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The dispersive properties of a dielectric-rod loaded waveguide immersed in a magnetized annular plasma

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Propagation properties of electromagnetic waves in a dielectric-rod waveguide immersed in a magnetized annular plasma are presented in this paper. The dispersion relations are derived and calculated. The results show that the dielectric-rod loading can make the structure less dispersive and the transmission frequency-band broadened.

Keywords: dielectric-rod, magnetized annular plasma, dispersion equation
PACC: 5000, 5200, 5240D

1. Introduction

In recent years, there has been growing interest in high-power and high-frequency microwave devices for generating radiation with millimetre and submillimetre wavelengths. An electron beam propagating through a slow-wave structure (SWS) coupled with the slow electromagnetic modes of the system can emit coherent short-wavelength radiation. The SWS may be a dielectric material (plasma and dielectric-rod, etc.). It is well known that the phase velocity of an electromagnetic wave is reduced after filling dielectric materials in the transmission area of all kinds of microwave devices. Meanwhile, the dispersion will change the propagation characteristic of the electromagnetic wave.

There are two methods to load the dielectric materials in the waveguide: namely, by using a dielectric waveguide and by placing a dielectric-rod along the axis of the waveguide. Although a lot of work has been done on the study of propagation properties of the dielectric waveguide filled with plasmas,[1−6] in this paper we will study the dielectric Cherenkov maser with an annular plasma, an axial dielectric rod in the presence of external finite magnetic field. For filling the waveguide with an axial dielectric rod[7−11] and the magnetized plasmas,[12−15] the dispersion of the SWS becomes weak, i.e. there is a small phase velocity change of the electromagnetic wave in the dielectric-rod-loaded waveguide within a broad frequency region. This leads to an effective interaction between the beam and the slow waves in the circuit with a relatively wider band. After filling the waveguide with dielectric materials, the position of the maximum E_z is changed and hence makes the frequency region broadened and the dispersive properties weakened.

2. "Cold" dispersion

Figure 1 shows the transverse section of the axial dielectric-rod-loaded waveguide structure immersed in a magnetized annular plasma.[11−14] The radius of the dielectric-rod is $r_{\rm d}$. R_c, R_2 and R_1 denote the outer radii of regions I ($R_2 < r \leq R_c$), II ($R_1 \leq r \leq R_2$) and III ($r_{\rm d} < r < R_1$), respectively. Region IV is the region ($0 < r \leq r_{\rm d}$). We study the structure by using the field theory, and assume that the electromagnetic wave propagation factor is $\exp\{{\rm j}\omega t - \gamma z + {\rm j}m\theta\}$, $k^2 = \omega^2\mu_0\varepsilon_0$, $\gamma = {\rm j}k_z$, k_z is the wave number of propagation along the axial z-direction.

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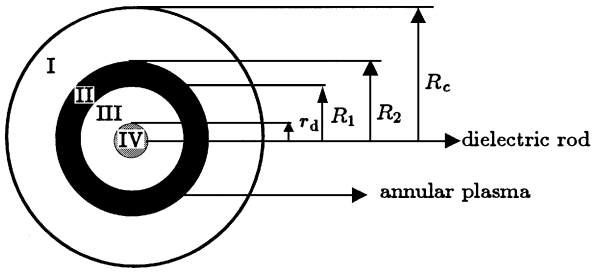


Fig.1. Transverse cross section of the dielectric-rod loaded waveguide immersed in a magnetized annular plasma.

2.1. Field components

In region I: $(R_2 < r \leq R_c)$

$$
\begin{aligned}
E_z^{\mathrm{I}} =& A_1 \mathrm{J}_0(k_c r) + A_2 \mathrm{N}_0(k_c r),\\
H_z^{\mathrm{I}} =& A_3 \mathrm{J}_0(k_c r) + A_4 \mathrm{N}_0(k_c r),\\
E_\theta^{\mathrm{I}} =& -\frac{1}{k_c^2}\{\mathrm{j}\omega\mu_0 k_c[A_3 \mathrm{J}_1(k_c r) + A_4 \mathrm{N}_1(k_c r)]\},\\
H_\theta^{\mathrm{I}} =& \frac{1}{k_c^2}\{\mathrm{j}\omega\varepsilon_0 k_c[A_1 \mathrm{J}_1(k_c r) + A_2 \mathrm{N}_1(k_c r)]\},\\
E_r^{\mathrm{I}} =& \frac{\mathrm{j}k_z}{k_c}[A_1 \mathrm{J}_1(k_c r) + A_2 \mathrm{N}_1(k_c r)],\\
H_r^{\mathrm{I}} =& \frac{\mathrm{j}k_z}{k_c}[A_3 \mathrm{J}_1(k_c r) + A_4 \mathrm{N}_1(k_c r)].
\end{aligned}
$$

In region II: $(R_1 < r \leq R_2)$

$$
\begin{aligned}
E_z^{\mathrm{II}} =& A_5 \mathrm{J}_0(p_1 r) + A_6 \mathrm{N}_0(p_1 r)\\
&+ A_7 \mathrm{J}_0(p_2 r) + A_8 \mathrm{N}_0(p_2 r),\\
H_z^{\mathrm{II}} =& A_5 h_1 \mathrm{J}_0(p_1 r) + A_6 h_1 \mathrm{N}_0(p_1 r)\\
&+ A_7 h_2 \mathrm{J}_0(p_2 r) + A_8 h_2 \mathrm{N}_0(p_2 r),\\
E_r^{\mathrm{II}} =& \frac{1}{D}\{\mathrm{j}k_z K^2[A_5 p_1 \mathrm{J}_0(p_1 r) + A_6 p_1 \mathrm{N}_0(p_1 r)\\
&+ A_7 p_2 \mathrm{J}_0(p_2 r) + A_8 p_2 \mathrm{N}_0(p_2 r)]\\
&- \omega\mu_0 k_g^2[A_5 h_1 p_1 \mathrm{J}_0(p_1 r) + A_6 h_1 p_1 \mathrm{N}_0(p_1 r)\\
&+ A_7 h_2 p_2 \mathrm{J}_0(p_2 r) + A_8 h_2 p_2 \mathrm{N}_0(p_2 r)]\},\\
E_\theta^{\mathrm{II}} =& \frac{1}{D}\{-k_z k_g^2[A_5 p_1 \mathrm{J}_0(p_1 r) + A_6 p_1 \mathrm{N}_0(p_1 r)\\
&+ A_7 p_2 \mathrm{J}_0(p_2 r) + A_8 p_2 \mathrm{N}_0(p_2 r)]\\
&- \mathrm{j}\omega\mu_0 K^2[A_5 h_1 p_1 \mathrm{J}_0(p_1 r) + A_6 h_1 p_1 \mathrm{N}_0(p_1 r)\\
&+ A_7 h_2 p_2 \mathrm{J}_0(p_2 r) + A_8 h_2 p_2 \mathrm{N}_0(p_2 r)]\},\\
H_r^{\mathrm{II}} =& \frac{1}{D}\{\omega\varepsilon_0\varepsilon_g k_z^2[A_5 p_1 \mathrm{J}_0(p_1 r) + A_6 p_1 \mathrm{N}_0(p_1 r)\\
&+ A_7 p_2 \mathrm{J}_0(p_2 r) + A_8 p_2 \mathrm{N}_0(p_2 r)]\\
&+ \mathrm{j}k_z K^2[A_5 h_1 p_1 \mathrm{J}_0(p_1 r) + A_6 h_1 p_1 \mathrm{N}_0(p_1 r)\\
&+ A_7 h_2 p_2 \mathrm{J}_0(p_2 r) + A_8 h_2 p_2 \mathrm{N}_0(p_2 r)]\},\\
H_\theta^{\mathrm{II}} =& \frac{1}{D}\{\mathrm{j}\omega\varepsilon_0(\varepsilon_1 K^2 - \varepsilon_g k_g^2)[A_5 p_1 \mathrm{J}_0(p_1 r)\\
&+ A_6 p_1 \mathrm{N}_0(p_1 r) + A_7 p_2 \mathrm{J}_0(p_2 r) + A_8 p_2 \mathrm{N}_0(p_2 r)]\\
&- k_z k_g^2[A_5 h_1 p_1 \mathrm{J}_0(p_1 r) + A_6 h_1 p_1 \mathrm{N}_0(p_1 r)\\
&+ A_7 h_2 p_2 \mathrm{J}_0(p_2 r) + A_8 h_2 p_2 \mathrm{N}_0(p_2 r)]\}.
\end{aligned}
$$

In region III: $(r_\mathrm{d} < r \leq R_1)$

$$
\begin{aligned}
E_z^{\mathrm{III}} =& A_9 \mathrm{J}_0(k_c r) + A_{10} \mathrm{N}_0(k_c r),\\
H_z^{\mathrm{III}} =& A_{11} \mathrm{J}_0(k_c r) + A_{12} \mathrm{N}_0(k_c r),\\
E_\theta^{\mathrm{III}} =& -\frac{\mathrm{j}\omega\mu_0}{k_c}[A_{11} \mathrm{J}_1(k_c r) + A_{12} \mathrm{N}_1(k_c r)],\\
H_\theta^{\mathrm{III}} =& \frac{\mathrm{j}\omega\varepsilon_0}{k_c}[A_9 \mathrm{J}_1(k_c r) + A_{10} \mathrm{N}_1(k_c r)],\\
E_r^{\mathrm{III}} =& \frac{\mathrm{j}k_z}{k_c}[A_9 \mathrm{J}_1(k_c r) + A_{10} \mathrm{N}_1(k_c r)],\\
H_r^{\mathrm{III}} =& \frac{\mathrm{j}k_z}{k_c}[A_{11} \mathrm{J}_1(k_c r) + A_{12} \mathrm{N}_1(k_c r)].
\end{aligned}
$$

In region IV: $(0 < r \leq r_\mathrm{d})$

$$
\begin{aligned}
E_z^{\mathrm{IV}} =& A_{13} \mathrm{J}_0(pr),\\
H_z^{\mathrm{IV}} =& A_{14} \mathrm{J}_0(pr),\\
E_\theta^{\mathrm{IV}} =& -\frac{\mathrm{j}\omega\mu_0}{p} A_{14} \mathrm{J}_1(pr),\\
H_\theta^{\mathrm{IV}} =& \frac{\mathrm{j}\omega\varepsilon_0}{p} A_{13} \mathrm{J}_1(pr),\\
E_r^{\mathrm{IV}} =& \frac{jk_z}{p} A_{13} \mathrm{J}_1(pr),\\
H_r^{\mathrm{IV}} =& \frac{jk_z}{p} A_{14} \mathrm{J}_1(pr).
\end{aligned}
$$

Here $k_c^2 = \gamma^2 + k^2$, $p^2 = \gamma^2 + k^2\varepsilon_\mathrm{d}$, $k^2 = \omega^2\mu_0\varepsilon_0$, $\gamma = \mathrm{j}kz$, ε_d is the dielectric-rod relative permittivity.

2.2. Boundary condition

At $r = R_c$, the tangential E-fields and H-fields are continuous

$$E_z^{\mathrm{I}} = 0, \quad E_\theta^{\mathrm{I}} = 0.$$

At $r = R_2$, we have

$$E_z^{\mathrm{I}} = E_z^{\mathrm{II}}, \quad E_\theta^{\mathrm{I}} = E_\theta^{\mathrm{II}},$$

$$H_z^{\mathrm{I}} = H_z^{\mathrm{II}}, \quad H_\theta^{\mathrm{I}} = H_\theta^{\mathrm{II}}.$$

At $r = R_1$,

$$E_z^{\mathrm{II}} = E_z^{\mathrm{III}}, \quad H_z^{\mathrm{II}} = H_z^{\mathrm{III}},$$

$$E_\theta^{\mathrm{II}} = E_\theta^{\mathrm{III}}, \quad H_\theta^{\mathrm{II}} = H_\theta^{\mathrm{III}}.$$

At $r = r_\mathrm{d}$,

$$E_z^{\mathrm{III}} = E_z^{\mathrm{IV}}, \quad H_z^{\mathrm{III}} = H_z^{\mathrm{IV}},$$

$$E_\theta^{\mathrm{III}} = E_\theta^{\mathrm{IV}}, \quad H_\theta^{\mathrm{III}} = H_\theta^{\mathrm{IV}}.$$

2.3. Dispersion equation

By using field components of every region to replace the components of boundary condition, and through complicated mathematic processes, we can obtain the following equation:

$$[D] \cdot \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_{14} \end{bmatrix} = 0, \tag{1}$$

where $[D]$ is a matrix consisting of coefficients in front of the parameters $(A_1, A_2, \cdots, A_{14})$ after substituting field components into the boundary condition. It can be expressed in the following forms:

$$[D] = \begin{bmatrix} a_{1,1} & \cdots & a_{1,14} \\ \vdots & \vdots & \vdots \\ a_{14,1} & \cdots & a_{14,14} \end{bmatrix}. \tag{2}$$

In order to gain correct results of Eq.(1), the determinant value of the coefficients matrix (2) must be zero, so we have

$$|D| = 0. \tag{3}$$

Every coefficient of expression (3) contains k_z and ω, so expression (3) is the "cold" dispersion equation. The elements in matrix (2) has been given in Ref.[14] and the Appendix of this paper.

3. Numerical results and discussion

The dispersion equation (3) is a very complicated transcendental equation including integrals associated with Bessel functions and the summations of an infinite series. From the numerical solution of Eq.(3), the dispersive relations and electric fields of the electromagnetic wave in the high frequency structure are obtained in the case of different rod radius and relative permittivity. Figures 2 and 3 show the dispersion curves for the dielectric-rod supported waveguide, which are obtained by solving Eq.(3) numerically for the following parameters: the plasma density is $n_{\rm p} = 4 \times 10^{11}{\rm cm}^{-3}$, external magnetic field is B_0=1.0T, the inner and outer radii of the annular plasma are R_1=7.5mm and R_2=8.5mm, and the radii of the dielectric-rod is $r_{\rm d}$=3.0mm, "A" denotes the case without dielectric-rod loading.

It can be seen clearly from Fig.2 that the cut-off frequency of the system is getting lower as the dielectric-rod relative permittivity increases, and the system can propagate fast waves and slow waves. In other words, we can obtain a broader band amplifier in the dielectric-loaded SWS. At the same time, we can find that the dispersion of the structure is getting weaker.

The influence of the rod radius on the dispersion is illustrated in Fig.3, in which the physical parameters are: the relative permittivity of the dielectric-rod is $\varepsilon_{\rm d}$=27, the radius $r_{\rm d}$=2.0, 2.5, 3.0 or 3.5mm. From Fig.3, we can see that the dispersive curves of different relative permittivity of the dielectric-rod are similar to that of different rod radius; with increasing rod radius, the frequency band is broadened and dispersion weakened.

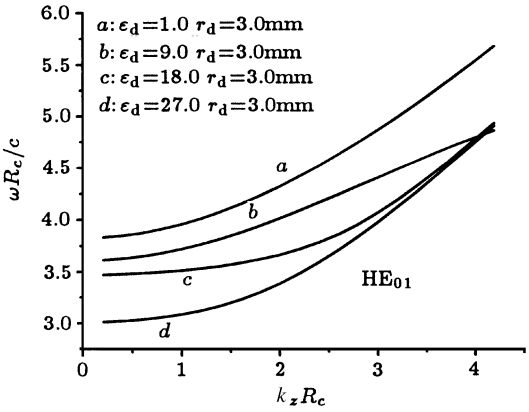


Fig.2. Dispersive curves of the dielectric-rod loaded SWS for different relative permittivity.

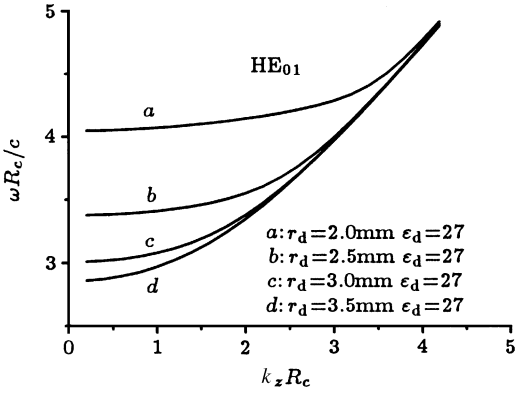


Fig.3. Dispersive curves of the dielectric-rod loaded SWS for different radius of the rod.

Figure 4 displays the influence of the relative permittivity $\varepsilon_{\rm d}$ on the distribution of the longitudinal normalized electric field $E_z(r)/(\beta_0 P^{1/2})$. The profiles of E_z are shown for the several values of dielectric permittivity $\varepsilon_{\rm d}$. regions I, II, III, IV are marked in

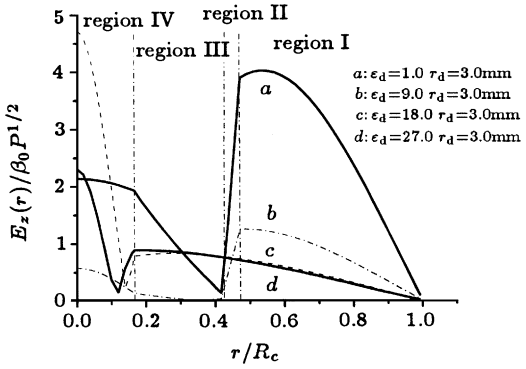


Fig.4. Longitudinal electric field profiles for the several values of dielectric-rod permittivity.

Fig.4, it is evident from Fig.4 that the electric field amplitude decreases with the relative permittivity ε_d. At the surfaces of the rod and annular plasma it reaches extrema. For the dielectric materials loaded decrease the longitudinal electric field makes the intensity of beam–wave interaction weaken.

4. Conclusion

In this paper, we have presented a dielectric-rod loaded waveguide immersed in a magnetized annular plasma as a slow-wave structure for a travelling wave tube or backward wave tube and we put the emphasis on the effect of the dielectric-rod parameters on the dispersion property of the structure. It is shown that the dielectric-rod loading can make the SWS less dispersive and the frequency broad, but reduces the beam–wave interaction. In order to enhance the gain, frequency-band and output power of the tube, we can load dielectric-rod, annular plasma and let the annular electron beam inject into the tube at the position of the maximum longitudinal electric field shown in Fig.4.

Appendix

The elements in matrix (2):

$$
\begin{aligned}
a_{1,1} &= \mathrm{J}_0(k_c R_c),\\
a_{1,2} &= \mathrm{N}_0(k_c R_c),\\
a_{1,3} &= \cdots = a_{1,14} = 0;
\end{aligned}
$$

$$
\begin{aligned}
a_{2,1} &= a_{2,2} = 0,\\
a_{2,3} &= -\frac{\omega}{k_c}\mathrm{J}_1(k_c R_c),\\
a_{2,4} &= -\frac{\omega}{k_c}\mathrm{N}_1(k_c R_c),\\
a_{2,5} &= \cdots = a_{2,14} = 0;
\end{aligned}
$$

$$
\begin{aligned}
a_{3,1} &= \mathrm{J}_0(k_c R_2),\\
a_{3,2} &= \mathrm{N}_0(k_c R_2),\\
a_{3,3} &= a_{3,4} = 0,\\
a_{3,5} &= -\mathrm{J}_0(P_1 R_2),\\
a_{3,6} &= -\mathrm{N}_0(P_1 R_2),\\
a_{3,7} &= -\mathrm{J}_0(P_2 R_2),\\
a_{3,8} &= -\mathrm{N}_0(P_2 R_2),\\
a_{3,9} &= \cdots = a_{3,14} = 0;
\end{aligned}
$$

$$
\begin{aligned}
a_{4,1} &= 0,\\
a_{4,2} &= 0,\\
a_{4,3} &= \omega \mathrm{J}_0(k_c R_2),\\
a_{4,4} &= \omega \mathrm{N}_0(k_c R_2),\\
a_{4,5} &= -H_1 \mathrm{J}_0(P_1 R_2),\\
a_{4,6} &= -H_1 \mathrm{N}_0(P_1 R_2),\\
a_{4,7} &= -H_2 \mathrm{J}_0(P_2 R_2),\\
a_{4,8} &= -H_2 \mathrm{N}_0(P_2 R_2),\\
a_{4,9} &= \cdots = a_{4,14} = 0;
\end{aligned}
$$

$$
\begin{aligned}
a_{5,1} &= a_{5,2} = 0,\\
a_{5,3} &= -\frac{\omega}{k_c}\mathrm{J}_1(k_c R_2),\\
a_{5,4} &= -\frac{\omega}{k_c}\mathrm{N}_1(k_c R_2),\\
a_{5,5} &= -\mathrm{J}_1(P_1 R_2)(k_z k_g^2 P_1 + K^2 P_1 H_1)/D,\\
a_{5,6} &= -\mathrm{N}_1(P_1 R_2)(k_z k_g^2 P_1 + K^2 P_1 H_1)/D,\\
a_{5,7} &= -\mathrm{J}_1(P_2 R_2)(k_z k_g^2 P_2 + K^2 P_2 H_2)/D,\\
a_{5,8} &= -\mathrm{N}_1(P_2 R_2)(k_z k_g^2 P_2 + K^2 P_2 H_2)/D,\\
a_{5,9} &= \cdots = a_{5,14} = 0;
\end{aligned}
$$

$$
\begin{aligned}
a_{6,1} &= -\frac{\omega}{k_c}\mathrm{J}_1(k_c R_2),\\
a_{6,2} &= -\frac{\omega}{k_c}\mathrm{N}_1(k_c R_2),\\
a_{6,3} &= a_{6,4} = 0,\\
a_{6,5} &= -P_1 \mathrm{J}_1(P_1 R_2)\left[\omega(\varepsilon_1 K^2 - \varepsilon_g k_g^2) + \frac{k_z k_g^2 H_1}{\omega}\right] \Big/ D,\\
a_{6,6} &= -P_1 \mathrm{N}_1(P_1 R_2)\left[\omega(\varepsilon_1 K^2 - \varepsilon_g k_g^2) + \frac{k_z k_g^2 H_1}{\omega}\right] \Big/ D,\\
a_{6,7} &= -P_2 \mathrm{J}_1(P_2 R_2)\left[\omega(\varepsilon_1 K^2 - \varepsilon_g k_g^2) + \frac{k_z k_g^2 H_2}{\omega}\right] \Big/ D,\\
a_{6,8} &= -P_2 \mathrm{N}_1(P_2 R_2)\left[\omega(\varepsilon_1 K^2 - \varepsilon_g k_g^2) + \frac{k_z k_g^2 H_2}{\omega}\right] \Big/ D,\\
a_{6,9} &= \cdots = a_{6,14} = 0;
\end{aligned}
$$

$$
\begin{aligned}
a_{7,1} &= \cdots = a_{7,4} = 0,\\
a_{7,5} &= -\mathrm{J}_0(P_1 R_1),\\
a_{7,6} &= -\mathrm{N}_0(P_1 R_1),\\
a_{7,7} &= -\mathrm{J}_0(P_2 R_1),\\
a_{7,8} &= -\mathrm{N}_0(P_2 R_1),\\
a_{7,9} &= \mathrm{J}_0(k_c R_1),\\
a_{7,10} &= \mathrm{N}_0(k_c R_1),\\
a_{7,11} &= \cdots = a_{7,14} = 0;
\end{aligned}
$$

$$
\begin{aligned}
a_{8,1} &= \cdots = a_{8,4} = 0,\\
a_{8,5} &= -H_1 \mathrm{J}_0(P_1 R_1),\\
a_{8,6} &= -H_1 \mathrm{N}_0(P_1 R_1),\\
a_{8,7} &= -H_2 \mathrm{J}_0(P_2 R_1),
\end{aligned}
$$

$$
\begin{aligned}
a_{8,8} &= -H_2\mathrm{N}_0(P_2R_1),\\
a_{8,9} &= a_{8,10} = 0,\\
a_{8,11} &= \omega\mathrm{J}_0(k_cR_1),\\
a_{8,12} &= \omega\mathrm{N}_0(k_cR_1),\\
a_{8,13} &= a_{8,14} = 0;\\
a_{9,1} &= \cdots = a_{9,4} = 0,\\
a_{9,5} &= -\mathrm{J}_1(P_1R_1)(k_zk_g^2P_1 + K^2P_1H_1)/D,\\
a_{9,6} &= -\mathrm{N}_1(P_1R_1)(k_zk_g^2P_1 + K^2P_1H_1)/D,\\
a_{9,7} &= -\mathrm{J}_1(P_2R_1)(k_zk_g^2P_2 + K^2P_2H_2)/D,\\
a_{9,8} &= -\mathrm{N}_1(P_2R_1)(k_zk_g^2P_2 + K^2P_2H_2)/D,\\
a_{9,9} &= a_{9,10} = 0,\\
a_{9,11} &= -\frac{\omega}{k_c}\mathrm{J}_1(k_cR_1),\\
a_{9,12} &= -\frac{\omega}{k_c}\mathrm{N}_1(k_cR_1),\\
a_{9,13} &= a_{9,14} = 0;\\
a_{10,1} &= \cdots = a_{10,4} = 0,\\
a_{10,5} &= -P_1\mathrm{J}_1(P_1R_1)\left[\omega(\varepsilon_1K^2 - \varepsilon_gk_g^2) + \frac{k_zk_g^2H_1}{\omega}\right]\Big/D,\\
a_{10,6} &= -P_1\mathrm{N}_1(P_1R_1)\left[\omega(\varepsilon_1K^2 - \varepsilon_gk_g^2) + \frac{k_zk_g^2H_1}{\omega}\right]\Big/D,\\
a_{10,7} &= -P_2\mathrm{J}_1(P_2R_1)\left[\omega(\varepsilon_1K^2 - \varepsilon_gk_g^2) + \frac{k_zk_g^2H_2}{\omega}\right]\Big/D,\\
a_{10,8} &= -P_2\mathrm{N}_1(P_2R_1)\left[\omega(\varepsilon_1K^2 - \varepsilon_gk_g^2) + \frac{k_zk_g^2H_2}{\omega}\right]\Big/D,\\
a_{10,9} &= -\frac{\omega}{k_c}\mathrm{J}_1(k_cR_1),\\
a_{10,10} &= -\frac{\omega}{k_c}\mathrm{N}_1(k_cR_1),\\
a_{10,11} &= \cdots = a_{10,14} = 0;\\
a_{11,1} &= \cdots = a_{11,8} = 0,\\
a_{11,9} &= -\mathrm{J}_0(k_cr_\mathrm{d}),\\
a_{11,10} &= -\mathrm{N}_0(k_cr_\mathrm{d}),\\
a_{11,11} &= = a_{11,12} = 0,\\
a_{11,13} &= \mathrm{J}_0(pr_\mathrm{d}),\\
a_{11,14} &= 0;\\
a_{12,1} &= \cdots = a_{12,10} = 0,\\
a_{12,11} &= -\mathrm{J}_0(k_cr_\mathrm{d}),\\
a_{12,12} &= -\mathrm{N}_0(k_cr_\mathrm{d}),\\
a_{12,13} &= 0,\\
a_{12,14} &= \mathrm{J}_0(pr_\mathrm{d});\\
a_{13,1} &= \cdots = a_{13,10} = 0,\\
a_{13,11} &= -\frac{\omega}{k_c}\mathrm{J}_1(k_cr_\mathrm{d}),\\
a_{13,12} &= -\frac{\omega}{k_c}\mathrm{N}_1(k_cr_\mathrm{d}),\\
a_{13,13} &= 0,\\
a_{13,14} &= -\frac{\omega}{p}\mathrm{J}_1(pr_\mathrm{d});\\
a_{14,1} &= \cdots = a_{14,8} = 0,\\
a_{14,9} &= \frac{\omega}{k_c}\mathrm{J}_1(k_cr_\mathrm{d}),\\
a_{14,10} &= \frac{\omega}{k_c}\mathrm{N}_1(k_cr_\mathrm{d}),\\
a_{14,11} &= a_{14,12} = 0,\\
a_{14,13} &= -\frac{\omega}{p}\mathrm{J}_1(pr_\mathrm{d}),\\
a_{14,14} &= 0.
\end{aligned}
$$

Here

$$
\begin{aligned}
&H_1' = \mathrm{j}\omega\mu_0H_1, \quad H_2' = \mathrm{j}\omega\mu_0H_2,\\
&h_1' = \mathrm{j}\omega\mu_0h_1, \quad h_2' = \mathrm{j}\omega\mu_0h_2,\\
&D = K^4 - k_g^4, \quad K^2 = \gamma^2 + k^2\varepsilon_1, \quad \gamma = \mathrm{j}k_z,\\
&k_g^2 = \omega^2\mu_0\varepsilon_0\varepsilon_g = k^2\varepsilon_g, \quad \varepsilon_g = |\varepsilon_2|,\\
&\varepsilon_2 = -\mathrm{j}\varepsilon_g, \quad h_{1,2} = \frac{(\gamma^2 + k^2\varepsilon_1)\dfrac{\varepsilon_3}{\varepsilon_1} - P_{1,2}^2}{\mathrm{j}\omega\mu_0\gamma\dfrac{\varepsilon_2}{\varepsilon_1}},
\end{aligned}
$$

$$
\begin{aligned}
P_{1,2}^2 = &\frac{1}{2\varepsilon_1}[\gamma^2(\varepsilon_1 + \varepsilon_3) + k^2(\varepsilon_1\varepsilon_3 + \varepsilon_1^2 + \varepsilon_2^2)]\\
&\pm \frac{1}{2\varepsilon_1}\{[\gamma^2(\varepsilon_3 - \varepsilon_1) + k^2(\varepsilon_1\varepsilon_3 - \varepsilon_1^2 - \varepsilon_2^2)]^2\\
&+ 4k^2\gamma^2\varepsilon_2^2\varepsilon_3\}^{1/2}.
\end{aligned}
$$

$$
\varepsilon_1 = 1 - \frac{\xi^2(1 - \mathrm{j}\delta)}{(1 - \mathrm{j}\delta)^2 - \tau^2}, \quad \varepsilon_2 = \frac{\mathrm{j}\tau\xi^2}{(1 - \mathrm{j}\delta)^2 - \tau^2},
$$

$$
\varepsilon_3 = 1 - \frac{\xi^2}{1 - \mathrm{j}\delta}, \quad \xi = \frac{\omega_\mathrm{p}}{\omega}, \quad \tau = \frac{\omega_\mathrm{c}}{\omega},
$$

$$
\delta = \frac{\gamma_\mathrm{eff}}{\omega}, \quad B_0 = B_0\vec{e}_z.
$$

$$
\omega_\mathrm{p} = \left(\frac{n_\mathrm{e}e^2}{m\varepsilon_0}\right)^{1/2}, \quad \omega_\mathrm{c} = \frac{eB_0}{m}.
$$

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