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Preparation of multicomponent motional coherent and squeezed coherent states of a trapped ion*

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In this paper, we present a scheme for preparation of multicomponent motional coherent and squeezed coherent states of the quantized centre of mass of an ion trapped in a one-dimensional harmonic potential and driven by two travelling-wave laser beams tuned to the nth red and blue vibrational sidebands, respectively. In addition, our scheme also provides experimental possibility for quantum state engineering.

Keywords: quantum state engineering, squeezed coherent state, coherent state
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1. Introduction

The generation and engineering of an arbitrary state of a quantum system lie at the heart of quantum optics. In the context of cavity quantum electrodynamics (QED) in an optical or a microwave regime, a number of schemes have been proposed for generating various nonclassical states of a single-mode electromagnetic field.[1−8] Recently, the experimental realization of superposition of two coherent states separated by a small distance in a cavity field has been reported.[9]

On the other hand, recent remarkable advance in laser cooling and ion trapping has opened prospects in quantum state engineering.[10,11] The internal (electronic) and external (vibrational centre-of-mass) degrees of freedom of a trapped ion can be coupled via the momentum exchange between the ion and a travelling-wave (TW) driving laser field, which makes it possible to manipulate the motional state of the ion. The extremely weak coupling between the vibrational motion of the trapped ion and the external environment provides the possibility of generating various nonclassical vibrational states of a trapped ion with a high degree of stability, such as Fock,[12] squeezed,[13] Schrödinger cat,[14] pair coherent and $SU(1,1)$ intelligent states.[15] To date, motional Schrödinger cat,[16] Fock, squeezed, and coherent states[17] have been observed.

In this paper, we show how to prepare the multicomponent motional coherent and squeezed coherent quantum states with arbitrary coefficients on a straight line in phase space by means of a discrete superposition of the desired states under the condition of two lasers tuned to one of the vibrational sidebands of the ionic transition. We observe that when two lasers are both resonant with a two-level ion, such a system acts like a classical microwave resource, and that if the two lasers are tuned to the first lower (red) and upper (blue) vibrational sidebands of the trapped ion or the second red and blue vibrational sidebands, respectively, then dynamic behaviours of the corresponding combined ion–field system can completely be described by the generalized Jaynes–Cummings models.[18−21] Furthermore, the properly adjusted laser beams allow one to achieve the desired vibrational states. It is pointed out, however, that it is a major asset of our proposal that only a small number of laser pulses and measurements are required to obtain a good approximation of quantum state. In addition, a unified standardized expression

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of the unitary time-evolution operator is derived from the ion–laser interaction Hamiltonian (as will be discussed below in detail). It is also shown that this procedure is a powerful method in the case of not only the first (second) vibrational sideband, but also all the other ones ($n \geq 3$).

The paper is organized as follows. In section 2, we derive the time evolution of an ion trapped in a harmonic potential and driven by two TW laser beams tuned to the nth red and blue vibrational sidebands, respectively. In section 3, we discuss some specific cases for the different vibrational sidebands. In section 4, we present a scheme to prepare the multicomponent motional coherent and squeezed coherent quantum states with arbitrary coefficients on a line in phase space for the ion motion. A summary appears in section 5.

2. The time evolution of an ion trapped in a harmonic potential

Consider an ion trapped in a one-dimensional (1D) harmonic potential and driven by two TW laser beams tuned to the nth red and blue vibrational sidebands, respectively. In the rotating-wave approximation (RWA), the Hamiltonian for this system is given by (assuming $\hbar$=1)

$$\begin{aligned} H &= H_0 + H_{\rm int}, \\ H_0 &= \nu a^+ a + \omega_1 A_{11} + \omega_2 A_{22}, \\ H_{\rm int} &= \lambda[E^+(x,t)A_{12} + {\rm h.c.}], \end{aligned} \tag{1}$$

where h.c. stand for the Hermitian conjugation terms. The first three terms in H_0 of Eq.(1) describe the free motion of electronic and the vibrational centre-of-mass degrees of freedom of the ion and the last one refers to the atom–field interaction. ν is the vibrational frequency of a harmonic trap potential. $A_{ij} = |i\rangle\langle j|$ and $A_{ii}(i \neq j, i, j = 1, 2)$ are respectively ionic level occupation and transition operator. ω_i is the eigenfrequency corresponding to the ith atomic level. a^+ and a are the creation and annihilation operator respectively for the vibrational quanta. λ is dipole matrix element characterizing the transition for the two-level ion. Without any loss of generality, we choose λ to be real. $E^+(x,t)$ is the positive-frequency part of a classical driving laser field, i.e.

$$E^+(x,t) = E_{0{\rm r}}{\rm e}^{-{\rm i}[\omega_{\rm r}t - k_{\rm r}\cdot x + \varphi_{\rm r}]} + E_{0{\rm b}}{\rm e}^{-{\rm i}[\omega_{\rm b}t - k_{\rm b}\cdot x + \varphi_{\rm b}]}, \tag{2}$$

where E_{0l}, φ_l, and k_l(l=r,b) are the amplitudes, phases, and wave vectors of the driving laser fields, respectively. $\omega_{\rm r} = \omega_{21} - n\nu$ and $\omega_{\rm b} = \omega_{21} + n\nu$ are frequencies of the driving laser fields irradiating a two-level ion in its well-resolved upper and lower nth vibrational sidebands. $\omega_{21} = \omega_2 - \omega_1$ is the transition frequency of the two-level ion between the ground electronic state $|1\rangle$ and the excited electronic state $|2\rangle$. The centre-of-mass position operator x can be expressed by $x = \sqrt{\dfrac{1}{2\nu M}}(a + a^+)$, with M being the mass of the trapped ion.

In the resolved sideband limit the vibrational frequency ν is much larger than other characteristic frequencies of the problem. According to Vogel and de Matos Filho,[22] if the ion is in the well-resolved sideband limit and the driving laser is resonant with one of the vibrational sidebands, then the interactions of the ion with lasers can be treated using a nonlinear Jaynes–Cummings model.[22,23] In this case the Hamiltonian for such a system is rewritten in the interaction picture as

$$\begin{aligned} H_{\rm int} =& \lambda\Bigg[E_{0{\rm r}}{\rm e}^{-\frac{1}{2}\eta_{\rm r}^2}{\rm e}^{-{\rm i}\varphi_{\rm r}}\sum_{j=0}^{\infty}\frac{({\rm i}\eta_{\rm r})^{2j+n}}{j!(j+n)!}(a^+)^j a^{j+n} \\ &+ E_{0{\rm b}}{\rm e}^{-\frac{1}{2}\eta_{\rm b}^2}{\rm e}^{-{\rm i}\varphi_{\rm b}}\sum_{j=0}^{\infty}\frac{({\rm i}\eta_{\rm b})^{2j+n}}{j!(j+n)!}(a^+)^{j+n}a^j\Bigg] \\ &\times A_{21} + {\rm h.c.}. \end{aligned} \tag{3}$$

We assume that the two lasers have the same amplitudes $E_{0{\rm r}} = E_{0{\rm b}} = E_0$ and wave vectors $k_{\rm r} = k_{\rm b} = k_0$. The phase $\phi = \dfrac{1}{2}(\varphi_{\rm r} + \varphi_{\rm b})$ contains the phases $\varphi_{\rm r}$ and $\varphi_{\rm b}$ of the two lasers detuned to the blue and red sidebands of the electronic transition. Thus the complex-valued effective Rabi frequency is defined by

$$\Omega_n = \lambda E_0{\rm e}^{-\frac{1}{2}\eta^2}{\rm e}^{-{\rm i}\phi}\frac{({\rm i}\eta)^n}{n!}, \tag{4}$$

where the Lamb–Dicke parameter $\eta = \dfrac{k}{\sqrt{2\nu M}}$.

We consider the behaviour of the ion in the Lamb–Dicke regime, i.e. $\eta \ll 1$. In this limit we can expand the Hamiltonian $H_{\rm int}$ in Eq.(3) up to the first order in η. Then the Hamiltonian can be reduced to

$$H_{\rm int} = (\Omega_n A_{21} + \Omega_n^* A_{12})x_\varphi, \tag{5}$$

where the notation x_φ is the generalized, phase-dependent operator acting on the centre-of-mass position of the trapped ion.[10] It can be expressed in terms of a and a^+ as

$$x_\varphi = {\rm e}^{{\rm i}\varphi}a^n + {\rm e}^{-{\rm i}\varphi}(a^+)^n, \tag{6}$$

where the phase $\varphi = \frac{1}{2}(\varphi_{\rm b} - \varphi_{\rm r})$ can be easily changed by altering the phase difference of the lasers.

Based on the interaction Hamiltonian in Eq.(5) one can obtain a unitary time-evolution operator $U_{\rm int}(t) = \exp(-{\rm i}tH_{\rm int})$ as follows (see the Appendix for the detailed derivation of the following expression):

$$U_{\rm int}(t) = \cos(x_\varphi|\Omega_n|t) - {\rm i}[{\rm e}^{-{\rm i}(\phi-\frac{1}{2}n\pi)}A_{21} + {\rm e}^{{\rm i}(\phi-\frac{1}{2}n\pi)}A_{12}]\sin(x_\varphi|\Omega_n|t), \quad (7)$$

which is a unified and standardized formula with respect to all the vibrational sidebands. The electronic matrix elements of the time-evolution operator (7) can be cast in a closed form as follows:

$$U_{11} = U_{22} = \frac{1}{2}[S_n^+(\xi) + S_n(\xi)], \quad (8)$$

$$U_{12} = \frac{1}{2}{\rm e}^{{\rm i}(\phi-\frac{1}{2}n\pi)}[S_n^+(\xi) - S_n(\xi)], \quad (9)$$

$$U_{21} = \frac{1}{2}{\rm e}^{-{\rm i}(\phi-\frac{1}{2}n\pi)}[S_n^+(\xi) - S_n(\xi)], \quad (10)$$

where $U_{ij} = \langle {\rm i}|U_{\rm int}(t)|j\rangle$ $(i,j{=}1{,}2)$, $S_n(\xi) = \exp[\xi(a^+)^n - \xi^* a^n]$, the complex-valued amplitude ξ is determined by the interaction time t and the phase φ as

$$\xi = {\rm i}{\rm e}^{-{\rm i}\varphi}|\Omega_n|t. \quad (11)$$

3. The discussion on some specific cases

As stated above, it is not difficult to show that the dynamics of a two-level ion trapped in Paul potential is the same as that of an individual two-level system interacting with radiation field in the cavity QED.

With the help of Eqs.(6)–(11), in the case of $n = 0$, the time-evolution operator, the phase-dependent position operator and the effective Rabi frequency are respectively simplified as

$$|\Omega_0| = \lambda E_0 {\rm e}^{-\frac{1}{2}\eta^2}, \quad (12)$$

$$x_\varphi^{(0)} = 2\cos\varphi = \text{constant}, \quad \chi = 2\lambda E_0 \cos\varphi {\rm e}^{-\frac{1}{2}\eta^2}, \quad (13)$$

$$U_{\rm int}^{(0)}(t) = \cos(\chi t) - {\rm i}({\rm e}^{-{\rm i}\phi}A_{21} + {\rm e}^{{\rm i}\phi}A_{12})\sin(\chi t). \quad (14)$$

Assume that an initial quantum state $|\Psi_{\rm e}(0)\rangle$ of the trapped ion to be in the excited electronic state $|2\rangle$, i.e. $|\Psi_{\rm e}(0)\rangle = |2\rangle$. After the interaction of the ion with two classical driving laser fields for a time t, the ionic state is

$$|\Psi_{\rm e}(t)\rangle = \cos(\chi t)|2\rangle - {\rm i}{\rm e}^{{\rm i}\phi}\sin(\chi t)|1\rangle, \quad (15)$$

therefore, the result indicates that under the action of $U_{\rm int}^{(0)}(t)$ the ion will be converted into an arbitrary superposition of $|2\rangle$ and $|1\rangle$, which is in a complete analogy to a classical microwave source.

In the $n = 1$ case, on the basis of Eqs.(6)–(11), the corresponding physical quantities are reduced to[14]

$$|\Omega_1| = \lambda E_0 \eta {\rm e}^{-\frac{1}{2}\eta^2}, \quad (16)$$

$$x_\varphi^{(1)} = a{\rm e}^{{\rm i}\varphi} + a^+{\rm e}^{-{\rm i}\varphi}, \quad (17)$$

$$\alpha = \frac{\rm i}{2}\lambda E_0 \eta t {\rm e}^{-\frac{1}{2}\eta^2}{\rm e}^{-{\rm i}\varphi}, \quad (18)$$

$$S_1(\alpha) \equiv D(\alpha) = \exp(\alpha^* a^+ - \alpha a), \quad (19)$$

where $D(\alpha)$ is a displacement operator, α is the relevant displacement amplitude. At the same time, it is straightforward to obtain the following matrix elements:

$$U_{11} = U_{22} = \frac{1}{2}[D^+(\alpha) + D(\alpha)],$$
$$U_{12} = \frac{1}{2}{\rm e}^{{\rm i}(\phi-\frac{1}{2}\pi)}[D^+(\alpha) - D(\alpha)],$$
$$U_{21} = \frac{1}{2}{\rm e}^{-{\rm i}(\phi-\frac{1}{2}\pi)}[D^+(\alpha) - D(\alpha)]. \quad (20)$$

In the $n = 2$ case, according to the same manner as above, we have

$$|\Omega_2| = \frac{1}{2}\lambda E_0 \eta^2 {\rm e}^{-\frac{1}{2}\eta^2},$$
$$x_\varphi^{(2)} = a^2{\rm e}^{{\rm i}\varphi} + (a^+)^2{\rm e}^{-{\rm i}\varphi},$$
$$\beta = \frac{i}{2}\lambda E_0 \eta^2 t {\rm e}^{-\frac{1}{2}\eta^2}{\rm e}^{-{\rm i}\varphi},$$
$$S_2(\beta) \equiv S(\beta) = \exp[\beta(a^+)^2 - \beta^* a^2], \quad (21)$$

where β and $S(\beta)$ are squeezed amplitude and squeezed operator, respectively. Correspondingly, the electronic matrix elements can be put into the following form:

$$U_{11} = U_{22} = \frac{1}{2}[S^+(\beta) + S(\beta)],$$
$$U_{12} = \frac{1}{2}{\rm e}^{{\rm i}\phi}[S(\beta) - S^+(\beta)],$$
$$U_{21} = \frac{1}{2}{\rm e}^{-{\rm i}\phi}[S(\beta) - S^+(\beta)]. \quad (22)$$

However, it is worthwhile pointing out that making full use of the above relations (20) or (22) we also can easily examine the dynamic behaviour of the system such as the photon number distribution (PND), the photon number inversion (PNI) and entanglement between the vibrational ions and photon.[24–28] Similarly, the system dynamic which takes into account the condition $n \geq 3$ (n is an integer) can also be explored,

which is currently underway, and corresponding results will be published elsewhere.

4. Multicomponent motional coherent and squeezed coherent states on a straight line

Let us suppose that an initial electronic quantum state $|\Psi_e(0)\rangle$ of the trapped ion to be in an arbitrary superposition of $|1\rangle$ and $|2\rangle$, i.e.

$$|\Psi_e(0)\rangle = \frac{1}{\sqrt{1+|\varepsilon_1|^2}}(|1\rangle + \varepsilon_1|2\rangle), \qquad (23)$$

where the parameter ε_1 controls the weights of the two electronic levels. However, it is worthwhile to note that for the following derivation we will exclude $\varepsilon'_N \neq \pm 1$ cases, i.e. we will not consider a specific case where the superposition of two electronic states $|1\rangle$ and $|2\rangle$ has the same probability. The above-mentioned electronic superposition state can be achieved by applying a laser field resonant with the two-level electronic transition, which corresponds to the $n = 0$ case in the last section. For the initial motional quantum state we take a vacuum state $|\Psi_m(0)\rangle = |0\rangle$, so that the initial state for such a system is

$$\begin{aligned}|\Psi(0)\rangle =& |\Psi_e(0)\rangle|\Psi_m(0)\rangle \\ =& \frac{1}{\sqrt{1+|\varepsilon_1|^2}}(|1\rangle + \varepsilon_1|2\rangle)|0\rangle. \end{aligned} \qquad (24)$$

Now we can make use of the laser fields to irradiate the ions for an interaction time t, that is to say, the time of bichromatic excitation is $t_0, \ldots, t_0 + t$. In order to detect the electronic state, we employ an electronic V scheme, where the upper levels $|2\rangle$ and $|3\rangle$ are coupled to the common ground level $|1\rangle$. $|2\rangle \to |1\rangle$ is a weak electronic transition, while $|3\rangle \to |1\rangle$ is a strong one. After the interaction of the ion with the above-mentioned two lasers, i.e. in the time intervals $t_0 + t, \ldots, t_1$, a laser resonant with the transition $|3\rangle \to |1\rangle$ is used to probe the appearance of fluorescence. The presence of fluorescence is correlated with the ion being in the electronic state $|1\rangle$, while the absence of fluorescence is correlated with the ion in the state $|2\rangle$. In the following we consider only these events in the presence of the observable fluorescence and for simplicity of analysis we choose the integer $n = 2$. After an interaction time t_1 the state evolution of the system can be determined by

$$\begin{aligned}|\Psi(t_1)\rangle =& U_{\rm int}(t)|\Psi(0)\rangle \\ =& \frac{1}{2\sqrt{1+|\varepsilon_1|^2}}\Big\{[(1-\varepsilon'_1)|\beta\rangle \\ &+ (1+\varepsilon'_1)|-\beta\rangle]|1\rangle \\ &+ \Big[\Big(\varepsilon_1 - \frac{\varepsilon_1'^*}{\varepsilon_1^*}\Big)|\beta\rangle \\ &+ \Big(\varepsilon_1 + \frac{\varepsilon_1'^*}{\varepsilon_1^*}\Big)|-\beta\rangle\Big]|2\rangle\Big\}, \end{aligned} \qquad (25)$$

where $\varepsilon'_1 = \varepsilon_1 e^{i(\phi-\pi)}$. If we can observe a fluorescence event, which implies that the ion is in the ground $|1\rangle$, then the vibrational motion state at time t_1 collapses to

$$|\Psi_m^{(1)}\rangle = \Theta_1[(1-\varepsilon'_1)|\beta\rangle + (1+\varepsilon'_1)|-\beta\rangle], \qquad (26)$$

with Θ_1 being a normalization factor.

The probability of observing the fluorescence, i.e. the desired-state preparation probability, is

$$P_1 = \frac{1}{4(\Theta_1)^2(1+|\varepsilon'_1|^2)}. \qquad (27)$$

By the same routine, we then send a second ion, which is similarly prepared but has a different parameter ε_2 and interacts with the bichromatic laser fields for a time t_2. In this case the emergent states, provided again that the fluorescence is detectable, can be proved to be

$$\begin{aligned}|\Psi_m^{(2)}\rangle =& \Theta_2[(1-\varepsilon'_1)(1-\varepsilon'_2)|2\beta\rangle \\ &+ 2(1-\varepsilon'_1\varepsilon'_2)|0\rangle \\ &+ (1+\varepsilon'_1)(1+\varepsilon'_2)|-2\beta\rangle], \end{aligned} \qquad (28)$$

where Θ_2 is a normalization factor and $\varepsilon'_2 = \varepsilon_2 e^{i(\phi-\pi)}$.

The probability of observing the fluorescence is

$$P_2 = \frac{(\Theta_1)^2}{4(\Theta_2)^2(1+|\varepsilon'_2|^2)}. \qquad (29)$$

As usual, after such N cycles including electronic-state preparation, bichromatic-laser interaction together with observation of the fluorescence, the resulting conditioned vibrational motional quantum state can be derived:

$$|\Psi_m^{(N)}\rangle = \Theta_N \sum_{k=0}^{N} C_k^{(N)}|(2k-N)\beta\rangle, \qquad (30)$$

with Θ_N being a normalization factor for the Nth ion and $C_k^{(N)}$ (the target-coherent-state coefficients) being a function of the ion-state parameter $\varepsilon'_N = \varepsilon_N e^{i(\phi-\pi)}$, i.e., $C_k^{(N)} = C_k^{(N)}(\varepsilon'_1, \varepsilon'_3, \cdots, \varepsilon'_N)$. Here we assume that each time the fluorescence is observed: that is to say, the experiment must be repeated until the fluorescence output arrives at the observable intensity. The probability of measuring the target-state preparation can be computed as a direct product of the probability P_N which is associated with each ion.

In what follows we show how one can obtain N parameters ε'_N by virtue of the target state (30) which yields a given arbitrary set of coefficients $\{C_k^{(N)}\}$. Firstly, we specify the superposition states with N constituent coherent states

$$|\Psi_m^{(N-1)}\rangle = \Theta_{N-1} \sum_{k=0}^{N-1} C_k^{(N-1)} |(2k-N+1)\beta\rangle, \quad (31)$$

which can be turned into Eq.(30) via sending the Nth ion (with an appropriate ε_N) interacting with the bichromatic-laser fields and detecting the fluorescence. From Eqs.(25), (30) and (31) it is easy to check that for the coefficients $C_k^{(N)}$ and the parameter ε'_N

$$\begin{aligned}
C_0^{(N)} &= (1+\varepsilon'_N) C_0^{(N-1)}, \\
C_1^{(N)} &= (1-\varepsilon'_N) C_0^{(N-1)} + (1+\varepsilon'_N) C_1^{(N-1)}, \\
&\vdots \\
C_k^{(N)} &= (1-\varepsilon'_N) C_{k-1}^{(N-1)} + (1+\varepsilon'_N) C_k^{(N-1)}, \quad k \neq 0, N, \\
&\vdots \\
C_N^{(N)} &= (1-\varepsilon'_N) C_{N-1}^{(N-1)}. \qquad (32)
\end{aligned}$$

The new coefficients $C_k^{(N-1)}$ can be expressed in terms of the old ones $C_k^{(N)}$ as

$$C_k^{(N-1)} = \frac{1}{\varepsilon'_N + 1} \sum_{m=0}^{k} \left(\frac{\varepsilon'_N - 1}{\varepsilon'_N + 1} \right)^m C_{k-m}^{(N)}. \quad (33)$$

Substituting Eq.(33) into the last equation of the set (32), it is easy to obtain a characteristic equation for ε'_N as follows:

$$\sum_{m=0}^{N} \left(\frac{\varepsilon'_N - 1}{\varepsilon'_N + 1} \right)^m C_{N-m}^{(N)} = 0, \quad (34)$$

where $\varepsilon'_N \neq \pm 1$, otherwise Eq.(34) has no meaning.

We can solve this equation numerically and choose one value ε'_N from the N roots of Eq.(34). Inserting the obtained ε'_N into Eq.(33), we derive the coefficients $C_k^{(N-1)}$ for the state $|\Psi_m^{(N-1)}\rangle$. We take $|\Psi_m^{(N-1)}\rangle$ as a new state and carry out the identical calculations. Then we acquire the parameters ε'_{N-1} and the state $|\Psi_m^{(N-2)}\rangle$ which contains $N-1$ components. The above-mentioned procedure is repeated until we end up with the vacuum $|\Psi_m^{(0)}\rangle = |0\rangle$. Therefore, we can obtain the values of N parameters ε'_1, ε'_2, ε'_3 $\cdots$, ε'_N, each defining the internal electronic states in a sequence of N ions.

It should be noted that the present scheme is a probabilistic one. In order to obtain a desired superposition of $N+1$ coherent states, one has to find the ion in the state $|1\rangle$ from among N successive measurements. The probability of success in this event is

$$\begin{aligned}
P &= \prod_{k=1}^{k=N} P_k = \left(\frac{1}{4}\right)^N (\Theta_N)^{-2} \prod_{k=0}^{k=N} \frac{1}{1+|\varepsilon'_k|^2}, \\
(\Theta_N)^{-2} &= \sum_{k,l=0}^{N} C_k^{(N)} [C_l^{(N)}]^* e^{-2(k-l)^2|\beta|^2}. \qquad (35)
\end{aligned}$$

From Eq.(35) one can verify that the probability is exponentially decreased with increase of the integer N. Thus the method is restricted to a small value of N. However, it has been shown that in many cases the superposition of even a small number of properly chosen coherent states can approximate to a nonclassical field state to a high degree of accuracy. Furthermore, we can choose an approximate characteristic root ε'_N to maximize P_k and thus P.

Finally, it is worthwhile pointing out that in a manner similar to the above-described process, it is easy to generate multicomponent coherent states in the $n=1$ case.

5. Conclusions

In summary, we have proposed a scheme for preparing superposition of arbitrary discrete coherent and squeezed coherent states on a line in phase space. Our scheme also provides experimental possibility for quantum state engineering. In contrast to the previous engineering methods,[1, 7] it is worthwhile mentioning that one of the key advantages of such an approach is that a good approximation of quantum state can be achieved by requiring only a small number of laser pulses and measurements. In view of the decoherence mechanism,[29] a reduction of the number of operations for the preparation of quantum state and experimental implementation of quantum-state engineering may become crucial. Consequently, we believe our scheme may be realized presently or soon.

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Appendix

In this Appendix, we present the detailed derivation of Eqs.(5)–(7) in the main text. In the Lamb–Dicke limit $\eta \ll 1$ Hamiltonian (3) can be retained up to the order $O(\eta)$, we first simplify Hamiltonian (3) into the following form:

$$\begin{aligned} H_{\rm int} =& \lambda[E_{0\rm r}{\rm e}^{-\frac{1}{2}\eta_{\rm r}^2}{\rm e}^{-{\rm i}\varphi_{\rm r}}\frac{({\rm i}\eta_{\rm r})^n}{n!}a^n \\ &+ E_{0\rm b}{\rm e}^{-\frac{1}{2}\eta_{\rm b}^2}{\rm e}^{-{\rm i}\varphi_{\rm b}}\frac{({\rm i}\eta_{\rm b})^n}{n!}(a^+)^n]A_{21} + {\rm h.c.}. \end{aligned} \tag{A1}$$

Assuming $E_{0\rm r} = E_{0\rm b} = E_0$ (real) and $k_{\rm r} = k_{\rm b} = k_0$, then yielding $\eta_{\rm r} = \eta_{\rm b} = \eta$ with $\eta = \dfrac{k}{\sqrt{2\nu M}}$ (M is the mass of the trapped ion), we further reduce $H_{\rm int}$ (A1) to be of the form

$$H_{\rm int} = (\Omega_n A_{21} + \Omega_n^* A_{12})x_\varphi, \tag{A2}$$

where the complex-valued effective Rabi frequency and phase-dependent position operator acting on the center-of-mass position of the trapped ion are respectively defined as

$$\begin{aligned} \Omega_n =& \lambda E_0{\rm e}^{-\frac{1}{2}\eta^2}{\rm e}^{-{\rm i}\phi}\frac{({\rm i}\eta)^n}{n!}, \\ \phi =& \frac{1}{2}(\varphi_{\rm r} + \varphi_{\rm b}), \\ x_\varphi =& {\rm e}^{{\rm i}\varphi}a^n + {\rm e}^{-{\rm i}\varphi}(a^+)^n, \\ \varphi =& \frac{1}{2}(\varphi_{\rm b} - \varphi_{\rm r}). \end{aligned} \tag{A3}$$

Using the relation (A2), it is now straightforward to obtain

$$\begin{aligned} (H_{\rm int})^{2n} =& \begin{pmatrix} |\Omega_n|^{2n}(x_\varphi)^{2n} & 0 \\ 0 & |\Omega_n|^{2n}(x_\varphi)^{2n} \end{pmatrix}, \\ (H_{\rm int})^{2n+1} =& \begin{pmatrix} 0 & \dfrac{\Omega_n}{|\Omega_n|}|\Omega_n|^{2n+1}(x_\varphi)^{2n+1} \\ \dfrac{\Omega_n^*}{|\Omega_n|}|\Omega_n|^{2n+1}(x_\varphi)^{2n+1} & 0 \end{pmatrix}. \end{aligned} \tag{A4}$$

Combining the above relations (A3) and (A4), the time-evolution operator can be expressed as

$$\begin{aligned} U_{\rm int}(t) =& \exp(-{\rm i}H_{\rm int}t) \\ =& \sum_{n=0}^{\infty}\frac{(-{\rm i}H_{\rm int}t)^{2n}}{(2n)!} + \sum_{n=0}^{\infty}\frac{(-{\rm i}H_{\rm int}t)^{2n+1}}{(2n+1)!} \\ =& \cos(x_\varphi|\Omega_n|t) \\ & - {\rm i}\left[\frac{\Omega_n}{|\Omega_n|}A_{21} + \frac{\Omega_n^*}{|\Omega_n|}A_{12}\right]\sin(x_\varphi|\Omega_n|t) \\ =& \cos(x_\varphi|\Omega_n|t) - {\rm i}[{\rm e}^{-{\rm i}(\phi-\frac{1}{2}n\pi)}A_{21}. \\ & + {\rm e}^{{\rm i}(\phi-\frac{1}{2}n\pi)}A_{12}]\sin(x_\varphi|\Omega_n|t). \end{aligned} \tag{A5}$$

In terms of the formulae $\cos\beta = \dfrac{1}{2}({\rm e}^{{\rm i}\beta} + {\rm e}^{-{\rm i}\beta})$ and $\sin\beta = \dfrac{1}{2{\rm i}}({\rm e}^{{\rm i}\beta} - {\rm e}^{-{\rm i}\beta})$, the time-evolution operator $U_{\rm int}(t)$ can be easily rewritten in a more convenient form:

$$\begin{aligned} U_{\rm int}(t) =& \frac{1}{2}[S_n(\xi) + S_n^+(\xi)] \\ & - \frac{1}{2}[S_n(\xi) - S_n^+(\xi)] \\ & \times [{\rm e}^{-{\rm i}(\phi-\frac{1}{2}n\pi)}A_{21} + {\rm e}^{{\rm i}(\phi-\frac{1}{2}n\pi)}A_{12}], \end{aligned} \tag{A6}$$

where

$$S_n(\xi) = [S_n(\xi)^+]^+ = \exp[\xi(a^+)^n - \xi^* a^n]$$

and

$$\xi = {\rm i}{\rm e}^{-{\rm i}\varphi}|\Omega_n|t.$$

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