi \\ &= \sum_{n=1}^{N} \frac{1}{2L} p_n^2 + \sum_{n=1}^{N-1} \frac{1}{2C}(q_n - q_{n+1})^2 \\ &\quad + \frac{1}{2C}(q_N - q_1)^2. \end{aligned} \tag{3}$$

According to the canonical quantization principle, we can quantize the system with operators (q_n, p_n), which satisfy the following commutation relations:

$$\begin{aligned} &[\hat{q}_j, \hat{q}_k] = [\hat{p}_j, \hat{p}_k] = 0, \\ &[\hat{q}_j, \hat{p}_k] = \mathrm{i}\hbar\delta_{jk}. \end{aligned}$$

For the sake of convenience, we delete the superscripts of relative operators in the following discussion such as adopting q_n instead of $\hat{q}_n$.

Equation (3) represents harmonic oscillators that are coupled to each other. In order to diagonalize Eq.(3), we can introduce the following unitary operator U in the coordinate representation:[19]

$$U = \prod_{n=2}^{N} \left(\frac{\omega}{\Omega_n}\right)^{1/4} \int_{-\infty}^{+\infty} \mathrm{d}^N q |uq\rangle\langle q|, \tag{4}$$

where $|q\rangle = |q_1, q_2, \ldots, q_N\rangle$ is the N-mode coordinate eigenstate and

$$\begin{aligned} &\omega^2 = \frac{2}{LC}, \\ &\Omega_n^2 = \omega^2(1 - \cos\varphi_n), \\ &\varphi_n = \frac{2(n-1)}{N}\pi, \quad n = 2, 3, \ldots, N. \end{aligned} \tag{5}$$

The matrix u is

$$u = RS. \tag{6}$$

If N is an even number, the matrix R is

$$R_{\mathrm{e}} = \sqrt{\frac{2}{N}} \begin{pmatrix} \frac{1}{\sqrt{2}} & \cos\varphi_2 & \cdots & \cos\varphi_{N/2} & -\frac{1}{\sqrt{2}} & \sin\varphi_{N/2} & \cdots & \sin\varphi_2 \\ \frac{1}{\sqrt{2}} & \cos 2\varphi_2 & \cdots & \cos 2\varphi_{N/2} & \frac{1}{\sqrt{2}} & \sin 2\varphi_{N/2} & \cdots & \sin 2\varphi_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{1}{\sqrt{2}} & \cos(N-1)\varphi_2 & \cdots & \cos(N-1)\varphi_{N/2} & -\frac{1}{\sqrt{2}} & \sin(N-1)\varphi_{N/2} & \cdots & \sin(N-1)\varphi_2 \\ \frac{1}{\sqrt{2}} & \cos N\varphi_2 & \cdots & \cos N\varphi_{N/2} & \frac{1}{\sqrt{2}} & \sin N\varphi_{N/2} & \cdots & \sin N\varphi_2 \end{pmatrix}. \tag{7}$$

If N is an odd number, the matrix R is

$$R_{\mathrm{o}} = \sqrt{\frac{2}{N}} \begin{pmatrix} \frac{1}{\sqrt{2}} & \cos\varphi_2 & \cdots & \cos\varphi_{\frac{N+1}{2}} & \sin\varphi_{\frac{N+1}{2}} & \cdots & \sin\varphi_2 \\ \frac{1}{\sqrt{2}} & \cos 2\varphi_2 & \cdots & \cos 2\varphi_{\frac{N+1}{2}} & \sin 2\varphi_{\frac{N+1}{2}} & \cdots & \sin 2\varphi_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{1}{\sqrt{2}} & \cos(N-1)\varphi_2 & \cdots & \cos(N-1)\varphi_{\frac{N+1}{2}} & \sin(N-1)\varphi_{\frac{N+1}{2}} & \cdots & \sin(N-1)\varphi_2 \\ \frac{1}{\sqrt{2}} & \cos N\varphi_2 & \cdots & \cos N\varphi_{\frac{N+1}{2}} & \sin N\varphi_{\frac{N+1}{2}} & \cdots & \sin N\varphi_2 \end{pmatrix}. \tag{8}$$

The matrix S is

$$S=\begin{pmatrix} 1 & & & \\ & \frac{1}{\sqrt[4]{1-\cos\varphi_2}} & & \\ & & \frac{1}{\sqrt[4]{1-\cos\varphi_3}} & \\ & & & \ddots \\ & & & \frac{1}{\sqrt[4]{1-\cos\varphi_N}} \end{pmatrix}. \tag{9}$$

According to Ref.[5], we can conveniently show that U is really a unitary operator, that is $UU^{+}=U^{+}U=1\text{and}U^{+}=U^{-1}$. The transformations of the coordinates and momenta are

$$Q_1=UQ_1U^{-1}=\sum_j R_{1j}^{-1}q_j, \tag{10}$$

$$P_1=UP_1U^{-1}=\sum_j R_{1j}^{-1}p_j, \tag{11}$$

$$Q_n=UQ_nU^{-1}=\sum_j \sqrt[4]{1-\cos\varphi_n}R_{nj}^{-1}q_j, \tag{12}$$

$$P_n=UP_nU^{-1}=\sum_j \frac{1}{\sqrt[4]{1-\cos\varphi_n}}R_{nj}^{-1}p_j. \tag{13}$$

We also prove $\det R_{\mathrm{e}}=\det R_{\mathrm{o}}=1$, $\tilde{R}=R^{-1}$. We define the annihilation and creation operators a and a^{+} by the relations

$$Q_n=\sqrt{\frac{\hbar}{2L_n\Omega_n}}(a_n^{+}+a_n),$$

$$P_n=\mathrm{i}\sqrt{\frac{L_n\Omega_n\hbar}{2}}(a_n^{+}-a_n),$$

where

$$L_n=\frac{L}{\sqrt{1-\cos\varphi_n}},$$

$n=2,3,\ldots,N$. Then Eq.(3) can become

$$H'=\frac{1}{2L}P_1^2+\sum_{n=2}^{N}\left(a_n^{+}a_n+\frac{1}{2}\right)\hbar\Omega_n. \tag{14}$$

H' includes N Hamiltonians of independent harmonic oscillators and a free particle. Thus the term $(2L)^{-1}P_1^2$ can be separated off, and we can directly write down the discrete energy level of H'

$$E_n=\sum_{n=2}^{N}\left(m_n+\frac{1}{2}\right)\hbar\Omega_n.$$

The term $(2L)^{-1}P_1^2$ represents the continuous energy part of H, which originates from the total momentum of the system. Further, we have to analyse Ω_n in more detail. Without question, Ω_n represents quantum frequency of a distributed parameter circuit, which is completely determined by the eigenparameter of the circuit. Since n has totally $(n-1)$ values, there should be $(n-1)$ mutually independent normal modes in a transmission line, of which each mode frequency bears no relationship to its position.

Comparing Eqs.(12) and (13), we can see that the operator U causes not only rotation transformation but also squeezing transformation. For example, as the factor $\sqrt[4]{1-\cos\varphi_n}$ appears in Uq_nU^{-1}, its inverse $1\sqrt[4]{1-\cos\varphi_n}$ appears in Up_nU^{-1}.

3. Quantum fluctuation

From Eqs.(10), (11) and (7) or (8), we obtain the mean values and mean-square values of the Q_j and P_j in the vacuum state. They are, respectively,

$$\overline{Q_j}=0,$$
$$\overline{P_j}=0,\quad (j=2,3,\ldots,N)$$
$$\overline{Q_j^2}=\frac{\hbar}{2L_j\Omega_j},$$
$$\overline{P_j^2}=\frac{L_j\Omega_j\hbar}{2},\quad (j=2,3,\ldots,N)$$

and

$$\overline{P_1}=0,\quad \overline{P_1^2}=0.$$

From the above equations, the quantum fluctuations of charge and current for the transmission line in the vacuum state are

$$\overline{[\Delta(q_j-q_{j+1})]^2}=\frac{1}{2C}\sum_{n=2}^{N}(u_{jn}-u_{j+1n})^2g_n, \tag{15}$$

$$\overline{(\Delta p_j)^2}=\sum_{n=2}^{N}\tilde{u}_{jn}^{-2}h_n,\quad j=2,3,\ldots,N.$$

Here

$$g_n=\frac{\hbar}{2L_n\Omega_n},\quad h_n=\frac{L_n\Omega_n\hbar}{2}.$$

From the results we find that there exist quantum fluctuations for both charge and current in each cell. The quantum fluctuations of the charge and current in each cell are interrelated. Notice that, in fact, the subscript j of Eqs.(15) and (16) depicts the position of each

cell in the circuit. Combining this with Eqs.(5) and (6), we can easily find that the quantum fluctuations of distributed parameter also have distributed qualities. That is, the quantum fluctuations depend on both the circuit parameters and the positions. In addition, the subscript of formula n depicts a different normal mode in distributed parameter circuits. So, the quantum fluctuations of charges and currents in multimode transmission are a superposition of each quantum fluctuation under every single mode transmission. Thus, we can easily find that less noise is produced under single-mode transmission.

4. Conclusion

In this paper, on the basis of the equations of motion for a transmission line, the quantum fluctuations of charge and current in a circuit in the vacuum state are investigated. It is found that the quantum fluctuations in distributed circuits also have distributed qualities in that it is determined not only by the circuit parameters but also by the position and the mode of signals.

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