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The KP and ZK equations for electrostatic waves with grain charge fluctuation*

Xue Ju-Kui(薛具奎)† and Lang He(郎 和)

College of Physics and Electronic Engineering, Northwest Normal University, Lanzhou 730070, China

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The propagation of three-dimensional nonlinear dust-acoustic and dust-Coulomb waves in unmagnetized/magnetized dusty plasmas consisting of electrons, ions, and charged dust particles is investigated. The grain charge fluctuation effect is also incorporated through the current balance equation. By using the perturbation method, a Kadomtsev–Petviashvili equation and a Zakharov–Kuznetsov equation governing the nonlinear waves in the unmagnetized and magnetized systems are obtained respectively. It has been shown that with the combined effects of grain charge fluctuation, the transverse perturbation, and the external magnetic field would modify the wave structures. Waves in those systems are unstable to the high-order long-wave perturbations.

Keywords: transverse perturbation, dust charge fluctuation, dust-acoustic and dust-Coulomb waves
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1. Introduction

Including heavily charged dust grains, the dynamics of a dusty plasma is very different, and the corresponding physical effects are basically distinct from those of an ordinary three-component plasma system. The presence of the charged dust grains in a plasma can modify the collective behaviour of the plasma, as well as excite new modes.[1−7,20] The charge on the dust grain, however, varies with space and time due to the electron and ion currents flowing into or out of the dust grain, as well as other processes such as the photo-emission of electrons. So the dust charge becomes a new dynamic variable and its dynamic nature is important for studying the behaviour of dusty plasmas. Recent linear and nonlinear studies [8−14] have indicated that the dust charge fluctuation could affect the wave-mode properties and lead to some new aspects of properties in tenuous and dense dust plasmas. The more interesting feature of charge fluctuations has been the dust-Coulomb wave in a dense dust plasma. Whilst the dust-acoustic wave can exist even in the constant dust charge model, the charge fluctuation is essential for the propagation of the dust-Coulomb wave. Therefore, the dust charge fluctuation plays a significant role for plasma collective motion. But all previous investigations of charge fluctuation effects have focused on the unmagnetized one-dimensional (1D) case. However, a purely unmagnetized 1D picture cannot explain the observed wave phenomena [15] in the low-altitude and higher-altitude auroral regions and a magnetized higher-dimensional model must be considered. The wave structure and stability in this higher-dimensional system will be modified because the anisotropy is introduced into the system. For example, at least some transverse perturbations will always exist in the higher-dimensional system even if the system is unmagnetized. Therefore, in the present paper, we study the three-dimensional (3D) nonlinear waves under the dust charge fluctuation effect. Using the perturbation method, a 3D Kadomtsev–Petviashvili (KP) equation in an unmagnetized dust plasma and a 3D Zakharov–Kuznetsov (ZK) equation in a magnetized dust plasma are deduced. The combined effects of grain charge fluctuation, the transverse perturbation, and the external magnetic field on the nonlinear wave structures and stability are discussed in detail.

2. Governing equations and derivation of the KP and ZK equations

We consider the propagation of dust-acoustic waves in a collisionless magnetized dusty plasma con-

*Project supported by the National Natural Science Foundation of China (Grant No 10347006).
†E-mail: xuejk@163.com

sists of hot electrons, ions, and finite-sized, negatively charged warm dust grains. The inertia of ions and electrons are neglected, thus, the ions and electron densities are governed by Boltzmann distribution and defined by their respective temperatures which can be assumed to be constant,

$$\begin{cases} n_{\rm e} = n_{\rm e0}\exp\left(\dfrac{e\phi}{\kappa T_{\rm e}}\right), \\ n_{\rm i} = n_{\rm i0}\exp\left(\dfrac{e\phi}{\kappa T_{\rm i}}\right). \end{cases} \tag{1}$$

The dust grains, on the other hand, being the heaviest component, provide the inertia, and therefore the wave dynamics are governed by the full set of 3D dust fluid equations including the Poisson equation:

$$\begin{cases} \dfrac{\partial n_{\rm d}}{\partial t} + \dfrac{\partial (n_{\rm d}u_{\rm d})}{\partial x} + \dfrac{\partial (n_{\rm d}v_{\rm d})}{\partial y} + \dfrac{\partial (n_{\rm d}w_{\rm d})}{\partial z} = 0, \\ \dfrac{\partial u_{\rm d}}{\partial t} + u_{\rm d}\dfrac{\partial u_{\rm d}}{\partial x} + v_{\rm d}\dfrac{\partial u_{\rm d}}{\partial y} + w_{\rm d}\dfrac{\partial u_{\rm d}}{\partial z} \\ + \dfrac{q_{\rm d}}{m_{\rm d}}\dfrac{\partial \phi}{\partial x} + \dfrac{V_{\rm td}^2}{n_{\rm d}}\dfrac{\partial n_{\rm d}}{\partial x} + \dfrac{q_{\rm d}}{m_{\rm d}}B_0 v_{\rm d} = 0, \\ \dfrac{\partial v_{\rm d}}{\partial t} + u_{\rm d}\dfrac{\partial v_{\rm d}}{\partial x} + v_{\rm d}\dfrac{\partial v_{\rm d}}{\partial y} + w_{\rm d}\dfrac{\partial v_{\rm d}}{\partial z} \\ + \dfrac{q_{\rm d}}{m_{\rm d}}\dfrac{\partial \phi}{\partial y} + \dfrac{V_{\rm td}^2}{n_{\rm d}}\dfrac{\partial n_{\rm d}}{\partial y} - \dfrac{q_{\rm d}}{m_{\rm d}}B_0 u_{\rm d} = 0, \\ \dfrac{\partial w_{\rm d}}{\partial t} + u_{\rm d}\dfrac{\partial w_{\rm d}}{\partial x} + v_{\rm d}\dfrac{\partial w_{\rm d}}{\partial y} + w_{\rm d}\dfrac{\partial w_{\rm d}}{\partial z} \\ + \dfrac{q_{\rm d}}{m_{\rm d}}\dfrac{\partial \phi}{\partial z} + \dfrac{V_{\rm td}^2}{n_{\rm d}}\dfrac{\partial n_{\rm d}}{\partial z} = 0, \\ \dfrac{\partial^2 \phi}{\partial x^2} + \dfrac{\partial^2 \phi}{\partial y^2} + \dfrac{\partial^2 \phi}{\partial z^2} + 4\pi(q_{\rm d}n_{\rm d} + en_{\rm i} - en_{\rm e}) = 0, \end{cases} \tag{2}$$

where $V_{\rm td} = (\kappa T_{\rm d}/m_{\rm d})^{1/2}$ is the dust thermal speed, $q_{\rm d}$ represents the charge and includes the sign of the dust charges, B_0 is the uniform magnetic field directed along the z-axis. Dust charge variation effects come in through the charge current balance equation,[12]

$$\frac{\partial q_{\rm d}}{\partial t} + u_{\rm d}\frac{\partial q_{\rm d}}{\partial x} + v_{\rm d}\frac{\partial q_{\rm d}}{\partial y} + w_{\rm d}\frac{\partial q_{\rm d}}{\partial z} = I_{\rm e} + I_{\rm i}, \tag{3}$$

which is valid for grain charging arising from plasma currents due to electrons and ions reaching the grain surface. When the streaming velocities of the electrons and ions are much smaller than their respective thermal velocities, the expressions for the electron and ion currents for spherical grains of radius R are given by

$$\begin{cases} I_{\rm e} = -\pi R^2 e\left(\dfrac{8\kappa T_{\rm e}}{\pi m_{\rm e}}\right)n_{\rm e}\exp\left(\dfrac{e\psi}{\kappa T_{\rm e}}\right), \\ I_{\rm i} = \pi R^2 e\left(\dfrac{8\kappa T_{\rm i}}{\pi m_{\rm i}}\right)n_{\rm i}\left(1 - \dfrac{e\psi}{\kappa T_{\rm e}}\right), \end{cases} \tag{4}$$

where $\psi = q_{\rm d}/R$ denotes the dust grain surface potential relative to the plasma potential.

2.1. Unmagnetized dust (B_0=0) and derivation of KP equation

To derive the KP equation that governs the behaviours of small but finite-amplitude waves in an unmagnetized (B_0=0) dust plasma, we assume that the wave propagates in the z-direction. However, there are higher-order transverse perturbations in the x–y plane. Hence, the independent variables can be extended as [16,17] $\xi = \varepsilon x$, $\eta = \varepsilon y$, $\zeta = \varepsilon^{1/2}(z - v_0 t)$ and $\tau = \varepsilon^{3/2}t$, where ε is a small parameter measuring the strength of the nonlinearity, v_0 is the phase velocity of the wave. The dependent variables are expanded as

$$\begin{cases} n_{\rm d} = n_{\rm d0} + \varepsilon n_{\rm d1} + \varepsilon^2 n_{\rm d2} + \cdots, \\ u_{\rm d} = \varepsilon^{3/2}u_{\rm d1} + \varepsilon^{5/2}u_{\rm d2} + \cdots, \\ v_{\rm d} = \varepsilon^{3/2}v_{\rm d1} + \varepsilon^{5/2}v_{\rm d2} + \cdots, \\ w_{\rm d} = \varepsilon w_{\rm d1} + \varepsilon^2 w_{\rm d2} + \cdots, \\ q_{\rm d} = q_{\rm d0} + \varepsilon q_{\rm d1} + \varepsilon^2 q_{\rm d2} + \cdots, \\ \phi = \varepsilon\phi_1 + \varepsilon^2\phi_2 + \cdots \end{cases} \tag{5}$$

Using the expansions of Eq.(5) and with the help of Eqs.(1) and (4), and omitting the damping due to charge fluctuation effects which were shown to be weak,[13,14] the charging equation (3) yields $O(\varepsilon)$ and $O(\varepsilon^2)$,

$$\begin{cases} \omega_1 q_{\rm d1} + R\omega_2\phi_1 = 0, \\ \dfrac{1}{2R}\omega_3\dfrac{e}{\kappa T_{\rm e}}q_{\rm d1}^2 + \dfrac{R}{2}\dfrac{e}{\kappa T_{\rm e}}\omega_4\phi_1^2 \\ + \dfrac{e}{\kappa T_{\rm e}}\omega_5\phi_1 q_{\rm d1} + R\omega_2\phi_2 + \omega_1 q_{\rm d2} = 0. \end{cases} \tag{6}$$

The charging frequencies appearing in Eq.(6) are defined through

$$\omega_1 = \frac{R}{\sqrt{2\pi}}\left[\frac{\omega_{\rm pi}}{\lambda_{\rm Di}} + \frac{\omega_{\rm pe}}{\lambda_{\rm De}}\exp\left(\frac{e\psi_0}{\kappa T_{\rm e}}\right)\right], \tag{7}$$

$$\omega_2 = \frac{R}{\sqrt{2\pi}}\left[\frac{\omega_{\rm pi}}{\lambda_{\rm Di}}\left(1 - \frac{e\psi_0}{\kappa T_{\rm i}}\right) + \frac{\omega_{\rm pe}}{\lambda_{\rm De}}\exp\left(\frac{e\psi_0}{\kappa T_{\rm e}}\right)\right], \tag{8}$$

$$\omega_3 = \omega_1 - \chi, \tag{9}$$

$$\omega_4 = \omega_2 - \chi\left(1 - \frac{e\psi_0}{\kappa T_{\rm i}}\right)\left(1 + \frac{T_{\rm e}}{T_{\rm i}}\right), \tag{10}$$

$$\omega_5 = \omega_1 - \chi\left(1 + \frac{T_{\rm e}}{T_{\rm i}}\right), \tag{11}$$

where

$$\omega_{{\rm p}j} = (4\pi n_{j0}e^2/m_j)^{1/2}, \quad \lambda_{{\rm D}j} = (\kappa T_j/4\pi n_{j0}e^2)^{1/2},$$

and the subscript "j" refers to "i" and "e", respectively. $\chi = \dfrac{R}{\sqrt{2\pi}}\dfrac{\omega_{\rm pi}}{\lambda_{\rm Di}}$.

Substituting Eqs.(5) into Eqs.(2) and collecting the terms in the different powers of ε, to lowest order in ε, we obtain

$$\begin{cases} n_{\rm d1} = \dfrac{n_{\rm d0}}{v_0} w_{\rm d1}, \\ w_{\rm d1} = \dfrac{q_{\rm d0}}{m_{\rm d} v_0}\phi_1 + \dfrac{V_{\rm td}^2}{n_{\rm d0} v_0} n_{\rm d1}, \\ \dfrac{\partial u_{\rm d1}}{\partial \zeta} = \dfrac{q_{\rm d0}}{m_{\rm d} v_0}\dfrac{\partial \phi_1}{\partial \xi} + \dfrac{V_{\rm td}^2}{n_{\rm d0} v_0}\dfrac{\partial n_{\rm d1}}{\partial \xi}, \\ \dfrac{\partial v_{\rm d1}}{\partial \zeta} = \dfrac{q_{\rm d0}}{m_{\rm d} v_0}\dfrac{\partial \phi_1}{\partial \eta} + \dfrac{V_{\rm td}^2}{n_{\rm d0} v_0}\dfrac{\partial n_{\rm d1}}{\partial \eta}, \\ \phi_1 = 4\pi\lambda_{\rm D}^2(q_{\rm d0} n_{\rm d1} + n_{\rm d0} q_{\rm d1}), \end{cases} \tag{12}$$

and the phase speed of the wave

$$v_0^2 = \frac{C_{\rm DA}^2}{1 + f\delta} + V_{\rm td}^2, \tag{13}$$

where $\lambda_{\rm D}^2 = (\lambda_{\rm Di}^{-2} + \lambda_{\rm De}^{-2})^{-1}$ is the global Debye length, $C_{\rm DA} = \omega_{\rm pD}\lambda_{\rm D}$ is the dust-acoustic speed, $\omega_{\rm pD} = (4\pi n_{\rm d0} q_{\rm d0}^2/m_{\rm d})^{1/2}$ is the dust plasma frequency, $f = 4\pi n_{\rm d0} R\lambda_{\rm D}^2$ is the dust fugacity parameter, and $\delta = \omega_2/\omega_1$. Equation (13) contains the characteristic speed of both the dust-acoustic waves and dust-Coulomb waves for a tenuous plasma ($f\delta \ll 1$) and dense plasma ($f\delta \gg 1$), respectively.

For the next higher order in ε, Eq.(2) can be written as

$$\begin{cases} \dfrac{\partial n_{\rm d1}}{\partial \tau} - v_0\dfrac{\partial n_{\rm d2}}{\partial \zeta} + n_{\rm d0}\dfrac{\partial w_{\rm d2}}{\partial \zeta} + \dfrac{\partial (n_{\rm d1} w_{\rm d1})}{\partial \zeta} + n_{\rm d0}\left(\dfrac{\partial u_{\rm d1}}{\partial \xi} + \dfrac{\partial v_{\rm d1}}{\partial \eta}\right) = 0, \\ \dfrac{\partial w_{\rm d1}}{\partial \tau} - v_0\dfrac{\partial w_{\rm d2}}{\partial \zeta} + w_{\rm d1}\dfrac{\partial w_{\rm d1}}{\partial \zeta} + \dfrac{1}{m_{\rm d}}\left(q_{\rm d0}\dfrac{\partial \phi_2}{\partial \zeta} + q_{\rm d1}\dfrac{\partial \phi_1}{\partial \zeta}\right) + \dfrac{V_{\rm td}^2}{n_{\rm d0}}\left(\dfrac{\partial n_{\rm d2}}{\partial \zeta} - \dfrac{1}{n_{\rm d0}} n_{\rm d1}\dfrac{\partial n_{\rm d1}}{\partial \zeta}\right) = 0, \\ \dfrac{1}{4\pi}\dfrac{\partial^2 \phi_1}{\partial \zeta^2} + n_{\rm d1} q_{\rm d1} + \dfrac{1}{8\pi}\dfrac{e}{\kappa T_{\rm e}}\left(\dfrac{1}{\lambda_{\rm Di}^2}\dfrac{T_{\rm e}}{T_{\rm i}} - \dfrac{1}{\lambda_{\rm De}^2}\right)\phi_1^2 - \dfrac{1}{4\pi\lambda_{\rm D}^2}\phi_2 + n_{\rm d0} q_{\rm d2} + q_{\rm d0} n_{\rm d2} = 0. \end{cases} \tag{14}$$

Now, using Eqs.(6), (12), (14) and eliminating $n_{\rm d2}$, $w_{\rm d2}$, ϕ_2, one obtains the following KP equation:

$$\frac{\partial}{\partial \zeta}\left(\frac{\partial \phi_1}{\partial \tau} + A\phi_1\frac{\partial \phi_1}{\partial \zeta} + B\frac{\partial^3 \phi_1}{\partial \zeta^3}\right) + C\left(\frac{\partial^2 \phi_1}{\partial \xi^2} + \frac{\partial^2 \phi_1}{\partial \eta^2}\right) = 0, \tag{15}$$

where the coefficients A, B, and C are

$$\begin{aligned} A = & \frac{q_{\rm d0}}{2 m_{\rm d} v_0}\left\{\frac{(1+\delta f)}{C_{\rm DA}^2}(3v_0^2 - V_{\rm td}^2) - \frac{3\delta f}{1+\delta f} + \frac{m_{\rm d} e}{\kappa T_{\rm e} q_{\rm d0}}\frac{C_{\rm DA}^2}{(1+f\delta)^2} \right. \\ & \times \left[\lambda_{\rm D}^2\left(\frac{1}{\lambda_{\rm Di}^2}\frac{T_{\rm e}}{T_{\rm i}} - \frac{1}{\lambda_{\rm De}^2}\right) \right. \\ & \left.\left. - f\left(\frac{\omega_3}{\omega_1}\delta^2 + \frac{\omega_4}{\omega_1} - 2\frac{\omega_5}{\omega_1}\delta\right)\right]\right\}, \end{aligned} \tag{16}$$

$$B = \frac{\lambda_{\rm D}^2 C_{\rm DA}^2}{2 v_0 (1+f\delta)^2}, \tag{17}$$

$$C = -\frac{v_0}{2}. \tag{18}$$

2.2. Magnetized dust and derivation of ZK equation

When there is a strong magnetic field ($B_0 \neq 0$), anisotropy must be introduced into the system and the perpendicular velocity components $u_{\rm d}$ and $v_{\rm d}$ will appear at higher order in ε than does the parallel velocity component $w_{\rm d}$. Hence, the independent variables can be stretched as[16,18] $\xi = \varepsilon^{1/2}x$, $\eta = \varepsilon^{1/2}y$, $\zeta = \varepsilon^{1/2}(z - v_0 t)$ and $\tau = \varepsilon^{3/2}t$. The dependent variables are expanded as

$$\begin{cases} n_{\rm d} = n_{\rm d0} + \varepsilon n_{\rm d1} + \varepsilon^2 n_{\rm d2} + \cdots, \\ u_{\rm d} = \varepsilon^{3/2} u_{\rm d1} + \varepsilon^2 u_{\rm d2} + \cdots, \\ v_{\rm d} = \varepsilon^{3/2} v_{\rm d1} + \varepsilon^2 v_{\rm d2} + \cdots, \\ w_{\rm d} = \varepsilon w_{\rm d1} + \varepsilon^2 w_{\rm d2} + \cdots, \\ q_{\rm d} = q_{\rm d0} + \varepsilon q_{\rm d1} + \varepsilon^2 q_{\rm d2} + \cdots, \\ \phi = \varepsilon\phi_1 + \varepsilon^2\phi_2 + \cdots \end{cases} \tag{19}$$

Using Eq.(19) we can obtain the same expressions as Eqs.(6)–(11) from the charge equation (3). Then, substituting the perturbation expansions of Eq.(19) into Eq.(2) and equating the coefficients of different powers of ε on both sides, to the lowest order in ε, we have

$$\begin{cases} n_{\rm d1} = \dfrac{n_{\rm d0}}{v_0} w_{\rm d1}, \\ w_{\rm d1} = \dfrac{q_{\rm d0}}{m_{\rm d} v_0}\phi_1 + \dfrac{V_{\rm td}^2}{n_{\rm d0} v_0} n_{\rm d1}, \\ \dfrac{\partial u_{\rm d1}}{\partial \zeta} = \dfrac{q_{\rm d0} B_0}{m_{\rm d} v_0} v_{\rm d2}, \\ \dfrac{\partial v_{\rm d1}}{\partial \zeta} = -\dfrac{q_{\rm d0} B_0}{m_{\rm d} v_0} u_{\rm d2}, \\ \phi_1 = 4\pi\lambda_{\rm D}^2(q_{\rm d0} n_{\rm d1} + n_{\rm d0} q_{\rm d1}), \end{cases} \tag{20}$$

and the same phase speed of the wave as Eq.(13) is obtained. For the next higher order in ε, using Eq.(2), one can deduce

$$\begin{cases} \dfrac{\partial n_{\rm d1}}{\partial \tau} - v_0 \dfrac{\partial n_{\rm d2}}{\partial \zeta} + n_{\rm d0} \dfrac{\partial w_{\rm d2}}{\partial \zeta} + \dfrac{\partial (n_{\rm d1} w_{\rm d1})}{\partial \zeta} + n_{\rm d0} \left(\dfrac{\partial u_{\rm d2}}{\partial \xi} + \dfrac{\partial v_{\rm d2}}{\partial \eta} \right) = 0, \\ \dfrac{\partial w_{\rm d1}}{\partial \tau} - v_0 \dfrac{\partial w_{\rm d2}}{\partial \zeta} + w_{\rm d1} \dfrac{\partial w_{\rm d1}}{\partial \zeta} + \dfrac{1}{m_{\rm d}} \left(q_{\rm d0} \dfrac{\partial \phi_2}{\partial \zeta} + q_{\rm d1} \dfrac{\partial \phi_1}{\partial \zeta} \right) + \dfrac{V_{\rm td}^2}{n_{\rm d0}} \left(\dfrac{\partial n_{\rm d2}}{\partial \zeta} - \dfrac{1}{n_{\rm d0}} n_{\rm d1} \dfrac{\partial n_{\rm d1}}{\partial \zeta} \right) = 0, \\ \dfrac{1}{4\pi} \left(\dfrac{\partial^2 \phi_1}{\partial \xi^2} + \dfrac{\partial^2 \phi_1}{\partial \eta^2} + \dfrac{\partial^2 \phi_1}{\partial \zeta^2} \right) + n_{\rm d1} q_{\rm d1} + \dfrac{1}{8\pi} \dfrac{e}{\kappa T_{\rm e}} \left(\dfrac{1}{\lambda_{\rm Di}^2} \dfrac{T_{\rm e}}{T_{\rm i}} - \dfrac{1}{\lambda_{\rm De}^2} \right) \phi_1^2 - \dfrac{1}{4\pi \lambda_{\rm D}^2} \phi_2 + n_{\rm d0} q_{\rm d2} + q_{\rm d0} n_{\rm d2} = 0. \end{cases} \tag{21}$$

Now, using Eqs.(6), (20), (21) and eliminating $n_{\rm d2}$, $w_{\rm d2}$, ϕ_2, one obtains the following ZK equation:

$$\frac{\partial \phi_1}{\partial \tau} + A \phi_1 \frac{\partial \phi_1}{\partial \zeta} + B \frac{\partial^3 \phi_1}{\partial \zeta^3} + D \frac{\partial}{\partial \zeta} \left(\frac{\partial^2 \phi_1}{\partial \xi^2} + \frac{\partial^2 \phi_1}{\partial \eta^2} \right) = 0, \tag{22}$$

where the coefficients A and B are the same as those defined in Eqs.(16), (17), and the coefficient D is given by

$$D = \frac{v_0 C_{\rm DA}^2}{2\omega_{\rm cd}^2 (1 + f\delta)} \left[1 + \frac{V_{\rm td}^2}{C_{\rm DA}^2} (1 + \delta f) \right], \tag{23}$$

where $\omega_{\rm cd}^2 = \left(\dfrac{q_{\rm d0} B_0}{m_{\rm d}} \right)^2$ is the dust-cyclotron frequency. It is clear that the magnetic field effect is included only in coefficient D.

3. Solutions of the KP and ZK equation and discussion

3.1. KP equation

If there are no transverse direction perturbations, the KP equation (15) reduces to the ordinary Korteweg–de Vris (KdV) equation. Thus we can express the steady state solution $\phi_1 = \phi$ of this KdV equation as

$$\phi = \left(\frac{3U_0}{A} \right) {\rm sech}^2 \left[\sqrt{\frac{U_0}{4B}} (\xi - U_0 \tau) \right]. \tag{24}$$

It is noticed that if $A < 0$, there exist solitary waves with negative potential or compressive solitary waves, and if $A > 0$, there exist solitary waves with positive potential or rarefactive solitary waves. Equation (16) indicates that the sign of A is related to many parameters of dust, such as dust charging, ions, and electrons. For simplicity, we discuss the cases where the charging frequencies, ω_i (i=1, 2, 3, 4, 5), are of the same order of magnitude $\left(\text{i.e. } \dfrac{\omega_2}{\omega_1} \approx \dfrac{\omega_3}{\omega_1} \approx \dfrac{\omega_4}{\omega_1} \approx \dfrac{\omega_5}{\omega_1} \right)$, and $T_{\rm i} \approx T_{\rm e} = T$. These are approximations for most dusty plasmas of current interest. So, the nonlinear coefficient A (Eq.16) reduces to

$$A = \frac{3\omega_{\rm pd}^2}{2ev_0 (n_{\rm e0} - n_{\rm i0})} \times \left[1 - \frac{f}{1+f} - \frac{1}{3} \left(\frac{n_{\rm e0} - n_{\rm i0}}{n_{\rm e0} + n_{\rm i0}} \right)^2 \frac{1}{(1+f)^2} + \frac{2}{3} \left(\frac{V_{\rm td}}{C_{\rm DA}} \right)^2 (1+f)^2 \right]. \tag{25}$$

In obtaining Eq.(25), the equilibrium charge neutrality condition $n_{\rm d0} q_{\rm d0} + e n_{\rm i0} - e n_{\rm d0} = 0$ has been used. We further assume that the dust is cold. Then we obtain

$$A = \frac{3\omega_{\rm pd}^2}{2ev_0 (n_{\rm e0} - n_{\rm i0})} \times \left[1 - \frac{f}{1+f} - \frac{1}{3} \left(\frac{n_{\rm e0} - n_{\rm i0}}{n_{\rm e0} + n_{\rm i0}} \right)^2 \frac{1}{(1+f)^2} \right]. \tag{26}$$

For negatively charged dust grains $n_{\rm i0} \gg n_{\rm e0}$, hence, the case of tenuous plasmas $f \ll 1$ (the dust-acoustic wave is known to exist in this regime), we have $v_0 \to C_{\rm DA}$ and

$$A = \frac{3\omega_{\rm pd}^2}{2ev_0 (n_{\rm e0} - n_{\rm i0})} \times \left[1 - f - \frac{1}{3} \left(\frac{n_{\rm e0} - n_{\rm i0}}{n_{\rm e0} + n_{\rm i0}} \right)^2 \right] < 0. \tag{27}$$

That is, there exist solitary waves with negative potential or compressive solitary waves. On the other hand, for the case of dense plasmas $f \gg 1$ (the dust-Coulomb wave is known to exist in this regime), we have $v_0 \to \dfrac{C_{\rm DA}}{\sqrt{f}}$, and Eq.(26) reduces to

$$A = \frac{3\omega_{\rm pd}^2}{2ev_0 (n_{\rm e0} - n_{\rm i0})} \times \left[- \frac{1}{3} \left(\frac{n_{\rm e0} - n_{\rm i0}}{n_{\rm e0} + n_{\rm i0}} \right)^2 \frac{1}{f^2} \right] > 0. \tag{28}$$

Hence we have solitary waves with positive potential or rarefactive solitary waves.

When the transverse direction perturbations is taken into account, the solitary solution of the KP equation (15) is unstable because of the positive dispersion ($C < 0$). Therefore, the solitary waves in both tenuous plasma ($f \ll 1$) and dense plasma ($f \gg 1$) are unstable to the transverse perturbations.

3.2. ZK equation

The ZK equation (22) admits higher-dimensional solutions, such as plane solitons, 2D cylindrical solitons, and 3D spherical solitons, etc. For simplicity, here we just consider the plane solitary wave solutions, $\phi_1 = \phi(\chi)$, of the ZK equation, where χ is the coordinate,

$$\chi = \zeta \cos\alpha + \xi \sin\alpha - U\tau. \tag{29}$$

In Eq.(29), α is the angle between the direction of wave propagation and that of the magnetic field B_0, and U is the speed of the soliton in the ξ–η–ζ frame. Without loss of generality, we assume that the wave propagates in the ξ–ζ plane because of cylindrical symmetry about the ζ axis. Using the transformation (29) and assuming that ϕ, together with its χ-derivatives, vanishes for $\chi \to \infty$, the ZK equation (22) can be integrated immediately and yields

$$\phi(\xi,\eta,\zeta) = \frac{3U/\cos\alpha}{A} \times \mathrm{sech}^2\left[\left(\frac{1}{4}\frac{U/\cos\alpha}{B\cos^2\alpha + D\sin^2\alpha}\right)^{1/2}\chi\right]. \tag{30}$$

It is obvious that the angle of obliquity α affects both the soliton amplitude and soliton width, but the magnetic field affects only the soliton width. We also can see from Eq.(30) that the ZK equation admits both compressive and rarefractive solitary waves, as in the discussion of the KP equation. The stability of the ZK equation has been studied by many authors. It has been found that the ZK equation (22) is unstable to the higher-order long-wave perturbations.[19]

4. Conclusion

In conclusion, we have derived the 3D KP and ZK equations governing the nonlinear behaviour of the system, including charge fluctuations, transverse perturbations without magnetic field, and the strong magnetic field effects. For the pure cold case, the dust-acoustic wave ($f \ll 1$) governed by the KP and ZK equations produces compressive solitons, whilst the dust-Coulomb wave ($f \gg 1$) governed by the KP and ZK equations produces rarefactive solitons. Both the dust-acoustic wave and the dust-Coulomb wave are unstable to the high-order long-wave perturbations.

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