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Measurement of the decoherence of a mesoscopic superposition of motional states of a trapped ion*

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We propose a scheme to observe the decoherence of a mesoscopic superposition of two coherent states in the motion of a trapped ion. In the scheme the ion is excited by two perpendicular lasers tuned to the ion transition. The decoherence is revealed by the decrease of the correlation between two successive measurements of the internal state of the ion after relevant laser–ion interaction.

Keywords: decoherence, mesoscopic superposition, trapped ion
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The superposition principle, which allows a particle to be in a superposition of different states, plays a fundamental role in treating microscopic systems. However, such a superposition state does not exist in the macrocosm, as illustrated by the Schrödinger cat paradox.[1] In this famous hypothetical experiment, the life of the cat is dependent upon whether the radioactive atom decays or not. Thus the cat is in the superposition of being dead and alive. This situation is, of course, opposed to our perception of the real world, for we only observe a live or dead cat.

Recently, it has been shown that the dissipation is responsible for the absence of macroscopic coherence.[2] There exists a coupling between the observed system and outside world reservoir, which changes the quantum superposition into a classical mixture. The decay time is given by the energy dissipation time divided by a dimensionless number characterizing the distance between the two parts in the superposition. Therefore, superpositions of macroscopically distinguishable states would decay too fast to be observed. However, it is possible to realize the Schrödinger cat like states in mesoscopic systems, which have observable decoherence time scales. Creating such states and then studying their decoherence would provide not only a test for quantum measurement theory, but also a way to determine the delimitation between the microcosm and the macrocosm. Numerous schemes have been proposed to generate Schrödinger cat states for an electromagnetic field in the cavity QED[3–5] and for the motion in an ion trap.[6,7] Recently, cat states in the forms of both cavity field[8] and ion motion[9] have been observed. Furthermore, a scheme for monitoring the decoherence of cat states of the radiation field has been proposed[10] and realized.[8] The scheme is based on two-atom correlation measurements performed on circular Rydberg atoms interacting dispersively with the cavity field. We here propose a scheme to measure the decoherence of the cat states of the ion motion. Our scheme is based on the correlation between two successive measurements of the internal state of the ion after the respective laser-assisted Raman couplings between the internal and external states of the ion.

Suppose that a two-level ion is trapped in a two-dimensional (2D) anisotropic harmonic potential. The ion is driven by two laser beams, which are tuned to the ion transition frequency and propagate along the x and y directions in the trap plane, respectively. The Hamiltonian for such a system is

$$H = \nu_x a^+ a + \nu_y b^+ b + \frac{1}{2}\omega_0(|e\rangle\langle e| - |g\rangle\langle g|) + [\lambda E^-(x,y,t)|g\rangle\langle e| + \text{H.c.}], \quad (1)$$

where a^+ (b^+) and a (b) are the creation and annihi-

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lation operators for the vibrational motion of the ion in the x (y) direction, $|e\rangle$ and $|g\rangle$ are the internally excited and ground states for the two-level ion, ν_x and ν_y are the trap frequencies in the x and y directions, ω_0 and λ are the transition frequency and coupling strength between the states $|e\rangle$ and $|g\rangle$, respectively. $E^-(x,y,t)$ is the negative frequency part of the classical driving fields

$$E^-(x,y,t) = E_x e^{i(\omega_0 t - k_0 x + \phi_x)} + E_y e^{i(\omega_0 t - k_0 y + \phi_y)}, \tag{2}$$

where E_j and ϕ_j ($j = x, y$) indicate the amplitude and phase of the relevant driving field, and k_0 is the wave vector. In terms of the creation and annihilation operators the position operators x and y can be expressed as $x = \sqrt{1/(2\nu_x M)}(a + a^+)$, $y = \sqrt{1/(2\nu_y M)}(b + b^+)$ with M being the mass of the trapped ion.

In the resolved sideband limit the ion–laser interaction Hamiltonian can be described by the nonlinear Jaynes–Cummings model.[11] In the interaction picture the Hamiltonian (1) can be written as

$$H_i = \sum_{k=0}^{\infty} \left[\Omega_x e^{-i\phi_x} e^{-\eta_x^2/2} \frac{(i\eta_x)^{2k}}{(k!)^2} a^{+k} a^k + \Omega_y e^{-i\phi_y} e^{-\eta_y^2/2} \frac{(i\eta_y)^{2k}}{(k!)^2} b^{+k} b^k \right] |e\rangle\langle g| + \text{H.c.}, \tag{3}$$

where $\Omega_j = \lambda E_j$ is the Rabi frequency of the relevant driving field, and $\eta_j = \sqrt{k_0^2/2M\nu_j}$ is the Lamb-Dicke parameter. We consider the case where the ion is confined in a 2D highly anisotropic trap with $\nu_x \ll \nu_y$, i.e. the ion is tightly bound in the y direction. In the Lamb–Dicke regime, i.e. $\eta_y \ll \eta_x \ll 1$, we can expand Eq.(3) up to the second order in η_x and zero order in η_y.[12] Then we have

$$H_i = [\Omega_x e^{-i\phi_x}(1 - \eta_x^2 a^+ a) + \Omega_y e^{-i\phi_y}] |e\rangle\langle g| + \text{H.c.} \tag{4}$$

With the choice $\Omega_x = \Omega_y = \Omega$ and $\phi_x = \pi$, $\phi_y = 0$, we obtain

$$H_i = g a^+ a(|e\rangle\langle g| + |g\rangle\langle e|), \tag{5}$$

where $g = \Omega\eta_x^2$. This Hamiltonian describes the Raman coupling model,[13] and can be rewritten as

$$H_i = g a^+ a(|+\rangle\langle +| - |-\rangle\langle -|), \tag{6}$$

where

$$|+\rangle = \frac{1}{\sqrt{2}}(|g\rangle + |e\rangle),$$
$$|-\rangle = \frac{1}{\sqrt{2}}(|g\rangle - |e\rangle). \tag{7}$$

We assume the ion is initially in the ground state $|g\rangle$, which can be expressed in terms of $|+\rangle$ and $|-\rangle$

$$|g\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle). \tag{8}$$

Suppose that the vibrational motion is initially in the coherent state $|\alpha\rangle$. After an interaction time τ we obtain

$$|\psi_1(\tau)\rangle = \frac{1}{\sqrt{2}}[|\alpha e^{-ig\tau}\rangle|+\rangle + |\alpha e^{ig\tau}\rangle|-\rangle]. \tag{9}$$

Thereby, the motional coherent state is dephased by $-g\tau$ or $g\tau$, depending on whether the ion is in the state $|+\rangle$ or $|-\rangle$. We choose the interaction time τ appropriately so that $g\tau = \pi/2$. Then we have

$$\begin{aligned} |\psi_1(\tau)\rangle =& \frac{1}{\sqrt{2}}[|-i\alpha\rangle|+\rangle + |i\alpha\rangle|-\rangle] \\ =& \frac{1}{2}[(|i\alpha\rangle + |-i\alpha\rangle)|g\rangle \\ & - (|i\alpha\rangle - |-i\alpha\rangle)|e\rangle]. \end{aligned} \tag{10}$$

We now perform a measurement of the internal state on the ion. The detection of the state $|g\rangle$ or $|e\rangle$ leaves the vibrational motion in the state

$$|\psi\rangle_m = \frac{1}{N_1}(|i\alpha\rangle + e^{i\varphi_1}|-i\alpha\rangle), \tag{11}$$

where $N_1 = \sqrt{2[1 + \cos\varphi_1 e^{-2|\alpha|^2}]}$, and $\varphi_1 = 0$ or $\varphi_1 = \pi$, corresponding to the detection of $|g\rangle$ or $|e\rangle$, respectively. In the case of $|\alpha|^2 \gg 1$, the coherent state $|i\alpha\rangle$ is orthogonal to $|-i\alpha\rangle$, and thus $N_1 = \frac{1}{\sqrt{2}}$.

If we detect the ion in the excited state, we then excite the ion only with the laser in the y direction tuned to the ion transition frequency. In this case the Hamiltonian for this system is

$$H_y = \Omega_y e^{-i\phi_y}|e\rangle\langle g| + \text{H.c.} \tag{12}$$

We choose the interaction time t carefully so that $\Omega_y t = \pi/2$. Then the state $|e\rangle$ flips back to $|g\rangle$. On the other hand, if we detect the ion in the ground state this step is unnecessary.

We now drive the ion, again with the two laser beams propagating respectively along the x and y directions. At this stage we neglect the dissipation effects. Then the initial state in this cycle is

$$|\psi_2(0)\rangle = \frac{1}{2}(|i\alpha\rangle + e^{i\varphi_1}|-i\alpha\rangle)(|+\rangle + |-\rangle). \tag{13}$$

We also choose the ion–laser interaction time in such a way that the vibrational motion will be dephased by

-$\pi/2$ or $\pi/2$ conditional on the ion being in the state $|+\rangle$ or $|-\rangle$. This leads to

$$\begin{aligned}|\psi_2(\tau)\rangle =&\frac{1}{2}(|\alpha\rangle|+\rangle + |-\alpha\rangle|-\rangle \\ &+ \mathrm{e}^{\mathrm{i}\varphi_1}|-\alpha\rangle|+\rangle + \mathrm{e}^{\mathrm{i}\varphi_1}|\alpha\rangle|-\rangle) \\ =&\frac{1}{2\sqrt{2}}[|g\rangle(1+\mathrm{e}^{\mathrm{i}\varphi_1})(|\alpha\rangle + |-\alpha\rangle) \\ &+ |e\rangle(1-\mathrm{e}^{\mathrm{i}\varphi_1})(|\alpha\rangle - |-\alpha\rangle)]. \end{aligned} \tag{14}$$

Now we again measure the internal state of the ion. Then the probability of finding the ion in the ground state $|g\rangle$ or excited state $|e\rangle$ respectively is

$$\begin{aligned}P_g^{(2)} &= \frac{1}{2}(1+\cos\varphi_1), \\ P_{\mathrm{e}}^{(2)} &= \frac{1}{2}(1-\cos\varphi_1). \end{aligned} \tag{15}$$

From the expressions, it can be easily seen that this measurement should be the same as the previous one, which means a complete correlation between the two measurements. The correlation results from the presence of two paths through which we can obtain a definite pair of outcomes with the vibrational motion finally in a certain state. For example, the pair of outcomes $\{g,g\}$ with the ion motion finally in the coherent state $|\alpha\rangle$ can be obtained via the two ways:

$$|g\rangle|\alpha\rangle \to |+\rangle|-\mathrm{i}\alpha\rangle \to |g\rangle|-\mathrm{i}\alpha\rangle \to |-\rangle|\alpha\rangle \to |g\rangle|\alpha\rangle, \tag{16}$$

$$|g\rangle|\alpha\rangle \to |-\rangle|\mathrm{i}\alpha\rangle \to |g\rangle|\mathrm{i}\alpha\rangle \to |+\rangle|\alpha\rangle \to |g\rangle|\alpha\rangle. \tag{17}$$

These two paths are indistinguishable, which leads to the interference.

We now include the effect of the dissipation on the superposition of the two coherent states. We assume the decay rate for the motion is $\gamma = 1/t_m$, where t_m is the damping time of the vibrational energy. After the cat state has been generated it should evolve toward a classical mixture. The density operator describing such an evolution process is given by[10]

$$\begin{aligned}\rho_m(t) =&\frac{1}{2}[|\mathrm{i}\alpha\mathrm{e}^{-\gamma t/2}\rangle\langle\mathrm{i}\alpha\mathrm{e}^{-\gamma t/2}| \\ &+ |-\mathrm{i}\alpha\mathrm{e}^{-\gamma t/2}\rangle\langle-\mathrm{i}\alpha\mathrm{e}^{-\gamma t/2}| \\ &\times \mathrm{e}^{-2|\alpha|^2(1-\mathrm{e}^{-\gamma t})}(\mathrm{e}^{\mathrm{i}\varphi_1}|-\mathrm{i}\alpha\mathrm{e}^{-\gamma t/2}\rangle\langle\mathrm{i}\alpha\mathrm{e}^{-\gamma t/2}| \\ &+ \mathrm{e}^{-\mathrm{i}\varphi_1}|\mathrm{i}\alpha\mathrm{e}^{-\gamma t/2}\rangle\langle-\mathrm{i}\alpha\mathrm{e}^{-\gamma t/2}|)]. \end{aligned} \tag{18}$$

In the case $\gamma t \ll 1$, $\rho_m(t)$ can be simplified into

$$\begin{aligned}\rho_m(t) =&\frac{1}{2}[|\mathrm{i}\alpha\rangle\langle\mathrm{i}\alpha| + |-\mathrm{i}\alpha\rangle\langle-\mathrm{i}\alpha| \\ &+ \mathrm{e}^{-2|\alpha|^2\gamma t}(\mathrm{e}^{\mathrm{i}\varphi_1}|-\mathrm{i}\alpha\rangle\langle\mathrm{i}\alpha| \\ &+ \mathrm{e}^{-\mathrm{i}\varphi_1}|\mathrm{i}\alpha\rangle\langle-\mathrm{i}\alpha|)]. \end{aligned} \tag{19}$$

Thus the coherent superposition decays within a time scale $t_m/(2|\alpha|^2)$.

We assume the time interval between the two laser-assisted Raman coupling is T. Suppose the ion-laser interaction time τ satisfies the conditions $\tau \ll t_m/(2|\alpha|^2)$ and $\tau \ll t_{\mathrm{r}}$, with t_{r} being the radiative lifetime of the two-level ion. Then the decoherence during the ion–laser interaction can be neglected. Just before the second Raman coupling occurs the density operator for the system is

$$\rho(0) = \rho_m(T) \otimes \frac{1}{2}(|+\rangle + |-\rangle)(\langle+| + \langle-|). \tag{20}$$

The Raman coupling leads to

$$\begin{aligned}\rho(\tau) =&\frac{1}{2}[\mathrm{e}^{-\mathrm{i}\frac{\pi}{2}a^+a}\rho_m(T)\mathrm{e}^{\mathrm{i}\frac{\pi}{2}a^+a}|+\rangle\langle+| \\ &+ \mathrm{e}^{\mathrm{i}\frac{\pi}{2}a^+a}\rho_m(T)\mathrm{e}^{-\mathrm{i}\frac{\pi}{2}a^+a}|-\rangle\langle-| \\ &+ \mathrm{e}^{-\mathrm{i}\frac{\pi}{2}a^+a}\rho_m(T)\mathrm{e}^{-\mathrm{i}\frac{\pi}{2}a^+a}|+\rangle\langle-| \\ &+ \mathrm{e}^{\mathrm{i}\frac{\pi}{2}a^+a}\rho_m(T)\mathrm{e}^{\mathrm{i}\frac{\pi}{2}a^+a}|-\rangle\langle+|]. \end{aligned} \tag{21}$$

Thus the probability of detecting the ion in the state $|g\rangle$ or $|e\rangle$ is

$$P_{g,e}^{(2)} = \frac{1}{2}(1 + \mathrm{Re}\{\mathrm{Tr}[\mathrm{e}^{-\mathrm{i}\pi a^+a}\rho_m(T)\}). \tag{22}$$

Substituting Eq.(18) into Eq.(22) we obtain

$$P_{g,e}^{(2)} = \frac{1}{2}\left[1 \pm \frac{\mathrm{e}^{-2|\alpha|^2\mathrm{e}^{-\gamma T}} + \cos\varphi_1\mathrm{e}^{-2|\alpha|^2(1-\mathrm{e}^{-\gamma T})}}{1+\cos\varphi_1\mathrm{e}^{-2|\alpha|^2}}\right]. \tag{23}$$

The expression indicates how the coherence varies with the time T. During the time $0 < T < t_m/(2|\alpha|^2)$ the superposition decays rapidly. In the case $t_m \gg T \gg t_m/(2|\alpha|^2)$, we have $P_g^{(2)} = P_{\mathrm{e}}^{(2)} = \frac{1}{2}$. Then the probabilities display no interference due to the disappearance of the interference term of Eq.(19).

In summary, we have proposed a scheme to monitor the decoherence of a mesoscopic superposition of two coherent states in the motion of a trapped ion. We note that the decoherence of the ion motion should occur at a slower pace than that of cavity field due to the extremely weak coupling between the vibrational motion and external environment.[6] Thus we can observe the decoherence of superpositions of two coherent components with a larger spacing in the ion motion than in cavity QED. Another advantage of using a trapped ion is that the interaction time of the ion with laser fields can be more easily controlled than that of an atom with a cavity field.

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