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# Nonlinear cascade-configuration multi-wave mixing scheme based on electromagnetically induced transparency*

Li Jia-Hua(李家华)[a)†], Yang Wen-Xing(杨文星)[a)], and Peng Ju-Cun(彭菊村)[b)]

[a)] *Department of Physics, Huazhong University of Science and Technology, Wuhan* 430074, *China*

[b)] *Department of Physics, Xiaogan Normal University, Xiaogan* 432100, *China*

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A nonlinear optical cascade-configuration multi-wave mixing (CCMWM) scheme is presented and analysed for the generation of coherent light in a six-level atomic system in the context of electromagnetically induced transparency (EIT). A detailed semi-classical study of the propagation of the generated mixing and probe fields is demonstrated. We show by numerical simulations that EIT is capable of suppressing linear and nonlinear photon absorption. The analytical dependence of the generated mixing field on the probe field and the respective detuning is also predicted. Such a nonlinear optical process can be used for generating coherent short-wavelength radiation.

**Keywords:** optical multi-wave mixing, electromagnetically induced transparency, photon absorption rate

**PACC:** 4250

## 1. Introduction

Electromagnetically induced transparency (EIT) is the optical transparency of a three-level medium at a resonant transition induced by application of a coherent electromagnetic field at an adjacent transition.[1–9] The accompanying reduction of the group velocity of light and large enhancement of the nonlinearity of the coherent atomic medium at the few-photon level by many orders of magnitude in the EIT transparency window[3] has been intensively and actively investigated in connection with many potential applications, especially low-intensity nonlinear optics (i.e. it can reach a level of few photons), optical multi-wave mixing, ultra-slow photon entanglement, quantum information processing and quantum information storage and release.[10] In essence, the transparency may be viewed as resulting from a combination of the dynamic ac-Stark splitting or Autler–Townes splitting and the quantum interference between the two dressed states which are created by the strong coupling laser field. A few years ago, Schmdit and co-workers had investigated a nonlinear double-$\Lambda$ scheme without considering the wave-mixing process, it turned out that based on electromagnetically induced transparency the nonlinear susceptibility of the probe field can greatly increase by many orders of magnitude.[11] Li *et al* has demonstrated a clear experimental observation of an enhanced non-degenerate four-wave mixing (NDFWM) process, making use of the EIT effect in a three-level $\Lambda$-type system of Rb atoms with continuous-wave (cw) diode lasers.[2] Later on, Deng *et al* proposed an EIT-based double-$\Lambda$ system with the inclusion of the generated field, and by means of the time-dependent perturbation treatment they showed that the possible quantum constructive and destructive interference between different excitation pathways can result in enhancement and suppression of the Kerr nonlinearity.[12] More recently, Wu *et al* analysed a four-wave-mixing (FWM) scheme in a five-level atomic system and hyper-Raman scattering (HRS) in resonant coherent media by the use of EIT, which leads to suppression of both two-photon and three-photon absorptions in both FWM and HRS schemes and enables the four-wave mixing to proceed through real, resonant intermediate states without ab-

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†Corresponding author. E-mail: huajia_li@163.com

sorption loss.[13,14] In particular, with the advent of Bose–Einstein condensation in an atomic gas there has been much interest in studying four-wave mixing with matter wave both experimentally and theoretically in the framework of nonlinear atomic optics.[15–24]

In this context, in the present work we present and analyse an optical multi-wave mixing scheme for the generation of coherent light in the six-level atomic system by means of EIT, under the condition that we apply the weak probe pulsed laser to serve as the first step of pumping source. This paper is organized as follows. In section 2, the model is described and the equations of motion describing the atom-field interaction for the system under consideration are derived. A detailed semi-classical study of the propagating of the generated mixing and probe fields is demonstrated. In section 3, photon absorption rate in the steady-state case is discussed. The results of numerical simulations show that EIT is capable of suppressing linear and nonlinear photon absorption. In section 4, the influences of the probe field and the respective detuning on the generated coherent short-wave mixing field are examined. We can apply such a nonlinear optical process to producing the short-wavelength radiation. Section 5 is a brief summary of our results.

## 2. Theoretical model and equations of motion

Let us consider the energy scheme depicted in Fig.1. In this six-level atomic scheme a weak probe pulsed laser (pulse length $\tau$) is tuned near the $|0\rangle \rightarrow |2\rangle$ resonance. One of the two lower levels is coupled to the upper level ($|1\rangle \rightarrow |2\rangle$) by a strongly coherent control laser to create an Autler–Townes doublet. A near-resonant microwave field of frequency $\omega_2$ with a Rabi frequency $\Omega_2$ couples level $|3\rangle$ into level $|4\rangle$ through an allowed magnetic transition and other two signal laser fields are tuned to the transitions $|2\rangle \rightarrow |3\rangle$ and $|4\rangle \rightarrow |5\rangle$, respectively. To avoid significant group velocity mismatch and for mathematical simplicity, in the present analysis we will assume that all the fields involved are continuous-wave single-frequency laser (cw laser) with the exception of the probe field.

Under the electro-dipole approximation and the rotating-wave approximation, the semi-classical Hamiltonian describing the atom-field interaction for the system under consideration can be written as in the Schrödinger's picture:[25]

$$\begin{aligned} H = & \sum_{j=0}^{5} E_j^{(a)} |j\rangle\langle j| + \hbar(\Omega_{\rm p} {\rm e}^{{\rm i}\theta_{\rm p}} |2\rangle\langle 0| + {\rm h.c.}) \\ & + \hbar(\Omega_{\rm c} {\rm e}^{{\rm i}\theta_{\rm c}} |2\rangle\langle 1| + {\rm h.c.}) \\ & + \hbar(\Omega_1 {\rm e}^{{\rm i}\theta_1} |3\rangle\langle 2| + {\rm h.c.}) \\ & + \hbar(\Omega_2 {\rm e}^{{\rm i}\theta_2} |4\rangle\langle 3| + {\rm h.c.}) \\ & + \hbar(\Omega_3 {\rm e}^{{\rm i}\theta_3} |5\rangle\langle 4| + {\rm h.c.}) \\ & + \hbar(\Omega_{\rm m} {\rm e}^{{\rm i}\theta_{\rm m}} |5\rangle\langle 0| + {\rm h.c.}), \end{aligned} \tag{1}$$

where $\theta_n = k_n \cdot r - \omega_n t$ corresponds to the positive-frequency part of the respective optical field, $\Omega_n$ ($n$=c, p, 1, 2, 3, m) are one-half Rabi frequencies for the relevant laser-driven transitions, i.e., $\Omega_{\rm p} = D_{20}E_{\rm p}(\omega_{\rm p})/(2\hbar)$, $\Omega_{\rm c} = D_{21}E_{\rm c}(\omega_{\rm c})/(2\hbar)$, $\Omega_1 = D_{32}E_1(\omega_1)/(2\hbar)$, $\Omega_2 = D_{43}E_2(\omega_2)/(2\hbar)$, $\Omega_3 = D_{54}E_3(\omega_3)/(2\hbar)$ and $\Omega_{\rm m} = D_{50}E_{\rm m}(\omega_{\rm m})/(2\hbar)$, with $D_{kl}$ denoting the dipole moment element for the transition between levels $|k\rangle$ and $|l\rangle$, and $E_j^{(a)} = \hbar\varepsilon_j$ being the energy of the atomic state $|j\rangle$. For simplicity, in what follows we will take $\varepsilon_0 = 0$ for the ground state $|0\rangle$ as the energy origin. Turning to the interacting picture, the Hamiltonian can be rewritten as follows (assuming $\hbar = 1$):

$$\begin{aligned} H_0 = & (\omega_{\rm p} - \omega_{\rm c})|1\rangle\langle 1| + \omega_{\rm p}|2\rangle\langle 2| \\ & + (\omega_{\rm p} + \omega_1)|3\rangle\langle 3| + (\omega_{\rm p} + \omega_1 + \omega_2)|4\rangle\langle 4| \\ & + (\omega_{\rm p} + \omega_1 + \omega_2 + \omega_3)|5\rangle\langle 5|, \\ H_{\rm int} = & \Delta_{\rm c}|1\rangle\langle 1| + \Delta_{\rm p}|2\rangle\langle 2| + \Delta_1|3\rangle\langle 3| + \Delta_2|4\rangle\langle 4| \\ & + \Delta_{\rm m}|5\rangle\langle 5| + (\Omega_{\rm p} {\rm e}^{{\rm i}k_{\rm p}\cdot r}|2\rangle\langle 0| + \Omega_{\rm c} {\rm e}^{{\rm i}k_{\rm c}\cdot r}|2\rangle\langle 1| \\ & + \Omega_1 {\rm e}^{{\rm i}k_1\cdot r}|3\rangle\langle 2| + \Omega_2 {\rm e}^{{\rm i}k_2\cdot r}|4\rangle\langle 3| \\ & + \Omega_3 {\rm e}^{{\rm i}k_3\cdot r}|5\rangle\langle 4| + \Omega_{\rm m} {\rm e}^{{\rm i}k_{\rm m}\cdot r}|5\rangle\langle 0| + {\rm h.c.}), \end{aligned} \tag{2}$$

where $\Delta_{\rm p} = \delta_{\rm p} = \varepsilon_2 - \omega_{\rm p}$ is the single-photon detuning, $\Delta_{\rm c} = \delta_{\rm p} - \delta_{\rm c} = \varepsilon_1 - (\omega_{\rm p} - \omega_{\rm c})$ and $\Delta_1 = \delta_{\rm p} + \delta_1 = \varepsilon_3 - (\omega_{\rm p} + \omega_1)$ are the corresponding two-photon detunings, as well as $\Delta_2 = \delta_{\rm p} + \delta_1 + \delta_2 = \varepsilon_4 - (\omega_{\rm p} + \omega_1 + \omega_2)$, $\Delta_3 = \delta_{\rm p} + \delta_1 + \delta_2 + \delta_{\rm m} = \varepsilon_5 - (\omega_{\rm p} + \omega_1 + \omega_2 + \omega_3)$ are the three-photon and four-photon detunings, respectively. $\delta_j$ ($j$=c, p, 1, 2, m) is the related frequency difference between the laser field and the two-level transition, as shown in Fig.1.

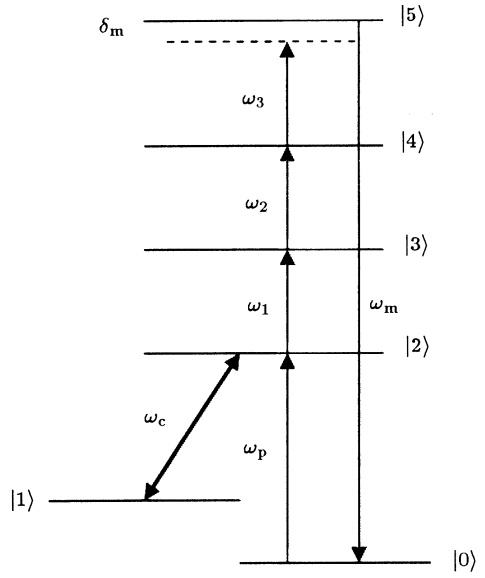



**Fig.1.** EIT-based six-level atomic diagram under consideration. A cw control field $\Omega_c$ creates a transparency window to suppress the nonlinear absorption of all orders for the probe and signal fields $\Omega_{p,1,2,3}$, which can be referred to as a multiphoton EIT.[26] The intensity of this control field $\Omega_c$ is relatively low in order to significantly reduce the propagation velocity of the probe wave $\Omega_p$, which may lead to significant enhancement of nonlinear optical processes and overall MWM conversion efficiency.[9]

Let us assume the wavefunction of the form

$$\begin{aligned}|\Psi\rangle =&A_0|0\rangle + A_1 e^{i(k_p-k_c)\cdot r}|1\rangle \\ &+ A_2 e^{ik_p\cdot r}|2\rangle + A_3 e^{i(k_p+k_1)\cdot r}|3\rangle \\ &+ A_4 e^{i(k_p+k_1+k_2)\cdot r}|4\rangle + A_5 e^{ik_m\cdot r}|5\rangle, \end{aligned} \quad (3)$$

where $A_j$ $(j = 0-5)$ stand for the probability amplitudes of the atomic states.

Making use of the Schrödinger equation in the interaction picture $i\partial_t|\Psi\rangle = H_{int}|\Psi\rangle$, the equations of motion for the probability amplitudes of the atomic wavefunctions can be readily obtained as

$$(\partial_t + \widetilde{\gamma}_1)A_1 = -i\Omega_c^* A_2, \quad (4a)$$
$$(\partial_t + \widetilde{\gamma}_2)A_2 = -i\Omega_p A_0 - i\Omega_c A_1 - i\Omega_1^* A_3, \quad (4b)$$
$$(\partial_t + \widetilde{\gamma}_3)A_3 = -i\Omega_1 A_2 - i\Omega_2^* A_4, \quad (4c)$$
$$(\partial_t + \widetilde{\gamma}_4)A_4 = -i\Omega_2 A_3 - i\Omega_3^* e^{-i\,\delta k\cdot r} A_5, \quad (4d)$$
$$(\partial_t + \widetilde{\gamma}_5)A_5 = -i\Omega_3 e^{i\,\delta k\cdot r} A_4 - i\Omega_m A_0, \quad (4e)$$

where we have introduced the new parameters $\widetilde{\gamma}_1 = \gamma_1 + i\Delta_c$, $\widetilde{\gamma}_2 = \gamma_2 + i\Delta_p$, $\widetilde{\gamma}_3 = \gamma_3 + i\Delta_1$, $\widetilde{\gamma}_4 = \gamma_4 + i\Delta_2$, $\widetilde{\gamma}_5 = \gamma_5 + i\Delta_m$ and wavenumber mismatch relation $\delta k = k_p + k_1 + k_2 + k_3 - k_m$ as well as energy relation $\omega_m = \omega_p + \omega_1 + \omega_2 + \omega_3$, $\gamma_j$ $(j = 1-5)$ being the decay rates of the states $|j\rangle$.

In order to correctly describe the propagation of the optical pulse, the atomic equations of motion must be simultaneously solved with Maxwell's equations for both the probe and the generated mixing fields in a self-consistent manner. In the limit of plane waves and slowly varying amplitude, the positive frequency parts of these fields satisfy

$$(\partial_z + 1/c\,\partial_t)E_p = 4i\pi k_p P_p^{(+)}, \quad (5a)$$
$$(\partial_z + 1/c\,\partial_t)E_m = 4i\pi k_m P_m^{(+)}, \quad (5b)$$

where $E_p(P_p^{(+)})$ and $E_m(P_m^{(+)})$ are the slowly varying envelopes of the probe and generated mixing field amplitudes (polarizations). In order to explore the propagation of the generated mixing field through a resonant medium, the polarization of the medium must be obtained. In the following, using the atomic wavefunction (4), we have

$$\begin{aligned}P(r,0) =&N\langle\Psi|\widehat{D}|\Psi\rangle \\ =&P_p^{(+)}(\omega_p)e^{ik_p\cdot r} + P_c^{(+)}(\omega_c)e^{ik_c\cdot r} \\ &+ P_1^{(+)}(\omega_1)e^{ik_1\cdot r} + P_2^{(+)}(\omega_2)e^{ik_2\cdot r} \\ &+ P_3^{(+)}(\omega_3)e^{ik_3\cdot r} + P_m^{(+)}(\omega_m)e^{ik_m\cdot r} + \text{h.c.}, \end{aligned} \quad (6)$$

where $N$ and $\widehat{D}$ are the atomic concentration and the dipole moment operator of the relevant transition, respectively. $P_n^{(+)}(\omega_n)$ represents the positive frequency part of the polarization at the corresponding field, i.e. $P_p^{(+)}(\omega_p) = ND_{02}A_2A_0^*$, $P_c^{(+)}(\omega_c) = ND_{12}A_2A_1^*$, $P_1^{(+)}(\omega_1) = ND_{23}A_3A_2^*$, $P_2^{(+)}(\omega_2) = ND_{34}A_4A_3^*$, $P_3^{(+)}(\omega_3) = ND_{45}A_5A_4^*$ and $P_m^{(+)}(\omega_m) = ND_{05}A_5A_0^*$. For both the probe pulsed laser field at frequency $\omega_p$ and the generated coherent MWM fields at frequency $\omega_m$, combining $P_p^{(+)}(\omega_p) = ND_{02}A_2A_0^*$ and $P_m^{(+)}(\omega_m) = ND_{05}A_5A_0^*$ with $\Omega_p = D_{20}E_p(\omega_p)/(2\hbar)$ and $\Omega_m = D_{50}E_m(\omega_m)/(2\hbar)$, the slowly varying envelope equations for the Rabi frequencies $\Omega_p$ and $\Omega_m$ are given by

$$(\partial_z + 1/c\,\partial_t)\Omega_p = ik_{02}A_2A_0^*, \quad (7a)$$
$$(\partial_z + 1/c\,\partial_t)\Omega_m = ik_{05}A_5A_0^*, \quad (7b)$$

where we have introduced the notations $k_{02} = 2\pi N\omega_p|D_{02}|^2/(\hbar c)$ and $k_{05} = 2\pi N\omega_m|D_{05}|^2/(\hbar c)$. Following the standard procedure described in Refs.[13,14,26-28], the probe pulsed laser is assumed to be sufficiently weak that the ratio $(\Omega_p/\Omega_c)^2$ is much less than unity, so that in essence all of the atomic population remains in the ground state $|0\rangle$, i.e. $A_0 \approx 1$. Again, we consider the control field $\omega_c$ and signal

fields $\omega_1$, $\omega_2$ and $\omega_3$ to be continuous single-frequency laser, but the probe field $\omega_{\rm p}$ and the generated mixing field $\omega_{\rm m}$ have a time-dependent beam profile. With these approximations, we will discuss the following two cases.

## 3. Photon absorption rate in steady-state case

Under the condition that the pulse's rise time satisfies adiabatic conditions, the method adopted in Refs.[26,28] can be applied. Therefore, based on Eqs.(4a)–(4e), the probability amplitudes of the atomic state are given by

$$A_2 \approx -\frac{i\widetilde{\gamma}_1\Omega_{\rm p}(\widetilde{\gamma}_3\widetilde{\gamma}_4\widetilde{\gamma}_5+\widetilde{\gamma}_3|\Omega_3|^2+\widetilde{\gamma}_5|\Omega_2|^2)}{\widetilde{\gamma}_5 M+N}, \quad (8a)$$

$$A_3 \approx -\frac{\widetilde{\gamma}_1\Omega_{\rm p}\Omega_1(\widetilde{\gamma}_4\widetilde{\gamma}_5+|\Omega_3|^2)}{\widetilde{\gamma}_5 M+N}, \quad (8b)$$

$$A_4 \approx -\frac{i\widetilde{\gamma}_1\widetilde{\gamma}_5\Omega_{\rm p}\Omega_1\Omega_2}{\widetilde{\gamma}_5 M+N}, \quad (8c)$$

$$A_5 \approx \frac{\widetilde{\gamma}_1\Omega_{\rm p}\Omega_1\Omega_2\Omega_3 {\rm e}^{{\rm i}\delta kz}}{\widetilde{\gamma}_5 M+N}, \quad (8d)$$

with

$$\begin{aligned} M =& \widetilde{\gamma}_1\widetilde{\gamma}_2\widetilde{\gamma}_3\widetilde{\gamma}_4+\widetilde{\gamma}_3\widetilde{\gamma}_4|\Omega_{\rm c}|^2+\widetilde{\gamma}_1\widetilde{\gamma}_4|\Omega_1|^2 \\ &+\widetilde{\gamma}_1\widetilde{\gamma}_2|\Omega_2|^2+|\Omega_{\rm c}|^2|\Omega_2|^2, \\ N =& \widetilde{\gamma}_1\widetilde{\gamma}_2\widetilde{\gamma}_3|\Omega_3|^2+\widetilde{\gamma}_3|\Omega_{\rm c}|^2|\Omega_3|^2+\widetilde{\gamma}_1|\Omega_1|^2|\Omega_3|^2. \end{aligned}$$

In the above discussion we have neglected the terms including $\Omega_{\rm m}$ due to the fact that $\Omega_{\rm m}$ is negligibly small. Photon absorption rate (PAR), i.e. steady-state atomic population, $P_n = |A_n|^2$, $n = 2-5$. To show explicitly the EIT-induced suppression of nonlinear photon absorption, we assume that all laser fields excluding the weak probe pulsed laser field $\omega_{\rm p}$, are on resonance with the corresponding atomic transition. Figure 2 presents the schematic plots of PAR versus the probe laser detuning $\delta_{\rm p}$ based on Eqs.(8a)–(8d). It turns out that at line centre (i.e. $\delta_{\rm p}=0$) EIT simultaneously suppresses single-photon absorption ($P_1=0$), two-photon absorption ($P_2=0$), three-photon absorption ($P_3=0$), four-photon absorption ($P_4=0$) and five-photon absorption ($P_5=0$).

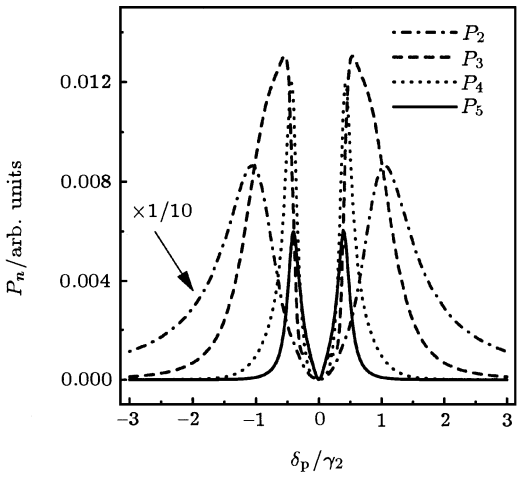



**Fig.2.** Schematic plot of photon absorption rate $P_n$ versus pump laser detuning $\delta_{\rm p}$. The fixed parameters used: $\delta_{\rm c}=\delta_1=\delta_2=\delta_{\rm m}=0$, $\delta k=0$, $\Omega_{\rm c}=\gamma_2$, $\Omega_{\rm p}=\Omega_1=\Omega_2=\Omega_3=0.3\gamma_2$, $\gamma_1=0.001\gamma_2$, $\gamma_3=0.2\gamma_2$, $\gamma_4=0.1\gamma_2$, and $\gamma_5=0.08\gamma_2$.

## 4. Propagation of generated coherent short-wavelength mixing field

Performing Fourier transformations

$$A_j(t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\alpha_j(\omega){\rm exp}(-{\rm i}\omega t){\rm d}\omega, \quad j=1-4$$

and

$$\Omega_k(t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}W_k(\omega){\rm exp}(-{\rm i}\omega t){\rm d}\omega,$$

$k$=p, m for both Eqs.(4a)–(4e) and (7a),(7b), here we take $\delta k=\delta k\cdot\hat{z}$ and straightforwardly obtain the following equations:

$$\Delta\widetilde{\omega}_1\alpha_1 = \Omega_{\rm c}^*\alpha_2, \quad (9a)$$

$$\Delta\widetilde{\omega}_2\alpha_2 = W_{\rm p}+\Omega_{\rm c}\alpha_1+\Omega_1^*\alpha_3, \quad (9b)$$

$$\Delta\widetilde{\omega}_3\alpha_3 = \Omega_1\alpha_2+\Omega_2^*\alpha_4, \quad (9c)$$

$$\Delta\widetilde{\omega}_4\alpha_4 = \Omega_2\alpha_3+\Omega_3^*{\rm e}^{-{\rm i}\,\delta kz}\alpha_5, \quad (9d)$$

$$\Delta\widetilde{\omega}_5\alpha_5 = \Omega_3{\rm e}^{{\rm i}\,\delta kz}\alpha_4+W_{\rm m}, \quad (9e)$$

where we have introduced the notations $\Delta\widetilde{\omega}_j=\omega+{\rm i}\widetilde{\gamma}_j$, $j=1-5$. The above equations (9a)–(9e) can be solved in terms of $W_{\rm p}$ and $W_{\rm m}$, giving the results

$$\alpha_2 = \frac{\Delta\widetilde{\omega}_1\Omega_1^*\Omega_2^*\Omega_3^*}{\Delta\widetilde{\omega}_5 S+G}W_{\rm m}{\rm e}^{-{\rm i}\,\delta kz}+\frac{F}{\Delta\widetilde{\omega}_5 S+G}W_{\rm p}, \quad (10a)$$

$$\alpha_5 = \frac{\Delta\widetilde{\omega}_1\Omega_1\Omega_2\Omega_3}{\Delta\widetilde{\omega}_5 S+G}W_{\rm p}{\rm e}^{{\rm i}\,\delta kz}+\frac{S}{\Delta\widetilde{\omega}_5 S+G}W_{\rm m}. \quad (10b)$$

Likewise,

$$(\partial_z-{\rm i}\omega/c)W_{\rm p} = {\rm i}k_{02}\alpha_2, \quad (11a)$$

$$(\partial_z-{\rm i}\omega/c)W_{\rm m} = {\rm i}k_{05}\alpha_5. \quad (11b)$$

Inserting Eqs.(10a), (10b) into Eqs.(11a), (11b), we have

$$[\partial_z-{\rm i}(\omega/c+f_2)]W_{\rm p} = {\rm i}g_2\widetilde{W}_{\rm m}, \quad (12a)$$

$$[\partial_z-{\rm i}(\omega/c+f_5-\delta k)]\widetilde{W}_{\rm m} = {\rm i}g_5\widetilde{W}_{\rm p}, \quad (12b)$$

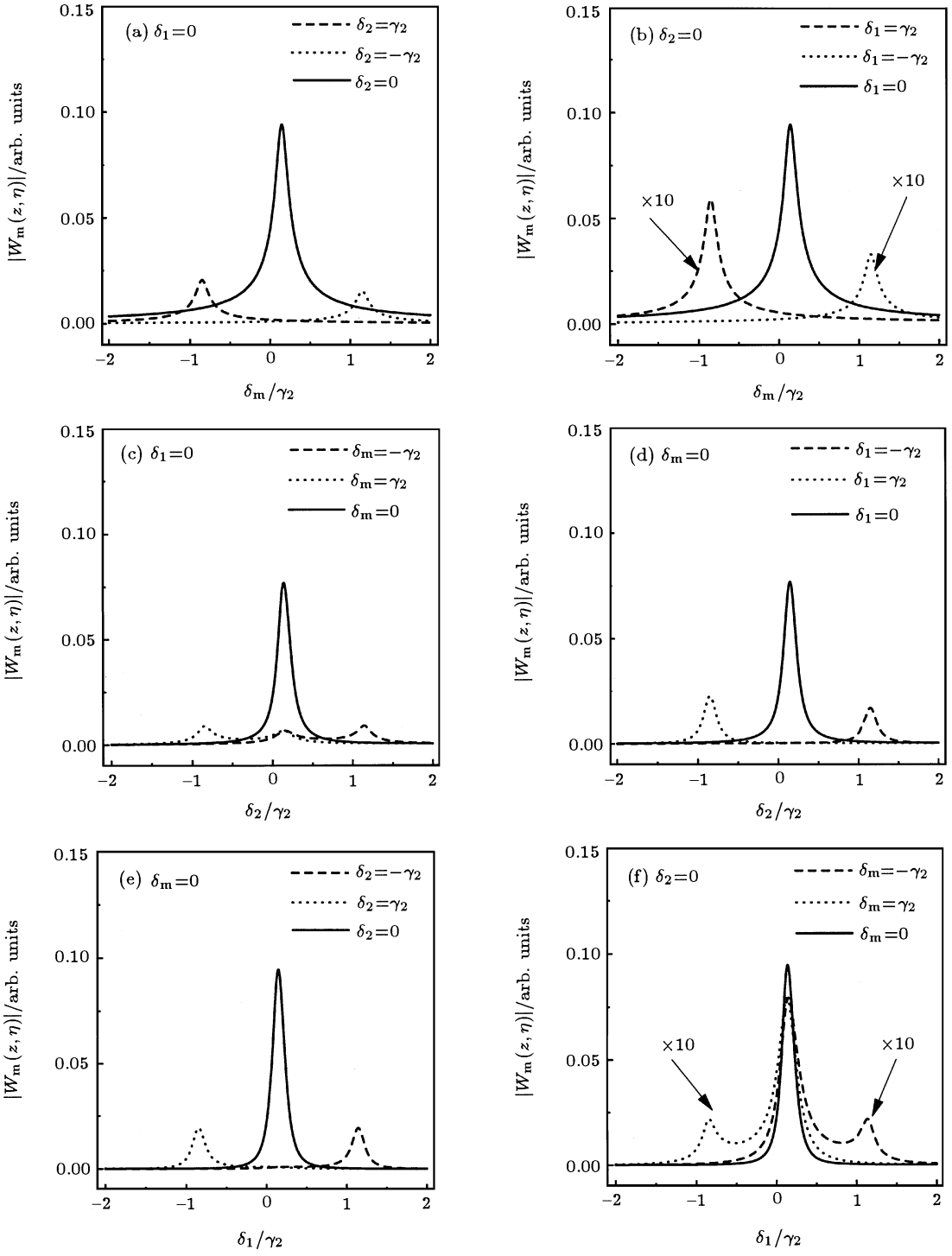



**Fig.3.** Plots of the generated mixing field $W_m$ as a function of the pump laser detuning $\delta_j$ ($j$=1, 2, m), for six different pump detunings. The fixed parameters used here are $\delta_p = \delta_c = 0$ (EIT condition), $\Omega_c = \gamma_2$, $\Omega_p = \Omega_1 = \Omega_2 = \Omega_3 = 0.03\gamma_2$, $\gamma_1 = 0.001\gamma_2$, $\gamma_3 = 0.2\gamma_2$, $\gamma_4 = 0.1\gamma_2$, $\gamma_5 = 0.08\gamma_2$, $\omega = 0.15\gamma_2$, $z = c/\gamma_2$, and $k_{02} = 100k_{05} = \gamma_2/z$.

where we have defined the new parameters

$$\widetilde{W}_m = W_m e^{i\delta kz},$$

$$f_2 = k_{02}\frac{F}{\Delta\widetilde{\omega}_5 S + G}, \quad g_2 = k_{02}\frac{\Delta\widetilde{\omega}_1\Omega_1^*\Omega_2^*\Omega_3^*}{\Delta\widetilde{\omega}_5 S + G},$$

$$f_5 = k_{05}\frac{S}{\Delta\widetilde{\omega}_5 S + G}, \quad g_5 = k_{05}\frac{\Delta\widetilde{\omega}_1\Omega_1\Omega_2\Omega_3}{\Delta\widetilde{\omega}_5 S + G},$$

$$S = \Delta\widetilde{\omega}_1\Delta\widetilde{\omega}_2\Delta\widetilde{\omega}_3\Delta\widetilde{\omega}_4 - \Delta\widetilde{\omega}_3\Delta\widetilde{\omega}_4|\Omega_c|^2 - \Delta\widetilde{\omega}_1\Delta\widetilde{\omega}_4|\Omega_1|^2 - \Delta\widetilde{\omega}_1\Delta\widetilde{\omega}_2|\Omega_2|^2 + |\Omega_c|^2|\Omega_2|^2,$$

$$G = \Delta\widetilde{\omega}_1|\Omega_1|^2|\Omega_3|^2 + \Delta\widetilde{\omega}_3|\Omega_c|^2|\Omega_3|^2 - \Delta\widetilde{\omega}_1\Delta\widetilde{\omega}_2\Delta\widetilde{\omega}_3|\Omega_3|^2,$$

$$F = \Delta\widetilde{\omega}_1\Delta\widetilde{\omega}_3\Delta\widetilde{\omega}_4\Delta\widetilde{\omega}_5 - \Delta\widetilde{\omega}_1\Delta\widetilde{\omega}_3|\Omega_3|^2 - \Delta\widetilde{\omega}_1\Delta\widetilde{\omega}_5|\Omega_2|^2.$$

After some algebra but with no further assumptions, for given $W_{\rm p}(0,\omega)$ and the CCMWM emission process $W_{\rm m}(0,\omega)=0$, the solutions to Eq.(12) are

$$W_{\rm m}(z,\omega)={\rm i}\frac{W_{\rm p}(0,\omega)g_5}{\Lambda}{\rm e}^{{\rm i}Dz}\sin(\Lambda z), \qquad (13)$$

where

$$D=\frac{\omega}{c}+\frac{f_2+f_5}{2}, \quad \Lambda=\left[\left(\frac{f_2-f_5}{2}\right)^2+g_2g_5\right]^{1/2}.$$

In order to show that the CCMWM scheme is capable of generating the mixing field at low pump intensities, we choose the Rabi frequencies $\Omega_{\rm p}$, $\Omega_1$, $\Omega_2$ and $\Omega_3$ of the probe and other three signal fields well below the saturation levels and present numerical simulations, as shown in Fig.3. The results show that $W_{\rm m}$ is maximized at (1) $\delta_1=0$, $\delta_2=0$ and $\delta_{\rm m}\approx 0.146\gamma_2$, (2) $\delta_1=0$, $\delta_{\rm m}=0$ and $\delta_2\approx 0.149\gamma_2$, (3) $\delta_2=0$, $\delta_{\rm m}=0$ and $\delta_1\approx 0.150\gamma_2$ under the usual EIT conditions, demonstrating the enhancement of the CCMWM via turning two signal fields to resonantly couple the transitions $|2\rangle\rightarrow|3\rangle\rightarrow|4\rangle\rightarrow|5\rangle$ while tuning the third signal field to non-resonant transition with approximate detuning $0.15\gamma_2$, manifested by the EIT-induced suppression of the two-photon, three-photon, four-photon and five-photon absorptions.

# 5. Conclusions

In summary, in the present work we present and analyze an optical cascade-configuration multi-wave mixing scheme for the generation of coherent light in the six-level atomic system by means of EIT, under the condition that we apply the weak probe pulsed laser. A detailed semi-classical study of the propagation of the generated mixing and probe fields is demonstrated. We show by numerical simulations that EIT is capable of suppressing linear and nonlinear photon absorption. The influences of the probe field and the respective detuning on the generated mixing field are discussed. In addition, we show that such a nonlinear optical process can be used for producing the coherent short-wavelength radiation.

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