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Output power spectrum of a single-mode laser driven by coloured pump and quantum noises with coloured correlation*

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By using the linear approximation method, the output power spectrum is calculated for a single-mode laser driven by coloured pump and quantum noises with coloured correlation. We have observed that the configuration of the output power spectrum is complicated: that is, it can be of single peak, two peaks or three peaks. The configurations of the power spectrum can be transformed from one into another by changing the cross-correlation time, the cross-correlation coefficient between the two noises, and pump noise intensity.

Keywords: linear approximation method, power spectrum, coloured correlation noises
PACC: 4250, 0540, 4260K

1. Introduction

Recently, the study of the influences of noises on laser systems has attracted considerable attention.[1−9] There are two kinds of noise: one is the quantum noise due to spontaneous emission, generally modelled as an additive noise; and the other is the pump noise due to external sources, generally modelled as a multiplicative noise. Since Fulinski and Telejke[10] proposed that two different noises can cross-correlated under certain conditions, Cao and Wu[11,12] have studied the one-dimensional stochastic systems in detail; Zhu *et al*[13−15] and Han *et al*,[16] taking into account the cross-correlation between the quantum and the pump noises, have found that the normalized intensity fluctuations of laser systems can be influenced by the cross-correlation between both the noises. Zhou *et al*[17] found by using a gain-noise model that the power spectrum of a single-mode laser driven by two white noises with a coloured cross-correlation can transform from single extremum into three extrema. Anthony *et al*[1] experimentally studied the power spectra of a single longitudinal and transverse mode ring dye laser that revealed the critical slowing down of the laser near the threshold. However, the power spectrum of a single-mode laser driven by both coloured pump and quantum noises with a coloured cross-correlation, whose self-correlation time and cross-correlation time are all different, is seldom reported.

Due to the nonlinear term in the laser Langevin equation, it is very difficult to solve the problem. However, the linear approximation is an effective method under the condition of the normalized intensity fluctuation $C(0) \ll 1$ in the laser system.[8] In this paper, by using the linear approximation, the output power spectrum is calculated in the frame of a gain-noise model for a single-mode laser driven by both the coloured pump and quantum noises with coloured correlation, and the influences of some parameters on the power spectrum are analysed. We found that there are some complex configurations in the power spectrum, i.e. it can be a single-peak, a two-peak or a three-peak spectrum. By changing the cross-correlation time, the cross-correlation intensity between the two noises, and

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the pump noise intensity, the power spectrum configurations can be transformed from one to another.

2. The output power spectrum of the gain-noise model for a single-mode laser

The intensity equation in a gain-noise model for a single-mode laser driven by the coloured cross-correlated pump and quantum noises, in which each is coloured, is given by

$$\frac{\mathrm{d}I}{\mathrm{d}t} = -2KI + \frac{2KI}{1+\beta I} + D + \frac{2I}{1+\beta I}\xi(t) + 2\sqrt{I}\eta(t), \quad (1)$$

where the pump noise $\xi(t)$ and the quantum noise $\eta(t)$ are correlated in the following form:

$$\begin{aligned}
&\langle \xi(t) \rangle = \langle \eta(t) \rangle = 0, \\
&\langle \xi(t)\xi(t') \rangle = \frac{Q}{2\tau_1} \exp\left(- \frac{|t-t'|}{\tau_1} \right), \\
&\langle \eta(t)\eta(t') \rangle = \frac{D}{2\tau_2} \exp\left(- \frac{|t-t'|}{\tau_2} \right), \\
&\langle \xi(t)\eta(t') \rangle = \langle \eta(t)\xi(t') \rangle = \frac{\lambda\sqrt{QD}}{2\tau_3} \exp\left(- \frac{|t-t'|}{\tau_3} \right), \\
&\quad (-1 \le \lambda \le 1). \qquad (2)
\end{aligned}$$

In Eqs.(1) and (2), I is the laser intensity; K represents the loss coefficient; $\beta = \tilde{A}/\Gamma$, $\tilde{A}$ and Γ represent the self-saturation and gain coefficients, respectively; Q and D are the intensities of the pump noise and the quantum noise, respectively; λ is the noise correlation coefficient between the two noises; and τ_1, τ_2, and τ_3 are the self-correlation and cross-correlation times of the pump noise and quantum noise, respectively.

Let

$$I = I_0 + \delta(t), \quad (3)$$

where $I_0 = (\Gamma - K)/\beta K$ is the deterministic steady-state intensity, and $\delta(t)$ is the perturbational term. We linearize Eq.(1) around the deterministic steady-state intensity I_0, thus the linear equation of the laser intensity is found to be

$$\frac{\mathrm{d}\delta(t)}{\mathrm{d}t} = -\gamma\delta(t) + D + \Delta\,\xi(t) + 2\sqrt{I_0}\eta(t), \quad (4)$$

where the attenuation coefficient $\gamma = 2K(\Gamma - K)/\Gamma$, $\Delta = 2I_0/(1+\beta I_0)$. Integrating Eq.(4), we have

$$\begin{aligned}
\delta(t) =& \exp(-\gamma t)\left\{ \frac{D}{\gamma}[\exp(\gamma t) - 1] \right. \\
&+ \Delta \int_0^t \exp(\gamma\, u)\xi(u)\mathrm{d}u \\
&\left. + 2\sqrt{I_0} \int_0^t \exp(\gamma u)\eta(u)\mathrm{d}u \right\}. \qquad (5)
\end{aligned}$$

The normalized intensity correlation function is defined as $C(t) = \lim_{t'\to\infty}\{[\langle I(t'+t)I(t')\rangle - \langle I(t')\rangle^2]/\langle I(t')\rangle^2\}$. Combining Eqs.(3) and (5), we obtain the normalized intensity correlation function

$$\begin{aligned}
C(t) =& \frac{1}{I_0^2}\left\{ \frac{Q\ \Delta^2}{2} \frac{\tau_1 \mathrm{e}^{-\frac{|t|}{\tau_1}} - \gamma^{-1}\mathrm{e}^{-\gamma|t|}}{\gamma^2\tau_1^2 - 1} \right. \\
&+ 2I_0 D \frac{\tau_2 \mathrm{e}^{-\frac{|t|}{\tau_2}} - \gamma^{-1}\mathrm{e}^{-\gamma|t|}}{\gamma^2\tau_2^2 - 1} \\
&\left. + 2\lambda\Delta\sqrt{I_0 DQ} \frac{\tau_3 \mathrm{e}^{-\frac{|t|}{\tau_3}} - \gamma^{-1}\mathrm{e}^{-\gamma|t|}}{\gamma^2\tau_3^2 - 1} \right\}. \quad (6)
\end{aligned}$$

It is obvious that there are four time scales in Eq.(6). Transforming the normalized intensity correlation function $C(t)$ into the power spectrum by Fourier transform, we obtain

$$\begin{aligned}
P =& \frac{1}{I_0^2} \cdot \frac{1}{\omega^2 + \gamma^2} \\
&\times \left[\frac{Q\,\Delta^2}{\omega^2\tau_1^2 + 1} + \frac{4\,I_0\,D}{\omega^2\tau_2^2 + 1} + \frac{4\,\lambda\,\Delta\sqrt{I_0 Q\,D}}{\omega^2\tau_3^2 + 1} \right]. \quad (7)
\end{aligned}$$

3. Discussion

According to the expression of the power spectrum, Eq.(7), the configuration of the power spectrum can be analysed. We find that the curve of power spectrum P versus ω has only a peak when $\lambda \ge 0$ (i.e. when the cross-correlation is positive or no cross-correction). The maximum is located at $\omega = 0$. When $\lambda < 0$, the P versus ω curve has multi-extremum, and the conditions of the extremum are

$$A\,y^5 + By^4 + Cy^3 + Ey^2 + Fy + G = 0, \quad (8)$$

where

$$\begin{aligned}
y =& \omega^2, \\
A =& 2[Q\Delta^2\tau_2^2\tau_3^2 + 4I_0 D\tau_3^2\tau_1^2 + 4\lambda\Delta\sqrt{I_0 DQ}\,]\,\tau_1^2\tau_2^2\tau_3^2, \\
B =& 4\tau_1^2\tau_2^2\tau_3^2[Q\Delta^2(\tau_2^2 + \tau_3^2) + 4I_0 D(\tau_3^2 + \tau_1^2) \\
&+ 4\lambda\Delta\sqrt{I_0 DQ}\,(\tau_1^2 + \tau_2^2)] \\
&+ (\gamma^2\tau_1^2 + 1)\tau_2^4\tau_3^4 Q\Delta^2 + (\lambda^2\tau_2^2 + 1)\tau_3^4\tau_1^4 4I_0 D \\
&+ (\lambda^2\tau_3^2 + 1)\tau_1^4\tau_2^4 4\lambda\Delta\sqrt{I_0 DQ},
\end{aligned}$$

$$
\begin{aligned}
C =& 2\{\tau_1^2(\tau_2^4 + 4\tau_2^2\tau_3^2 + \tau_3^4)Q\Delta^2 \\
&+ \tau_2^2(\tau_3^4 + 4\tau_3^2\tau_1^2 + \tau_1^4)4I_0D \\
&+ \tau_3^2(\tau_1^4 + 4\tau_1^2\tau_2^2 + \tau_2^4)4\lambda\Delta\sqrt{I_0DQ}(\gamma^2\tau_1^2 + 1) \\
&\times (\tau_2^2 + \tau_3^2)\tau_2^2\tau_3^2Q\Delta^2 \\
&+ (\gamma^2\tau_2^2 + 1)(\tau_3^2 + \tau_1^2)\tau_3^2\tau_1^2 4I_0D \\
&+ (\gamma^2\tau_3^2 + 1)(\tau_1^2 + \tau_1^2)\tau_1^2\tau_2^2 4\lambda\Delta\sqrt{I_0DQ}, \\
E =& 4[\tau_1^2(\tau_2^2 + \tau_3^2)Q\Delta^2 + \tau_2^2(\tau_3^2 + \tau_1^2)4I_0D \\
&+ \tau_3^2(\tau_1^2 + \tau_2^2)4\lambda\Delta\sqrt{I_0DQ}] \\
&+ (\gamma^2\tau_1^2 + 1)(\tau_2^4 + 4\tau_2^2\tau_3^2 + \tau_3^4)Q\Delta^2 \\
&+ (\gamma^2\tau_2^2 + 1)(\tau_3^4 + 4\tau_3^2\tau_1^2 + \tau_1^4)4I_0D \\
&+ (\gamma^2\tau_3^2 + 1)(\tau_1^4 + 4\tau_1^2\tau_2^2 + \tau_2^4)4\lambda\Delta\sqrt{I_0DQ}, \\
F =& 2\{\tau_1^2Q\Delta^2 + \tau_2^2 4I_0D + \tau_3^2 4\lambda\Delta\sqrt{I_0DQ} \\
&+ (\gamma^2\tau_1^2 + 1)(\tau_2^2 + \tau_3^2)Q\Delta^2 \\
&+ (\gamma^2\tau_2^2 + 1)(\tau_3^2 + \tau_1^2)4I_0D \\
&+ (\gamma^2\tau_3^2 + 1)(\tau_1^2 + \tau_2^2)4\lambda\Delta\sqrt{I_0DQ}, \\
G =& (\gamma^2\tau_1^2 + 1)Q\Delta^2 + (\gamma^2\tau_2^2 + 1)4I_0D \\
&+ (\gamma^2\tau_3^2 + 1)4\lambda\Delta\sqrt{I_0DQ}.
\end{aligned}
$$

Equation (8) is so complicated that it is impossible to obtain the analytic expression. For simplicity, by numerical calculation, we discuss only the influences on the P versus ω curve from the cross-correlation time τ_3 and the cross-correlation coefficient λ between the two noises, and the pump noise intensity Q, separately.

3.1. The influence of the cross-correlation time τ_3 between the two noises

The P versus ω curve is plotted as a function of τ_3 in Fig.1(a), from which we can see that, as τ_3 increases from 1.0 to 2.0, the profile of the power spectrum transforms from a three-peak into a two-peak one. When $\tau_3 = 1.0$, there are three peaks on the P versus ω curve, the central peak is sharp and located at $\omega = 0$, and the two peaks beside it are symmetrical about $\omega = 0$, the heights of them being smaller than the central one. With increasing τ_3, the heights of the two side peaks increase, while that of the central peak remains unchanged; when τ_3 increases to $\tau_3 = 2.0$, at $\omega = 0$ is the point of minimum, and the power spectrum is transformed into a two-peak one. Figure 1(b) shows the two sections of Fig.1(a) at $\tau_3 = 1.0$ and $\tau_3 = 2.0$, which exhibit the three-peak and two-peak profiles on the P versus ω curve more distinctly than Fig.1(a); at $\omega = 0$, the central maximum of the P versus ω curve changes into a minimum, and the value of the maximum at $\tau_3 = 1.0$ is equal to that of the minimum at $\tau_3 = 2.0$, but the values of the two side peaks at $\tau_3 = 1.0$ are far smaller than those at $\tau_3 = 2.0$.

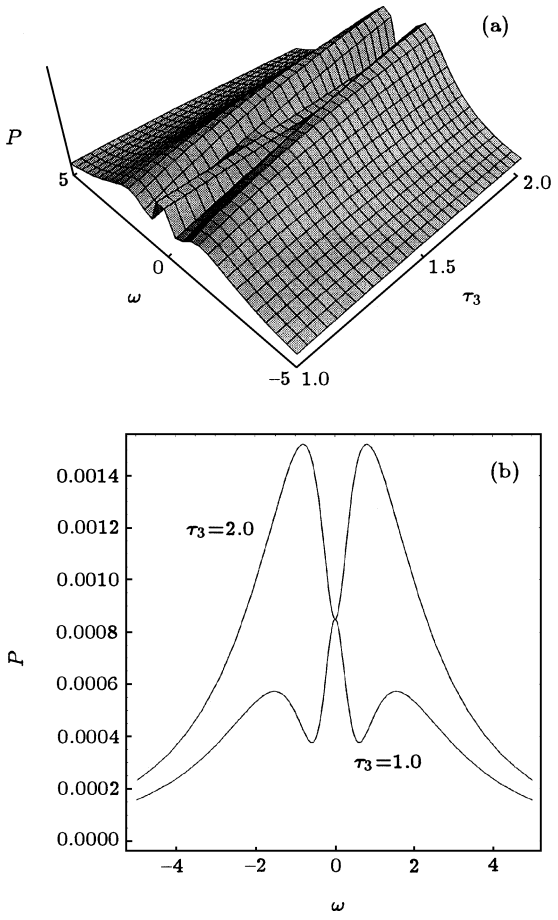


Fig.1. Output power P versus the frequency ω at different τ_3. I_0=2.0, D=0.5, Q=0.5, λ=−0.8, γ=10, β=1.0, τ_1=0.5, τ_2=2.0.

3.2. The influence of the cross-correlation coefficient λ between the two noises

The P versus ω curve is plotted as a function of the correlation coefficient λ in Fig.2(a), from which we can see that there is a single-peak on the P versus ω curve when the negative cross-correlation λ is feeble. As λ changes from 0 to -1, the profile of the power spectrum transforms from a single-peak into a three-peak one, the height of the central peak becomes small, and the side peaks appear. Figure 2(b) shows the sections in Fig.2 (a) at $\lambda = 0$ and $\lambda = -0.9$. At $\lambda = 0$ (i.e. no cross-correlation between the two noises), there is one peak on the P versus ω curve,

located at $\omega = 0$. At $\lambda = -0.9$, there are three peaks on the curve, with the central peak located at $\omega = 0$, the side peaks are symmetrical about $\omega = 0$, and the values of three maxima are almost equal. We also find that the values of the maxima at $\lambda = -0.9$ are far smaller than the value of the maximum at $\lambda = 0$. However, $\lambda = -0.65$ is the critical cross-correlation coefficient at which the profile of power spectrum starts to transform from a single-peak into a three-peak one.

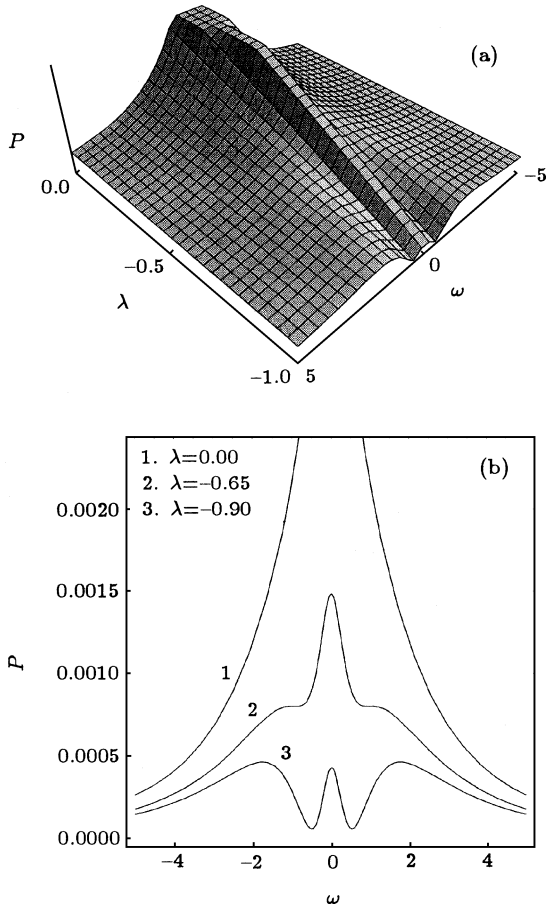


Fig.2. Output power P versus the frequency ω at different λ. I_0=2.0, D=0.5, Q=0.5, γ=10, β=1.0, τ_1=0.5, τ_2=2.0, τ_3=1.0.

3.3. The influence of the pump noise intensity Q

The P versus ω curve is plotted as a function of Q in Fig.3, from which we can see that there is a sharp single peak on the curve at $\omega = 0$ when $Q = 0$ (i.e. no pump noise). As Q increases from 0 to 1, the value of the central maximum reduces quickly, the side peaks appear (which are symmetrical about $\omega = 0$), the profile of the P versus ω curve transforms from single-peak into a three-peak one. We also find that the heights of the three peaks are almost equal when Q is large.

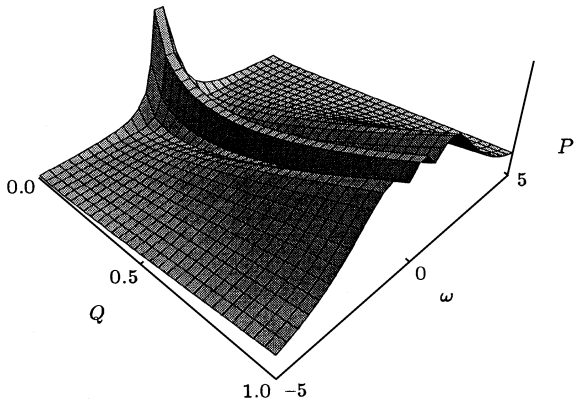


Fig.3. Output power P versus the frequency ω at different Q. I_0=2.0, D=0.5, $\lambda = -0.8$, γ=10, β=1.0, τ_1=0.5, τ_2=2.0, τ_3=1.0

4. Conclusion

In the gain-noise model for the single-mode laser system driven by coloured pump and quantum noises with coloured correlation, the configuration of the output power spectrum is complicated: it can be of single peak, two peaks or three peaks. The profile of the power spectrum can transform directly from single-peak into a three-peak one by changing the pump noise intensity Q and the cross-correlation coefficient λ, and can transform from two-peak into a three-peak one by changing the cross-correlation time τ_3. The reason for this is that there are four time scales in the normalized intensity correlation function, Eq.(6), and the power spectrum Eq.(7) corresponding to Eq.(6) is the sum of three terms, in which each term is the product of two Lorentzian functions with a maximum. Adding together the three terms makes the power spectrum become complicated, i.e. the configuration of power spectrum is determined by the "colour" and the cross-correlation between the two noises stated above.

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