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Efficient scheme for the preparation of arbitrary anticorrelated states of a two-mode field*

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A scheme is presented for generating any anticorrelated state of a two-mode field. In the scheme a sequence of suitably prepared three-level V-type atoms are sent across a two-mode cavity initially in the vacuum state. The detection of all the atoms in the ground state collapses the two-mode field to the desired state.

Keywords: anticorrelated state, two-mode field, three-level V-type atom
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Over the past few years, there has been much interest in the generation of nonclassical states of light. In cavity QED, schemes have been proposed for the generation of Schrödinger cat states of a cavity field.[1−3] On the other hand, Vogel *et al*[4] have proposed a method for creating an arbitrary state of a quantized single-mode electromagnetic field within the framework of the Jaynes–Cummings model. They have shown that any superposition of the first $N+1$ Fock states can be generated from the vacuum state by injecting N appropriately prepared two-level atom into a resonant single-mode cavity and detecting all of them in the ground state.

More recently, Deb *et al*[5] have generalized the method of Ref.[4] for producing two-mode anticorrelated field states which have constant total photon number in the two modes

$$|\phi_{\rm d}\rangle = \sum_{n=0}^{N} d_n |n, N-n\rangle, \tag{1}$$

where d_n is the desired coefficient. The drawback of this method is that it required that one of the two modes is initially prepared in the Fock state $|N\rangle$, which is experimentally problematic since the preparation of the Fock state $|N\rangle$ is difficult. In this paper we propose an alternative scheme for constructing such anticorrelated two-mode field states by injecting a sequence of three-level V-type atoms into a two-mode resonator and detecting them in the ground state. The distinct advantage of our scheme is that it does not require the cavity field be initially in the Fock state $|N\rangle$. Thus, it must be more experimentally feasible than that of Ref.[5].

We first consider a three-level V-type atom interacting with a two-mode cavity field. Let the two upper levels be denoted by 1 and 2 and the lower by 3. The transition between levels 1 and 2 is dipole forbidden. The Hamiltonian for this system is ($\hbar = 1$)[6−8]

$$\begin{aligned} H = &\sum_{i=1}^{2} \Omega_i a_i^+ a_i + \sum_{j=1}^{3} \omega_j A_j^+ A_j \\ &+ [(g_1 a_1 A_1^+ A_3 + g_2 a_2 A_2^+ A_3) + \text{H.c.}], \end{aligned} \tag{2}$$

where a_i^+ and a_i are the creation and annihilation operators for the cavity modes i=1, 2 with frequency Ω_i, respectively, A_j^+ and A_j are the creation and annihilation operators of the atom in the level j with frequency ω_j, and g_1 and g_2 are the corresponding atom-cavity-mode coupling constants, respectively.

In the spin-one operator representation the Hamiltonian of Eq.(2) takes the form[6]

$$\begin{aligned} H =& \omega_2 I + \frac{1}{2}(\omega_1 - \omega_3) J_z + \frac{1}{2}(\omega_1 + \omega_3 - 2\omega_2) J_z^2 \\ &+ \Omega_1 a_1^+ a_1 + \Omega_2 a_2^+ a_2 + \frac{1}{2} g_1 (a_1 J_+^2 + a_1^+ J_-^2) \\ &+ \frac{1}{2\sqrt{2}} g_2 [a_2 (J_+ - B_+) + (J_- - B_-) a_2^+], \end{aligned} \tag{3}$$

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where

$$J_{\pm} = J_x \pm \mathrm{i}J_y, \quad B_{\pm} = J_{\pm}J_z + J_z J_{\pm}, \tag{4}$$

with J_x, J_y, J_z being the spin-one operators. Under the exact two-photon resonance condition

$$\omega_1 - \omega_3 - \Omega_1 = \omega_2 - \omega_3 - \Omega_2 = \Delta, \tag{5}$$

the Hamiltonian becomes

$$H = \Omega_1 N_1 + \Omega_2 N_2 + C, \tag{6}$$

where the operators N_1 and N_2 are constants of motion

$$N_1 = a_1^+ a_1 + \frac{1}{2}(J_z + J_z^2), \quad N_2 = a_2^+ a_2 - J_z^2 + I, \tag{7}$$

C has the following matrix representation

$$C = \begin{pmatrix} 0 & 0 & g_1 a_1 \\ 0 & 0 & g_2 a_2 \\ g_1 a_1^+ & g_2 a_2^+ & -\Delta \end{pmatrix}. \tag{8}$$

The operators N_1, N_2, and C commute among themselves. For the sake of simplicity, we here consider the resonance case, i.e. $\Delta = 0$. The time evolution operator for such a system is given by

$$U(t) = \mathrm{e}^{-\mathrm{i}Ht} = \mathrm{e}^{-\mathrm{i}\Omega_1 N_1 t}\mathrm{e}^{-\mathrm{i}\Omega_2 N_2 t}\mathrm{e}^{-\mathrm{i}Ct}. \tag{9}$$

We consider N three-level V-type atoms prepared in the superpositions of the two excited states $|1\rangle$ and $|2\rangle$ crossing the two-mode resonator sequentially. Each such prepared atom increases the total photon number of the two modes by one if it is detected in the ground state after its interaction with the two-mode field. Therefore, after N such prepared atoms cross the initially empty cavity and are detected in the ground state $|3\rangle$ a two-mode field in the state of the following form is built

$$\left|\phi_f^N\right\rangle = \sum_{n=0}^{N} C_n^N \left|n, N-n\right\rangle. \tag{10}$$

Assume the Nth atom is initially in the superposition state

$$\left|\phi_a^N\right\rangle = \frac{1}{\sqrt{1+|\varepsilon_N|^2}}(|1\rangle_N + \varepsilon_N |2\rangle_N), \tag{11}$$

where ε_N is a complex parameter. In order to generate $\left|\phi_a^N\right\rangle$ from the ground state $|3\rangle_N$ we first send the Nth atom across a classical field tuned to the transition between the states $|3\rangle_N$ and $|1\rangle_N$. This classical field is far off-resonant with the transition between the states $|3\rangle_N$ and $|2\rangle_N$ and thus the state $|2\rangle_N$ is not affected during the interaction. Then, the Hamiltonian in the interaction picture is given by[9]

$$H_1 = |\alpha_1|\,(\mathrm{e}^{\mathrm{i}\theta_1}\,|1\rangle_N\langle 3| + \mathrm{e}^{-\mathrm{i}\theta_1}\,|3\rangle_N\langle 1|), \tag{12}$$

where $|\alpha_1|$ and θ_1 are the coupling strength and phase of the classical field, respectively. After an interaction time t_1 the atom evolves into

$$\left|\phi_{a,1}^N\right\rangle = \cos(|\alpha_1|\,t_1)\,|3\rangle_N - \mathrm{i}\sin(|\alpha_1|\,t_1)\mathrm{e}^{\mathrm{i}\theta_1}\,|1\rangle_N. \tag{13}$$

Choose the coupling strength $|\alpha_1|$ (or interaction time t_1) and phase θ_1 appropriately so that $\theta_1 = \pi/2$ and $\sin(|\alpha_1|\,t_1) = 1/\sqrt{1+|\varepsilon_N|^2}$. Then we have

$$\left|\phi_{a,1}^N\right\rangle = \frac{1}{\sqrt{1+|\varepsilon_N|^2}}(|1\rangle_N + |\varepsilon_N|\,|3\rangle_N). \tag{14}$$

Now we let this atom cross another classical field tuned to the transition between the states $|3\rangle$ and $|2\rangle$. In the interaction picture, the Hamiltonian is given by

$$H_2 = |\alpha_2|\,(\mathrm{e}^{\mathrm{i}\theta_2}\,|2\rangle_N\langle 3| + \mathrm{e}^{-\mathrm{i}\theta_2}\,|3\rangle_N\langle 2|), \tag{15}$$

where $|\alpha_2|$ and θ_2 are the coupling strength and phase of the second classical field, respectively. After an interaction time t_2 the atom evolves into

$$\begin{aligned}\left|\phi_{a,2}^N\right\rangle =& \frac{1}{\sqrt{1+|\varepsilon_N|^2}}\{|1\rangle_N + |\varepsilon_N|\,[\cos(|\alpha_2|\,t_2)\,|3\rangle_N \\ & - \mathrm{i}\sin(|\alpha_2|\,t_2)\mathrm{e}^{\mathrm{i}\theta_2}\,|2\rangle_N]\}.\end{aligned} \tag{16}$$

Choose the coupling strength $|\alpha_2|$ (or interaction time t_2) and phase θ_2 appropriately so that $\theta_2 = \phi_N + \pi/2$ and $\sin(|\alpha_2|\,t_1) = 1$, with $\varepsilon_N = |\varepsilon_N|\,\mathrm{e}^{\mathrm{i}\phi_N}$. Then we obtain the state of Eq.(11).

Before the Nth atom is injected into the cavity the field state is assumed to be

$$\left|\phi_f^{N-1}\right\rangle = \sum_{n=0}^{N-1} C_n^{N-1}\left|n, N-1-n\right\rangle. \tag{17}$$

After the interaction of the Nth atom with the cavity field the state of the whole system is given by

$$|\phi_N\rangle = U(\tau_N)\,|\phi_a^N\rangle\,\left|\phi_f^{N-1}\right\rangle, \tag{18}$$

where τ_N is the given interaction time. Substituting Eqs.(9), (11), and (17) into Eq.(18) we have

$$\begin{aligned}|\phi_N\rangle =& \frac{1}{\sqrt{1+|\varepsilon_N|^2}}\Big\{\sum_{n=0}^{N-1}(-i)C_n^{N-1}\\ &\times[A_n^N(\tau_N)\,|n+1,N-1-n\rangle\\ &+B_n^N(\tau_N)\,|n,N-n\rangle]\,|3\rangle\\ &+|\varphi_1\rangle\,|1\rangle+|\varphi_2\rangle\,|2\rangle\Big\},\end{aligned} \tag{19}$$

where

$$A_n^N(\tau_N) = \mathrm{e}^{-\mathrm{i}[(n+1)\Omega_1+(N-1-n)\Omega_2]\tau_N}\frac{\sin(\sqrt{C_n}\tau_N)}{\sqrt{C_n}}, \tag{20}$$

$$B_n^N(\tau_N) = \varepsilon_N\mathrm{e}^{-\mathrm{i}[n\Omega_1+(N-n)\Omega_2]\tau_N}\frac{\sin(\sqrt{D_N}\tau_N)}{\sqrt{D_N}}, \tag{21}$$

$$C_n = g_1^2(n+1)+g_2^2(N-1-n), \tag{22}$$

$$D_n = g_1^2 n+g_2^2(N-n), \tag{23}$$

$|\varphi_i\rangle$ denote the field states when the atom is in the excited state $|i\rangle$ with i=1,2, respectively. If we now find the atom in the ground state $|3\rangle$ the field collapses to the pure state $|\phi_f^N\rangle$ of Eq.(10) with the coefficients C_n^N which reads

$$C_n^N = N_N(A_{n-1}^N(\tau_N)C_{n-1}^{N-1}+\varepsilon_N B_n^N(\tau_N)C_n^{N-1}), \tag{24}$$

where N_N is a normalization factor.

In order to obtain the desired state $|\phi_{\mathrm{d}}\rangle$ of Eq.(1) we must fulfil $C_n^N = d_n$, i.e.

$$\begin{aligned}&N_N\varepsilon_N B_n^N(\tau_N)C_0^{N-1} = d_0,\\ &\cdots\cdots\\ &N_N[A_{n-1}^N(\tau_N)C_{n-1}^{N-1}+\varepsilon_N B_n^N(\tau_N)C_n^{N-1}] = d_n,\\ &\cdots\cdots\\ &N_N A_{N-1}^N(\tau_N)C_{N-1}^{N-1} = d_N.\end{aligned} \tag{25}$$

We first express the unknown values of C_n^{N-1} in terms of d_n,

$$\begin{aligned}C_n^{N-1} =& \frac{1}{N_N}\sum_{i=1}^{N-n}\left[\prod_{m=n}^{n+l-2}\frac{B_{m+1}^N}{A_m^N}\right]\\ &\times\frac{d_{n+l}}{A_{n+l-1}}(-\varepsilon_N)^{l-1}.\end{aligned} \tag{26}$$

Substituting thus obtained value C_0^{N-1} into the first set of Eq.(25) we obtain the characteristic equation for ε_N

$$d_0+\sum_{l=1}^{N}\left[\prod_{m=0}^{l-2}\frac{B_{m+1}^N}{A_m^N}\right]\frac{B_0^N d_l}{A_{l-1}^N}(-\varepsilon_N)^l = 0. \tag{27}$$

We can solve this equation and choose one value ε_N out of N roots. Substituting it into Eq.(26) we obtain the coefficients C_n^{N-1} of the state $|\phi_f^{N-1}\rangle$. Taking $|\phi_f^{N-1}\rangle$ as a new desired state and doing the same calculations we obtain the parameter ε_{N-1} for the $(N-1)$th atom and the coefficients C_n^{N-2} for the state $|\phi_f^{N-2}\rangle$. We repeat the procedure until we reach the vacuum state for the two-mode field. Then we obtain a string of parameters $\varepsilon_1,\varepsilon_2,\ldots,\varepsilon_N$ which define the initial states of a sequence of atoms we have to inject into the cavity in order to obtain the desired state $|\phi_f^N\rangle$ from the vacuum state.

The two-mode anticorrelated states are important quantum states. A good example is the $SU(2)$ coherent state

$$|\xi,N\rangle = (1+|\xi|^2)^{-N/2}\sum_{n=0}^{N}\left(\begin{array}{c}N\\ n\end{array}\right)^{1/2}\xi^n|n,N-n\rangle. \tag{28}$$

It has been shown that the $SU(2)$ coherent state may exhibit the nonclassical features such as sub-Poissonian statistic and anticorrelations.[10,11] Another typical example is the Bell states[12]

$$\frac{1}{\sqrt{2}}(|1,0\rangle\pm|0,1\rangle), \tag{29}$$

the Bell states are useful in quantum information processing.

In summary, we have presented a method for generating arbitrary anticorrelated states of a two-mode cavity field. It is based on the injection of a sequence of three-level V-type atoms into a two-mode resonator one by one and the detection of them in the ground state. Unlike the scheme of Ref.[5], our scheme does not require one mode of the field be initially prepared in the Fock state $|N\rangle$, so it is much more experimentally feasible than that of Ref.[5]. Our scheme may be useful in studying nonclassical features of a cavity field and quantum information.

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