Vol 13 No 10, October 2004
1009-1963/2004/13(10)/1639-05

Chinese Physics

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Using symbolic computation to construct travelling wave solutions to nonlinear partial differential equations*

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(Received 27 October 2003; revised manuscript received 31 May 2004)

Based upon the symbolic computation and the coupled projective Riccati equation, the tanh function method is further improved. As its applications, Wu–Zhang equation (which describes a (2+1)-dimensional dispersive long wave) and the (1+1)-dimensional dispersive long wave equation obtained from Wu–Zhang equation by scaling transformation and symmetry reduction are chosen to illustrate the validity of the proposed approach.

Keywords: Wu–Zhang equation, coupled projective Riccati equations, soliton wave solution, periodic wave solution, symbolic computation

PACC: 0340K, 0220, 0290

1. Introduction

The study of explicit solutions to various nonlinear partial differential equations (PDEs) has been very important in modern mathematics with ramifications in several areas, such as mathematics, physics and other sciences. There are several systemic approaches to explicit solutions to PDEs: for instance, the inverse scattering transformation, Hirota bilinear method, tanh function method, Jacobi elliptic functions method, etc.[1−20]

In Ref.[21], three sets of model equations have been derived for modelling nonlinear and dispersive long gravity waves travelling in two horizontal directions on shallow waters with uniform depth. Omitting the higher order terms, one of these equations, the Wu–Zhang equation, can be written as[21−23]

$$\begin{cases} u_t + uu_x + vu_y + w_x = 0, \\ v_t + uv_x + vv_y + w_y = 0, \\ w_t + (uw)_x + (vw)_y \\ +\dfrac{1}{3}(u_{xxx} + u_{xyy} + v_{xxy} + v_{yyy}) = 0, \end{cases} \tag{1}$$

where w is the elevation of the water wave, and u and v are, respectively, the surface velocities of water along the x-, y-directions. By scaling transformation and symmetry reduction, Eq.(1) can be reduced to the (1+1)-dimensional dispersive long wave equation[22,23]

$$\begin{cases} v_t + vv_y + w_y = 0, \\ w_t + (wv)_y + \dfrac{1}{3}v_{yyy} = 0. \end{cases} \tag{2}$$

A good understanding of all solutions of Eq.(1) is very helpful for coastal and civil engineers to apply the nonlinear water wave model in harbour and coastal design. Therefore, finding more types of solutions to Eqs.(1) and (2) is of fundamental interest in fluid dynamics.

Chen *et al*[22] have investigated the Painlevé property of Eq.(1) and obtained its two types of periodic wave solutions expressed by Jacobi elliptic functions using the WTC method, but found that they have no Painlevé property. Chen *et al*[23] have obtained new Jacobi elliptic functions solutions to Eq.(1) by solving Eq.(2).

In this paper, we find the travelling wave solutions to Eq.(1) and Eq.(2) directly by using our proposed method. As a result, their new solutions which contain soliton solutions and periodic solutions are obtained. The obtained solutions in this paper are not the special cases of the Chen results. This method is the extension of the one given in Refs.[8,10].

*Project supported by the State Key Programme of Basic Research of China (Grant No G1998030600).

2. The further improved tanh function method

In this section, a computational method is described step by step.

For the given PDEs

$$\begin{cases} F_1(u,v,u_t,v_t,u_x,v_x,u_{tt},v_{tt},u_{xx},v_{xx},\cdots)=0, \\ G_1(u,v,u_t,v_t,u_x,v_x,u_{tt},v_{tt},u_{xx},v_{xx},\cdots)=0, \end{cases} \tag{3}$$

we seek the following formal travelling wave solutions which are of important physical significance:

$$u(x,t)=u(\xi), \quad v(x,t)=v(\xi), \quad \xi=k(x-\lambda t), \tag{4}$$

where k and λ are constants to be determined later. Then Eq.(3) is reduced to the nonlinear ordinary differential equation

$$\begin{cases} F_2(u,v,u',v',u'',v'',\cdots)=0, \\ G_2(u,v,u',v',u'',v'',\cdots)=0, \end{cases} \tag{5}$$

where a prime denotes $\dfrac{\mathrm{d}}{\mathrm{d}\xi}$. We suppose that the solutions we are looking for can be written in the following form

$$\begin{cases} u=\sum\limits_{i=0}^{r} a_i g^i(\xi)+\sum\limits_{j=1}^{r} b_j g^{j-1}(\xi)f(\xi), \\ v=\sum\limits_{m=0}^{s} c_m g^m(\xi)+\sum\limits_{n=1}^{s} d_n g^{n-1}(\xi)f(\xi), \end{cases} \tag{6}$$

with

$$\begin{aligned} f(\xi)&=\frac{\tanh(\xi)}{A\tanh(\xi)+B\mathrm{sech}(\xi)+C}, \\ g(\xi)&=\frac{\mathrm{sech}(\xi)}{A\tanh(\xi)+B\mathrm{sech}(\xi)+C}, \end{aligned} \tag{7}$$

which satisfy

$$\begin{cases} f'(\xi)=\dfrac{g(\xi)}{C}((C^2-B^2)g(\xi)-ABf(\xi)+B), \\ g'(\xi)=\dfrac{g(\xi)}{C}(ABg(\xi)+(A^2-C^2)f(\xi)-A), \end{cases} \tag{8}$$

$$\begin{aligned} &(C^2-B^2)g^2(\xi) \\ =&1-2(Af(\xi)+Bg(\xi))+2ABf(\xi)g(\xi) \\ &+(A^2-C^2)f^2(\xi), \end{aligned} \tag{9}$$

where A, B and $C\neq 0$ are constants, a_i, b_j, c_m, d_n are constants to be determined later, r, s can be determined by balancing the highest order derivative term and the nonlinear term in Eq.(5).

The computational process is similar to the above one if we assume the functions f and g to be in the following forms:

$$\begin{aligned} f(\xi)&=\frac{\tan(\xi)}{A\tan(\xi)+B\sec(\xi)+C}, \\ g(\xi)&=\frac{\sec(\xi)}{A\tan(\xi)+B\sec(\xi)+C}, \end{aligned} \tag{10}$$

which satisfy

$$\begin{cases} f'(\xi)=\dfrac{g(\xi)}{C}((C^2-B^2)g(\xi)-ABf(\xi)+B), \\ g'(\xi)=\dfrac{g(\xi)}{C}(ABg(\xi)+(A^2+C^2)f(\xi)-A), \end{cases} \tag{11}$$

$$\begin{aligned} &(C^2-B^2)g^2(\xi) \\ =&1-2(Af(\xi)+Bg(\xi))+2ABf(\xi)g(\xi) \\ &+(A^2+C^2)f^2(\xi). \end{aligned} \tag{12}$$

Equations (8), (11) and expressions (9), (12) can obviously be written in the following unified forms:

$$\begin{cases} f'(\xi)=\dfrac{g(\xi)}{C}((C^2-B^2)g(\xi)-ABf(\xi)+B), \\ g'(\xi)=\dfrac{g(\xi)}{C}(ABg(\xi)+(A^2+\delta C^2)f(\xi)-A), \end{cases} \tag{13}$$

$$\begin{aligned} &(C^2-B^2)g^2(\xi) \\ =&1-2(Af(\xi)+Bg(\xi))+2ABf(\xi)g(\xi) \\ &+(A^2+\delta C^2)f^2(\xi), \end{aligned} \tag{14}$$

where $\delta=\pm 1$.

If $C=0$, we choose the following forms for expressions (7) and (10) respectively:

$$\begin{aligned} f_1(\xi)&=\frac{1}{A\tanh(\xi)+B\mathrm{sech}(\xi)}, \\ g_1(\xi)&=\frac{\mathrm{sech}(\xi)}{A\tanh(\xi)+B\mathrm{sech}(\xi)}, \end{aligned} \tag{15}$$

$$\begin{aligned} f_2(\xi)&=\frac{1}{A\tan(\xi)+B\sec(\xi)}, \\ g_2(\xi)&=\frac{\sec(\xi)}{A\tan(\xi)+B\sec(\xi)}, \end{aligned} \tag{16}$$

which satisfy

$$\begin{cases} f_i'(\xi)=-Ag_i^2(\xi)+\delta\dfrac{B}{A}g_i(\xi)(1-Bg_i(\xi)), \\ g_i'(\xi)=-Af_i(\xi)g_i(\xi), \end{cases} \tag{17}$$

$$f_i^2(\xi)=g_i^2(\xi)+\delta\frac{1}{A^2}(1-Bg_i(\xi))^2, \tag{18}$$

where $\delta=\pm 1$ and $i=1,2$ ($\delta=1$ when $i=1$).

With the aid of Maple, we can substitute Eqs.(6), (13), and (14) or Eqs.(6), (17), and (18) into Eq.(5) and collect the coefficients with the same power

$f^i(\xi)g^j(\xi)(i = 0,1; j = 0,1,\cdots,h)$. Set each of the obtained coefficients to be zero to obtain an overdetermined system of nonlinear algebraic equations w.r.t. the unknown variables k, λ, a_i, b_j, c_m and d_n. We apply Wu's elimination method[24] to solve the obtained algebraic equations, yielding the values of k, λ, a_i, b_j, c_m and d_n. Therefore the solutions to Eq.(3) are found by substituting these values into Eq.(6) along with Eqs.(7) and (10) or Eqs.(15) and (16).

The singular solutions of Eq.(3) are obtained if we rewrite the expressions (7), (15) and (10), (16).

3. The explicit travelling wave solutions to Eq.(1)

Let $u(x,y,t) = u(\xi)$, $v(x,y,t) = v(\xi)$, $w(x,y,t) = w(\xi)$, $\xi = k(x + ly - \lambda t)$, then Eq.(1) becomes

$$\begin{cases} -\lambda u' + uu' + lvu' + w' = 0, \\ -\lambda v' + uv' + lvv' + lw' = 0, \\ -\lambda w' + (uw)' + l(vw)' \\ +\dfrac{1}{3}k^2(u''' + l^2u''' + lv''' + l^3v''') = 0, \end{cases} \tag{19}$$

where $'$ denotes $\dfrac{\mathrm{d}}{\mathrm{d}\xi}$.

We suppose that Eq.(19) has the solutions expressed in the following forms

$$\begin{cases} u = a_0 + a_1 f(\xi) + a_2 g(\xi), \\ v = b_0 + b_1 f(\xi) + b_2 g(\xi), \\ w = c_0 + c_1 f(\xi) + c_2 g(\xi) + c_3 f(\xi)g(\xi) + c_4 g^2(\xi). \end{cases} \tag{20}$$

Substituting Eqs.(21, 13, 14) into Eq.(19) and collecting the coefficients with the same power $f^i(\xi)g^j(\xi)$ (i=0, 1; j=0, 1, 2, 3) and setting each of the obtained coefficients to be zero, we acquire a system of nonlinear algebraic equations, which here is omitted. Solving it by using of Wu's method and symbolic computation software Maple 8, we have the following cases.

When $\delta = 1$,

Case 1 $a_1 = b_1 = c_1 = c_3 = 0$, $a_2 = \dfrac{2\sqrt{3}kR}{3}$, $b_2 = \dfrac{2\sqrt{3}klR}{3}$, $c_2 = \dfrac{2Bk^2(1+l^2)}{3}$, $c_4 = -\dfrac{2k^2R^2(1+l^2)}{3}$, $c_0 = -\dfrac{(1+l^2)k^2A^2}{3R^2}$, $a_0 = -\dfrac{1}{3}\dfrac{-3\lambda R + 3lb_0R + kB\sqrt{3}(1+l^2)}{R}$,
where $R = \sqrt{A^2+B^2}$.

Case 2 c_1=c_2=c_3=c_4 $= 0$, $a_1 = -lb_1$, $a_0 = -lb_0 + \lambda$, $b_2 = -\dfrac{a_2}{l}$.

When $\delta = -1$,

Case 3 a_1=b_1=c_1 $= c_3 = 0$, $a_2 = \dfrac{2\sqrt{3}kT}{3}$, $b_2 = \dfrac{2\sqrt{3}klT}{3}$, $c_2 = -\dfrac{2Bk^2(1+l^2)}{3}$, $c_0 = \dfrac{(1+l^2)k^2A^2}{3T^2}$,

$$a_0 = \frac{1}{3}\frac{3\lambda T^2 - 3lb_0T^2 + \sqrt{3}kBT(1+l)^2}{T^2},$$

where $T = \sqrt{A^2+B^2}$.

Case 4 c_1=c_2=c_3=c_4 $= 0$, $a_1 = -lb_1$, $a_0 = -lb_0 + \lambda$, $b_2 = -\dfrac{a_2}{l}$.

Therefore, the solitary wave solutions of Eq.(1) are as follows:

$$\begin{cases} u_1 = -\dfrac{1}{3}\dfrac{-3\lambda R + 3lb_0R + kB\sqrt{3}(1+l^2)}{R} + \dfrac{2\sqrt{3}}{3}\dfrac{kR\,\mathrm{sech}(k(x+ly-\lambda t))}{A\tanh(k(x+ly-\lambda t)) + B\,\mathrm{sech}(k(x+ly-\lambda t))}, \\ v_1 = b_0 + \dfrac{2\sqrt{3}}{3}\dfrac{klR\,\mathrm{sech}(k(x+ly-\lambda t))}{A\tanh(k(x+ly-\lambda t)) + B\,\mathrm{sech}(k(x+ly-\lambda t))}, \\ w_1 = -\dfrac{(1+l^2)k^2A^2}{3R^2} + \dfrac{2}{3}\dfrac{Bk^2(1+l^2)\,\mathrm{sech}(k(x+ly-\lambda t))}{A\tanh(k(x+ly-\lambda t)) + B\,\mathrm{sech}(k(x+ly-\lambda t))} \\ \quad -\dfrac{2}{3}\dfrac{k^2R^2(1+l^2)\,\mathrm{sech}^2(k(x+ly-\lambda t))}{[A\tanh(k(x+ly-\lambda t)) + B\,\mathrm{sech}(k(x+ly-\lambda t))]^2}, \end{cases}$$

where $R = \sqrt{A^2+B^2}$ and $AB \neq 0$.

$$\begin{cases} u_2 = -lb_0 + \lambda - \dfrac{lb_1}{A\tanh(k(x+ly-\lambda t)) + B\,\mathrm{sech}(k(x+ly-\lambda t))} \\ \qquad + \dfrac{a_2\,\mathrm{sech}(k(x+ly-\lambda t))}{A\tanh(k(x+ly-\lambda t)) + B\,\mathrm{sech}(k(x+ly-\lambda t))}, \\ v_2 = b_0 + \dfrac{b_1}{A\tanh(k(x+ly-\lambda t)) + B\,\mathrm{sech}(k(x+ly-\lambda t))} \\ \qquad - \dfrac{a_2\,\mathrm{sech}(k(x+ly-\lambda t))}{l[A\tanh(k(x+ly-\lambda t)) + B\,\mathrm{sech}(k(x+ly-\lambda t))]}, \\ w_2 = c_0, \end{cases}$$

where a_2, b_1 are arbitrary constants and $AB \neq 0$.

The explicit periodic wave solutions become

$$\begin{cases} u_3 = \dfrac{1}{3}\dfrac{3\lambda T^2 - 3lb_0T^2 + \sqrt{3}kBT(1+l)^2}{T^2} + \dfrac{2\sqrt{3}}{3}\dfrac{kT\sec(k(x+ly-\lambda t))}{A\tan(k(x+ly-\lambda t)) + B\sec(k(x+ly-\lambda t))}, \\ v_3 = b_0 + \dfrac{2\sqrt{3}}{3}\dfrac{klT\sec(k(x+ly-\lambda t))}{A\tan(k(x+ly-\lambda t)) + B\sec(k(x+ly-\lambda t))}, \\ w_3 = \dfrac{(1+l^2)k^2A^2}{3T^2} - \dfrac{2}{3}\dfrac{Bk^2(1+l^2)\sec(k(x+ly-\lambda t))}{A\tan(k(x+ly-\lambda t)) + B\sec(k(x+ly-\lambda t))} \\ \qquad -\dfrac{2}{3}\dfrac{k^2T^2(1+l^2)\sec^2(k(x+ly-\lambda t))}{[A\tan(k(x+ly-\lambda t)) + B\sec(k(x+ly-\lambda t))]^2}, \end{cases}$$

where $T = \sqrt{A^2 - B^2}$ and $|\ A\ | > |\ B\ |$.

$$\begin{cases} u_4 = -lb_0 + \lambda - \dfrac{lb_1}{A\tan(k(x+ly-\lambda t)) + B\sec(k(x+ly-\lambda t))} \\ \qquad + \dfrac{a_2\sec(k(x+ly-\lambda t))}{A\tan(k(x+ly-\lambda t)) + B\sec(k(x+ly-\lambda t))}, \\ v_4 = b_0 + \dfrac{b_1}{A\tan(k(x+ly-\lambda t)) + B\sec(k(x+ly-\lambda t))} \\ \qquad - \dfrac{a_2\sec(k(x+ly-\lambda t))}{l[A\tan(k(x+ly-\lambda t)) + B\sec(k(x+ly-\lambda t))]}, \\ w_4 = c_0, \end{cases}$$

where a_2, b_1 are arbitrary constants and $AB \neq 0$.

4. The explicit travelling wave solutions to Eq.(2)

Let $v(y,t) = v(\xi)$, $w(y,t) = w(\xi)$, $\xi = k(y - \lambda t)$, then Eq.(2) becomes

$$\begin{cases} -\lambda v' + vv' + w' = 0, \\ -\lambda w + (vw)' + \dfrac{1}{3}v''' = 0, \end{cases} \tag{21}$$

where $'$ denotes $\dfrac{\mathrm{d}}{\mathrm{d}\xi}$.

We assume that Eq.(21) has the following solutions:

$$\begin{cases} v = a_0 + a_1 f(\xi) + a_2 g(\xi), \\ w = b_0 + b_1 f(\xi) + b_2 g(\xi) + b_3 f(\xi)g(\xi) + b_4 g^2(\xi). \end{cases} \tag{22}$$

Similarly, we can obtain the following cases.

When $\delta = 1$,

Case 1 $a_1 = b_1 = b_3 = 0$, $a_2 = \dfrac{2\sqrt{3}Mk}{3}$, $b_4 = -\dfrac{2}{3}M^2k^2$, $b_2 = \dfrac{2}{3}Bk^2$, $a_0 = \dfrac{3\lambda M^2 - \sqrt{3}kBM}{3M^2}$, $b_0 = \dfrac{(C^2 - A^2)k^2}{3M^2}$, where $M = \sqrt{A^2 + B^2 - C^2}$.

Case 2 $A = B = a_2 = b_0 = b_1 = b_2 = b_3 = 0$, $b_4 = \dfrac{2}{3}k^2C^2$, $a_1 = \dfrac{2\sqrt{3}}{3}kC$, $a_0 = \lambda$.

Case 3 $B = b_0 = b_1 = b_2 = b_3 = a_2 = 0$, $b_4 = \dfrac{2k^2(C^2 - A^2)}{3}$, $a_1 = \dfrac{2\sqrt{3}k(C^2 - A^2)}{3C}$, $a_0 = \dfrac{3\lambda C + 2\sqrt{3}kA}{3C}$.

When $\delta = -1$,

Case 4 $A = B = a_2 = b_0 = b_1 = b_2 = b_3 = 0$, $b_4 = -\dfrac{2}{3}k^2C^2$, $a_1 = \dfrac{2\sqrt{3}}{3}kC$, $a_0 = \lambda$.

Case 5 $B = b_0 = b_1 = b_2 = b_3 = a_2 = 0$, $b_4 = -\dfrac{2k^2(C^2 + A^2)}{3}$, $a_1 = \dfrac{2\sqrt{3}k(C^2 + A^2)}{3C}$, $a_0 = \dfrac{3\lambda C - 2\sqrt{3}kA}{3C}$.

From cases 1–3, we can obtain the solitary wave solutions of Eq.(2), namely

$$\begin{cases} v_1 = \dfrac{3\lambda M^2 - \sqrt{3}kBM}{3M^2} + \dfrac{2\sqrt{3}}{3}\dfrac{Mk\,\mathrm{sech}(k(y-\lambda t))}{A\tanh(k(y-\lambda t)) + B\mathrm{sech}(k(y-\lambda t)) + C}, \\ w_1 = \dfrac{(C^2 - A^2)k^2}{3M^2} + \dfrac{2}{3}\dfrac{k^2B\mathrm{sech}(k(y-\lambda t))}{A\tanh(k(y-\lambda t)) + B\mathrm{sech}(k(y-\lambda t)) + C} \\ \qquad -\dfrac{2}{3}\dfrac{k^2M^2\mathrm{sech}^2(k(x+ly-\lambda t))}{[A\tanh(k(y-\lambda t)) + B\mathrm{sech}(k(y-\lambda t)) + C]^2}, \end{cases}$$

where $M = \sqrt{A^2 + B^2 - C^2}$ and $A^2 + B^2 > C^2$.

$$\begin{cases} v_2 = \lambda + \dfrac{2\sqrt{3}}{3} k \tanh(k(y - \lambda t)), \\ w_2 = \dfrac{2}{3} k^2 \mathrm{sech}^2(k(y - \lambda t)); \end{cases}$$

$$\begin{cases} v_3 = \dfrac{3\lambda C + 2\sqrt{3} k A}{3C} + \dfrac{2\sqrt{3}}{3C} \dfrac{k(C^2 - A^2)\tanh(k(y - \lambda t))}{A\tanh(k(y - \lambda t)) + C}, \\ w_3 = \dfrac{2}{3} \dfrac{k^2(C^2 - A^2)\mathrm{sech}^2(k(y - \lambda t))}{[A\tanh(k(y - \lambda t)) + C]^2}. \end{cases}$$

From cases 4–5, the periodic solutions for Eq.(2) are as follows:

$$\begin{cases} v_4 = \lambda + \dfrac{2\sqrt{3}}{3} k \tan(k(y - \lambda t)), \\ w_4 = -\dfrac{2}{3} k^2 \mathrm{sech}^2(k(y - \lambda t)); \end{cases}$$

$$\begin{cases} v_5 = \dfrac{3\lambda C - 2\sqrt{3} k A}{3C} + \dfrac{2\sqrt{3}}{3C} \dfrac{k(C^2 + A^2)\tan(k(y - \lambda t))}{A\tan(k(y - \lambda t)) + C}, \\ w_5 = -\dfrac{2}{3} \dfrac{k^2(C^2 + A^2)\sec^2(k(y - \lambda t))}{[A\tan(k(y - \lambda t)) + C]^2}. \end{cases}$$

5. Conclusions

In this paper, we present a method and its constructive algorithm to seek for travelling wave solutions to PDEs. Abundant travelling wave solutions to Eqs.(1) and (2) are derived by using this method. Our method is easy to understand and can also apply to other PDEs. To ensure that the results are correct, we have verified the solutions by putting them back into the original equations using Maple 8.

References

[1] Ablowitz M J and Clarkson P A 1991 *Soliton, Nonlinear Evolution Equations and Inverse Scattering* (New York: Cambridge University Press)
[2] Conte R and Musette M 1992 *J. Phys A: Math. Gen.* **25** 5609
[3] Wadati M 1975 *J. Phys. Soc. Jpn.* **38** 673
[4] Gao Y T and Tian B 1997 *Comput Math. Appl.* **33** 35
[5] Gao Y T and Tian B 2002 *Phys. Lett.* A **301** 74
[6] Fan E G 2003 *Chaos, Solitons Fractals* **15** 559
[7] Fan E G 2000 *Phys. Lett.* A **277** 212
[8] Yan Z Y 2003 *Chaos, Solitons Fractals* **16** 759
[9] Xie F D *et al* 2001 *Phys. Lett.* A **285** 76
[10] Xie F D *et al* 2004 *Chaos, Solitons and Fractals* **19** 1113
[11] Zhang J F 2001 *Chin. Phys.* **10** 893
[12] Zhang J F 2002 *Chin. Phys.* **11** 425
[13] Liu S K *et al* 2001 *Acta Phys. Sin.* **50** 2068 (in Chinese)
[14] Liu S K *et al* 2001 *Phys. Lett.* A **289** 69
[15] Zhang S Q *et al* 2002 *Chin. Phys.* **11** 993
[16] Yao R X and Li Z B 2002 *Chin. Phys.* **11** 864
[17] Ruan H Y and Chen Y X 2000 *Acta Phys. Sin.* **49** 177 (in Chinese)
[18] Yan Q Y, Zhang Y F and Xei X P 2003 *Chin. Phys.* **12** 131
[19] Xu G Q and Li Z B 2002 *Acta Phys. Sin.* **51** 946 (in Chinese)
[20] Zhang W G 2003 *Chin. Phys.* **12** 144
[21] Wu T Y and Zhang J E 1996 On modeling nonlinear long wave *Mathematics is for Solving Problem* (ed. L P Look, V Rogthurd, M Tulin) (Philadelphia, PA: SIAM) 233
[22] Chen C L *et al* 2002 *Phys. Rev.* E **66** 036605
[23] Chen Y *et al* 2004 *Chin. Phys.* (in press)
[24] Wu W T 1994 *Algorithms and Computation* (Berlin: Springer)