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Mode correlation and coherent-mode decomposition of laser beams*

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Theory of the coherent-mode decomposition of laser beams is proposed. The new model for the flat-topped beams proposed by Li recently is taken as an example. The analytical expressions for the M^2-factor and mode coherence coefficients of flat-topped beams have been derived in the rectangular coordinate system, by means of which the mode correlation, mode structure, and coherent-mode decomposition of flat-topped beams can be analysed.

Keywords: coherent-mode decomposition, M^2-factor, mode coherence coefficients (MCCs), flat-topped beams

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1. Introduction

In many practical cases, laser beams generated from high-power lasers usually result in a complex multi-mode structure and possess partial coherence, because of the high gain, large aperture, amplification of spontaneous emission, nonlinear optical effects, diffraction of cavity mirrors, as well as the variation of laser controlling parameters. Du *et al*[1,2] proved theoretically and experimentally that different transverse modes of laser beams are more or less correlated to each other. The degree of correlation depends on the width of apertures and the types of lasers. Moreover, they developed a set of complex iteration method to calculate the mode structures and the corresponding mode coherence coefficients (MCCs) of laser beams, by means of which the beam propagation factor (M^2-factor) can be determined. However, an interesting question arising in the study of the characteristics of laser beams is the theory of the coherent-mode decomposition of laser beams, i.e. how to determine the mode structure, the MCCs and the weighting factors of laser beams from the given M^2-factor, which can be measured from experiments.

In this paper, the physical model and the main steps for the theory of the coherent-mode decomposition of laser beams are proposed. The new model for the flat-topped beams introduced by Li[3] recently is taken as an example to illustrate the method for the mode-decomposition of laser beams. The analytical expressions for the M^2-factor and MCCs of flat-topped beams are derived. Finally, the mode correlation, mode structure, and coherent-mode decomposition of flat-topped beams are analysed.

2. Physical model and method

First, laser beams are taken as a whole. Using the definition of second-order moments of intensity, the quantitative relationship between the beam propagation factor and the beam parameters is built. On the other hand, laser beams can be regarded as a coherent superposition of a series of orthogonal basis modes whose weighting factors can be determined from the MCCs. Consequently, the relationship between the beam propagation factor and the MCCs can be obtained. Then, making use of the orthogonality of the orthogonal basis modes, the MCCs are obtained from the field distributions of the laser beams and orthogonal basis modes. Thus, the relationship between the MCCs and the beam parameters is acquired. Because the beam propagation factor of a laser beam must be kept the same, whether it is regarded as a whole or as a coherent superposition of the orthogonal basis modes, the theory of the coherent-mode decomposi-

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tion of laser beams can be built. Finally, the problem that possibly appears about the non-unique solution for the coherent-mode decomposition can be solved by means of the measurement of other parameters according to the experimental condition.

3. Coherent-mode decomposition of flat-topped beams

Recently, a new type of beam model to describe the flat-topped beams was introduced by Li. The experimental method to realize this kind of flat-topped beams is also proposed.[3] Making use of this new model, a flat-topped beam can be considered as a linear combination of fundamental Gaussian beams with different beam widths. Consequently, the propagation properties of flat-topped beams can be studied by using the well-known propagation law of fundamental Gaussian beams. In this paper, the new model for the flat-topped beams proposed by Li is taken as an example to illustrate the method for the mode-decomposition of laser beams.

3.1. Beam propagation factor

Similar to the new model for the flat-topped beams proposed by Li in the cylindrical coordinate system, the field distribution of flat-topped beams on the plane of z=0 in the rectangular coordinate system can be expressed as

$$E(x)=\sum_{k=1}^{N}\alpha_k\exp\left(-k\beta\frac{x^2}{w_0^2}\right), \tag{1}$$

where N and w_0 are the beam order and waist width, respectively. α_k and β are the binomial coefficient and scaling factor, respectively. If the beam order N is an integer, α_k and β can be expressed as

$$\alpha_k=(-1)^{k+1}\frac{N!}{k!(N-k)!}, \tag{2}$$

$$\beta=\sum_{k=1}^{N}\frac{1}{k}. \tag{3}$$

For convenience, an unimportant amplitude factor is omitted in Eq.(1) and only the two-dimensional case is considered.

The intensity distribution of flat-topped beams at z=0 is obtained from Eq.(1)

$$\begin{aligned} I(x)=&E(x)E^*(x)\\ =&\sum_{k=1}^{N}\sum_{k'=1}^{N}\alpha_k\alpha_{k'}\exp\left[-(k+k')\beta\frac{x^2}{w_0^2}\right], \end{aligned} \tag{4}$$

where * denotes the complex conjugate.

The second-order moment of the intensity in the spatial domain is defined as[4]

$$\sigma_x^2=\frac{\int_{-\infty}^{+\infty}x^2I(x)\mathrm{d}x}{\int_{-\infty}^{+\infty}I(x)\mathrm{d}x}. \tag{5}$$

On substituting Eq.(4) into Eq.(5), and recalling the integral formula[5]

$$\int_0^{+\infty}\mathrm{e}^{-st}t^{\nu}\mathrm{d}t=\nu!s^{-\nu-1}, \tag{6}$$

lead to

$$\sigma_x^2=\frac{w_0^2}{\beta}\frac{\sum_{k=1}^{N}\sum_{k'=1}^{N}\alpha_k\alpha_{k'}(k+k')^{-3/2}}{\sum_{k=1}^{N}\sum_{k'=1}^{N}\alpha_k\alpha_{k'}(k+k')^{-1/2}}. \tag{7}$$

Calculations similar to the above yield the second-order moment of the variance in the spatial-frequency domain

$$\sigma_k^2=\frac{2\beta}{w_0^2}\frac{\sum_{k=1}^{N}\sum_{k'=1}^{N}\alpha_k\alpha_{k'}kk'(k+k')^{-3/2}}{\sum_{k=1}^{N}\sum_{k'=1}^{N}\alpha_k\alpha_{k'}(k+k')^{-1/2}}. \tag{8}$$

Thus, the M^2-factor can be expressed as

$$M^2=2\sigma_x\sigma_k=2\frac{\sqrt{\sum_{k=1}^{N}\sum_{k'=1}^{N}\alpha_k\alpha_{k'}kk'(k+k')^{-3/2}}\times\sqrt{\sum_{k=1}^{N}\sum_{k'=1}^{N}\alpha_k\alpha_{k'}(k+k')^{-3/2}}}{\sum_{k=1}^{N}\sum_{k'=1}^{N}\alpha_k\alpha_{k'}(k+k')^{-1/2}}. \tag{9}$$

It can be seen from Eq.(9) that the M^2-factor of flat-topped beams only depends on the beam order N. It is shown from the further numerical calculation results of Eq.(9) that the M^2-factor of the flat-topped beams increases with the beam order N. For the case of N=1, the beam propagation factor M^2=1, which corresponds to the special case of fundamental Gaussian beam.

3.2.Mode coherence coefficients

Under the quasi-monochromatic assumption, the partially coherent beams can be described by the mutual intensity in the space–time domain[1]

$$J(x_1, x_2) = \sum_{m=0}^{+\infty}\sum_{n=0}^{+\infty}\lambda_{mn}\phi_m(x_1)\phi_n^*(x_2), \qquad (10)$$

where λ_{mn} are the MCCs, m and n are the mode indices, $\phi(x)$ represents the field distribution, which can be expressed as a series of orthogonal basis modes, for example, the Hermite–Gaussian modes on the plane of z=0 in the rectangular coordinate system, that is,

$$\phi_i(x_j) = \left(\frac{\alpha^2}{\pi}\right)^{1/4}/\sqrt{2^i \mathrm{i}!}\exp(-\alpha^2 x_j^2/2)H_i(ax_j)$$
$$(i = m, n; \quad j = 1, 2), \qquad (11)$$

where α is related to the waist width w_{oh} of the basis Gaussian mode by

$$\alpha = \frac{\sqrt{2}}{w_{\mathrm{oh}}}. \qquad (12)$$

It is worth noting that w_{oh} is not the same as w_0 in the general case. Equation (10) implies that laser beams can be expressed as the superposition of the coherent modes characterized by the Hermite–Gaussian function. The MCCs can be used to describe the contents of the basis modes and the degree of correlation between different modes.

The mutual intensity distribution of flat-topped beams at z=0 can be expressed from Eq.(1) as

$$J(x_1, x_2) = E(x_1)E^*(x_2) = \sum_{k=1}^{N}\sum_{k'=1}^{N}\alpha_k\alpha_{k'} \times \exp\left[-\beta\left(k\frac{x_1^2}{w_0^2} + k'\frac{x_2^2}{w_0^2}\right)\right]. \qquad (13)$$

Recalling the orthogonality of the Hermite–Gaussian series, the MCCs is obtained from Eq.(10)

$$\lambda_{mn} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\phi_m^*(x_1)J(x_1, x_2)\phi_n(x_2)\mathrm{d}x_1\mathrm{d}x_2. \qquad (14)$$

On substituting Eqs.(11), (13) into Eq.(14) and using the formula[5]

$$\int_{-\infty}^{+\infty}H_m(t)\exp(-(x-t)^2/2u)\mathrm{d}t = \sqrt{2\pi u}(1-2u)^{m/2}H_m\left(\frac{x}{\sqrt{1-2u}}\right), \qquad (15)$$

and the properties of the Hermite function

$$H_m(0) = (-1)^{m/2}\frac{m!}{\left(\frac{m}{2}\right)!} \quad (m \text{ is even number});$$
$$H_m(0) = 0 \quad (m \text{ is odd number}), \qquad (16)$$

the analytical expression for the MCCs of flat-topped beams can be derived as

$$\lambda_{mn} = \frac{(-1)^{(m+n)/2}w_0^2\alpha\sqrt{\pi m!n!}}{2^{(m+n)/2-1}\left(\frac{m}{2}\right)!\left(\frac{n}{2}\right)!} \times \sum_{k=1}^{N}\sum_{k'=1}^{N}\frac{\alpha_k\alpha_{k'}}{(2k\beta + w_0^2\alpha^2)^{1/2}(2k'\beta + w_0^2\alpha^2)^{1/2}} \times \left[\frac{2k\beta - w_0^2\alpha^2}{2k\beta + w_0^2\alpha^2}\right]^{m/2}\left[\frac{2k'\beta - w_0^2\alpha^2}{2k'\beta + w_0^2\alpha^2}\right]^{n/2}$$
$$(m, n \text{ are even numbers});$$
$$\lambda_{mn} = 0 \qquad (m, n \text{ are odd numbers}). \qquad (17)$$

It can be shown from Eq.(17) that for the flat-topped beams, only the MCCs of even order modes are not zero, that is, only the even order Hermite–Gaussian modes can exists.

3.3.Analysis of mode correlation and mode structure

Formula (9) for the M^2-factor can be obtained by taking the flat-topped beams as a whole. On the other hand, the flat-topped beams can be expanded into a series of orthogonal basis modes. Then, the generalized formula of the corresponding M^2-factor is expressed in the rectangular coordinate system as[1]

$$M^2 = \sqrt{\left[\sum_{m=0}^{+\infty}(2m+1)C_{mm}\right]^2 - 4\left[\sum_{m=0}^{+\infty}\sqrt{(m+1)(m+2)}\mathrm{Re}C_{m,m+2}\right]^2}, \tag{18}$$

where

$$C_{mm} = \lambda_{mm} / \sum_{m=0}^{+\infty}\lambda_{mm} \tag{19}$$

and

$$C_{m,m+2} = \lambda_{m,m+2} / \sum_{m=0}^{+\infty}\lambda_{mm} \tag{20}$$

represent the weighting factors, λ_{mm} and $\lambda_{m,m+2}$ are given by Eq.(17). It can be seen from Eq.(18) that the M^2-factor of laser beams is not only dependent on the weighting factors of all basis modes but also dependent on the weighting factors of the MCCs of the mth and the $(m+2)$th.

The beam propagation factor of a flat-topped beam must be kept the same, whether it is regarded as a whole or as the result of the superposition of orthogonal basis modes. Thus, the theory of the coherent-mode decomposition of flat-topped beams can be obtained from Eqs.(17)–(20), i.e. the mode indices m, the MCCs λ_{mm}, $\lambda_{m,m+2}$, and the weighting factors C_{mm}, $C_{m,m+2}$ can be calculated from the given M^2-factor by a suitable choice of the truncation number, e.g., $C_{mn} \geq 1\%$. Concretely speaking, for a given M^2-factor, the beam order N of flat-topped beams is calculated by using Eq.(9). On substituting the beam order N into Eqs.(17), (19) and (20), C_{mm} and $C_{m,m+2}$ can be obtained. The suitable value of $w_0^2\alpha^2$ in Eq.(17) is chosen to make the M^2-factor calculated from Eq.(18) be the same as the given M^2-factor. Thus, λ_{mm}, $\lambda_{m,m+2}$, C_{mm} and $C_{m,m+2}$ calculated using the above method are the MCCs and the corresponding weighting factors of the flat-topped beams for the given M^2-factor. Some illustrative numerical examples for the coherent-mode decomposition of flat-topped beams are shown in Table 1.

Table 1. Some numerical examples of the mode indices and weighting factors calculated from the given M^2-factor of the flat-topped beams.

M^2	$\alpha^2 w_0^2$	m	C_{00} /%	C_{02} C_{20} /%	C_{22} /%	C_{24} C_{42} /%	C_{44} /%	C_{46} C_{64} /%	C_{66} /%
1.21	4.0	0;2	90.5	28.8	9.1	–1.5			
	10.0	0;2;4;6	66.1	40.6	25.0	13.9	7.7	3.0	1.1
1.34	4.0	0;2	88.9	30.1	10.2	–2.4			
	10.0	0;2;4;6	63.8	40.9	26.2	15.4	9.0	3.0	1.0
1.55	3.8	0;2;4	88.4	29.3	9.7	–3.4	1.2		
	10.0	0;2;4;6	61.9	40.7	26.8	16.6	10.3	3.2	1.0

It can be seen from Table 1 that for the same M^2-factor, the solution may not be unique mathematically owing to the possible choice of the parameter $w_0^2\alpha^2$ in a certain range, whose physical meaning is twice the ratio of the beam width of the flat-topped beams and the basis Gaussian mode. However, from the view point of physics, the solution must be unique for a specific experimental condition. Therefore, the parameter $w_0^2\alpha^2$ can be restrained according to the specific condition. For example, if a flat-topped beam is produced from a stable resonator characterized by the g parameters $g_j = 1 - L/R_j$ $(j=1, 2)$ (L the length of the cavity, R_j the radius of curvature of the cavity mirrors), w_{oh} can be regarded as the waist width of the resonator fundamental eigenmode, which is given by[6,7]

$$w_{\mathrm{oh}}^2 = \frac{\lambda L}{\pi}\frac{\sqrt{g_1 g_2(1-g_1 g_2)}}{|g_1+g_2-2g_1 g_2|}. \tag{21}$$

Thus, the parameter $w_0^2\alpha^2$ is determined from Eqs.(12) and (21), i.e.

$$w_0^2\alpha^2 = \frac{2w_0^2}{w_{\mathrm{oh}}^2} = \frac{2\pi w_0^2}{\lambda L}\frac{|g_1+g_2-2g_1 g_2|}{\sqrt{g_1 g_2(1-g_1 g_2)}}. \tag{22}$$

It can be shown from Table 1 that the cross correlation always exists between the different transverse modes which compose a flat-topped beam. For the same $w_0^2\alpha^2$, the number of the correlation modes increases with the value of M^2-factor. Equation (18) indicates that the contribution of the cross correlation is always to decrease the M^2-factor, independent of its

signature. It can be seen that the above conclusions hold true in cylindrical coordinate system by similar calculations. In that case, the flat-topped beams can be expressed as a superposition of the Laguerre–Gaussian modes.

For the case without correlation, Eq.(17) is simplified to $\lambda_{mm} = \lambda_{00} = \sqrt{\frac{\pi}{2}} w_0$ $(m = 0)$, $\lambda_{mm} = 0$ $(m \neq 0)$ which corresponds to the case of fundamental Gaussian beam (N=1).

4. Conclusions

A physical model and method for the coherent-mode decomposition of laser beams have been proposed in this paper. The new model of flat-topped beams introduced recently by Li is taken as an example to illustrate the method and the steps of the coherent-mode decomposition. The analytical expressions for the M^2-factor and the MCCs of flat-topped beams have been derived. The mode structure, mode correlation, and coherent-mode decomposition of flat-topped beams are analysed. Since the theory of the coherent-mode decomposition of laser beams needs to determine the contents of all modes from the given M^2-factor, the present model is an inverse problem. From the point of view of mathematics, it allows multi-solutions. However, from the point of view of physics, the solution must be unique for a specific experimental condition. Thus, the free parameter in the MCCs formula can be restrained according to the specific condition. It is worth pointing out that the method for the coherent-mode decomposition proposed in this paper is not only suitable for the non-truncation case but also can be generalized to fit the truncation case, which is often encountered in practice. Just note that for the non-truncation beams, the beam propagation factor is the well-known M^2-factor proposed by Siegman. However, for the truncation beams, the beam propagation factor can be generalized from the calculation method for the non-truncation beams.[8]

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