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Coherent Landau states with gauge invariance*

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Coherent states for the Landau system (planar charged particle moving in a uniform magnetic field) are developed. It is convenient to use these states to describe the circular motion of the charged particle. The results obtained do not depend on gauges.

Keywords: Landau system, coherent state, gauge invariance
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1. Introduction

The Landau system is a very important two-dimensional quantum mechanical system. The study of the system has attracted considerable interest in both quantum mechanics and condensed matter physics. Apart from its intrinsic mathematical interest, this problem has attracted much attention because it related with many applications, such as cyclotron, quantum Hall effect, mass-spectrometer, magnetic resonance, etc. Though the Landau system's quantum level structure is similar to that of the two-dimensional harmonic oscillator, the Landau system (Osc(1) dynamical degeneracy group) is algebraically, geometrically and physically richer than the two-dimensional harmonic oscillator (SU_2 dynamical degeneracy group).[1−5] The canonical momentum or canonical angular momentum is used to describe the Landau system in the general quantum references,[6−12] but both the canonical momentum and the canonical angular momentum are dependent on gauges. Recently, we have introduced the centre coordinate operators and initial mechanical momentum operator which are all invariants with the gauge invariance, and obtained four kinds of raising and lowering operators of the Landau system and phase Landau states.[13−16] In this paper, we introduce coherent states for the Landau system by using the centre coordinate operators and initial mechanical momentum operators. The results obtained do not depend on gauges.

2. Two pairs of raising and lowering operators

Taking a uniform magnetic field with a magnetic induction $B(B>0)$ in the Z-axis direction, the mechanical momentum operator and the Hamiltonian of a planar charged particle are then

$$\pi_x = p_x - qA_x, \quad \pi_y = p_y - qA_y, \tag{1}$$

$$H = \frac{1}{2\mu}\pi_x^2 + \frac{1}{2\mu}\pi_y^2, \tag{2}$$

where μ, q are the mass and charge of the particle respectively and $\boldsymbol{A} = A_x\boldsymbol{e}_1 + A_y\boldsymbol{e}_2$ is the potential vector satisfying $\boldsymbol{B} = \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\boldsymbol{e}_3$. Note that

$$[x, \pi_x] = [y, \pi_y] = \mathrm{i}\hbar, \tag{3}$$

$$[\pi_x, \pi_y] = \mathrm{i}\hbar qB. \tag{4}$$

The Heisenberg equation of motion is

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{1}{\mathrm{i}\hbar}[x, H] = \dfrac{1}{\mu}\pi_x, \\ \dfrac{\mathrm{d}\pi_x}{\mathrm{d}t} = \dfrac{1}{\mathrm{i}\hbar}[\pi_x, H] = \omega\pi_y, \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = \dfrac{1}{\mathrm{i}\hbar}[y, H] = \dfrac{1}{\mu}\pi_y, \\ \dfrac{\mathrm{d}\pi_y}{\mathrm{d}t} = \dfrac{1}{\mathrm{i}\hbar}[\pi_y, H] = -\omega\pi_x. \end{cases} \tag{5}$$

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Its solution can be written as[13]

$$\begin{cases} x = x_{\rm c} - \dfrac{1}{qB}\pi_{y0}\cos\omega t + \dfrac{1}{qB}\pi_{x0}\sin\omega t, \\ y = y_{\rm c} + \dfrac{1}{qB}\pi_{x0}\cos\omega t + \dfrac{1}{qB}\pi_{y0}\sin\omega t, \\ \pi_x = \pi_{x0}\cos\omega t + \pi_{y0}\sin\omega t, \\ \pi_y = \pi_{y0}\cos\omega t - \pi_{x0}\sin\omega t, \end{cases} \tag{6}$$

where $\omega = \dfrac{qB}{\mu}$; $x_{\rm c}$ is the centre x-coordinate operator; $y_{\rm c}$ is the centre y-coordinate operator; π_{x0} is the x-component of the initial mechanical operator; π_{y0} is the y-component of the initial momentum operator. $x_{\rm c}$, $y_{\rm c}$, π_{x0} and π_{y0} are all invariants with the gauge invariance. Their relationships can be written as

$$[x_{\rm c}, y_{\rm c}] = -{\rm i}\frac{\hbar}{qB}, \tag{7}$$

$$[\pi_{x0}, \pi_{y0}] = {\rm i}\hbar qB, \tag{8}$$

$$[x_{\rm c}, \pi_{x0}] = [x_{\rm c},\ \pi_{y0}] = [y_{\rm c}, \pi_{x0}] = [y_{\rm c}, \pi_{y0}] = 0. \tag{9}$$

Two pairs of lowering and raising operators can be introduced by use of $x_{\rm c}$, $y_{\rm c}$ π_{x0} and π_{y0} as follows:

$$A_1 = \left(\frac{|q|B}{2\hbar}\right)^{1/2}\left(y_{\rm c} + {\rm i}\frac{q}{|q|}x_{\rm c}\right),$$

$$A_1^+ = \left(\frac{|q|B}{2\hbar}\right)^{1/2}\left(y_{\rm c} - {\rm i}\frac{q}{|q|}x_{\rm c}\right), \tag{10}$$

$$A_2 = \left(\frac{1}{2\hbar|q|B}\right)^{1/2}\left(\pi_{x0} + {\rm i}\frac{q}{|q|}\pi_{y0}\right),$$

$$A_2^+ = \left(\frac{1}{2\hbar|q|B}\right)^{1/2}\left(\pi_{x0} - {\rm i}\frac{q}{|q|}\pi_{y0}\right). \tag{11}$$

We have the following relations among them:

$$[A_1,\ A_1^+] = [A_2,\ A_2^+] = 1, \tag{12}$$

$$[A_1,\ A_2] = [A_1^+,\ A_2^+] = [A_1,\ A_2^+] = [A_1^+,\ A_2] = 0, \tag{13}$$

$$x_{\rm c}^2 + y_{\rm c}^2 = \left(A_1^+A_1 + \frac{1}{2}\right)\frac{2\hbar}{|q|B}, \quad H = \left(A_2^+A_2 + \frac{1}{2}\right)\hbar\omega. \tag{14}$$

3. Coherent Landau states

We are interested in common eigenstates $|\alpha_1, \alpha_2\rangle$ of A_1 and A_2 and call them coherent Landau states. We have results for the state $|\alpha_1, \alpha_2\rangle$

$$< x_{\rm c} > = < \alpha_1, \alpha_2|x_{\rm c}|\alpha_1, \alpha_2 > = {\rm i}\frac{q}{|q|}\left(\frac{\hbar}{2|q|B}\right)^{1/2}(\alpha_1^* - \alpha_1), \tag{15}$$

$$< y_{\rm c} > = < \alpha_1, \alpha_2|y_{\rm c}|\alpha_1, \alpha_2 > = \left(\frac{\hbar}{2|q|B}\right)^{1/2}(\alpha_1^* + \alpha_1), \tag{16}$$

$$< \pi_{y0} > = < \alpha_1, \alpha_2|\pi_{y0}|\alpha_1, \alpha_2 > = {\rm i}\frac{q}{|q|}\left(\frac{\hbar|q|B}{2}\right)^{1/2}(\alpha_2^* - \alpha_2), \tag{17}$$

$$< x_{\rm c}^2 + y_{\rm c}^2 > = < a_1, a_2|x_{\rm c}^2 + y_{\rm c}^2|a_1, a_2 > = \left(|a_1|^2 + \frac{1}{2}\right)\frac{2\hbar}{|q|B}, \tag{18}$$

$$< E > = < \alpha_1, \alpha_2|H|\alpha_1, \alpha_2 > = \left(|\alpha_2|^2 + \frac{1}{2}\right)\hbar|\omega|, \tag{19}$$

$$< x > = < \alpha_1, \alpha_2|x|\alpha_1, \alpha_2 > = < x_{\rm c} > - \frac{1}{qB} < \pi_{y0} > \cos\omega t + \frac{1}{qB} < \pi_{x0} > \sin\omega t, \tag{20}$$

$$< y > = < \alpha_1, \alpha_2|y|\alpha_1, \alpha_2 > = < y_{\rm c} > + \frac{1}{qB} < \pi_{x0} > \cos\omega t + \frac{1}{qB} < \pi_{y0} > \sin\omega t, \tag{21}$$

$$< \pi_x > = \frac{{\rm d} < x >}{{\rm d}t}, \quad < \pi_y > = \frac{{\rm d} < y >}{{\rm d}t}, \tag{22}$$

$$(\Delta x_{\rm c})^2 = (\Delta y_{\rm c})^2 = \frac{\hbar}{2|q|B}, \tag{23}$$

$$(\Delta\pi_{x0})^2 = (\Delta\pi_{y0})^2 = (\Delta\pi_x)^2 = (\Delta\pi_y)^2 = \frac{\hbar|q|B}{2}, \tag{24}$$

$$(\Delta x)^2 = (\Delta y)^2 = \frac{\hbar}{|q|B}, \tag{25}$$

$$\left[\Delta\left(\pi_x - \frac{qB}{2}y\right)\right]^2 = \left[\Delta\left(\pi_y + \frac{qB}{2}x\right)\right]^2 = \frac{\hbar|q|B}{4}, \tag{26}$$

$$[\Delta(x_{\rm c}^2 + y_{\rm c}^2)]^2 = |\alpha_1|^2\frac{4\hbar^2}{q^2B^2},$$

$$(\Delta E)^2 = |\alpha_2|^2\hbar^2\omega^2, \tag{27}$$

$$(\Delta x_{\rm c})(\Delta y_{\rm c}) = \frac{\hbar}{2|q|B}, \tag{28}$$

$$(\Delta\pi_{x0})(\Delta\pi_{y0}) = (\Delta\pi_x)(\pi_y) = \frac{\hbar|q|B}{2}, \tag{29}$$

$$(\Delta x)\left[\Delta\left(\pi_x - \frac{qB}{2}y\right)\right] = (\Delta y)\left[\Delta\left(\pi_y + \frac{qB}{2}x\right)\right] = \frac{\hbar}{2}. \tag{30}$$

These results describe the circular motion of charged particles in a uniform magnetic field. Because x, y, $x_{\rm c}$, $y_{\rm c}$, π_x, π_y, π_{x0} and π_{y0} are all real physical

quantities satisfying the gauge transformation rule, results (15)–(30) do not depend on gauges. In coherent Landau states $|\alpha_1, \alpha_2\rangle$, we achieve the minimum uncertainty relations (29) and (30). In some applications such as in a cyclotron (where the centre position of circular motion for particles is known) or a mass-spectrometer (where initial mechanical momentum of circular motion for particles is known), it is convenient to use the coherent Landau states.

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