Vol 12 No 9, September 2003
1009-1963/2003/12(09)/0971-06

Chinese Physics

© 2003 Chin. Phys. Soc.
and IOP Publishing Ltd

Entangled states in the two-mode coherent fields interacting with a two-level atom*

Liu Xiao-Juan(刘小娟)[a)b)c)] and Fang Mao-Fa(方卯发)[a)b)†]

[a)] *Department of Physics, Hunan Normal University, Changsha* 410081, *China*
[b)] *Anhui Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Hefei* 230031, *China*
[c)] *Department of Physics, Xiangtan Normal College, Xiangtan* 411202, *China*

(Received 24 January 2003; revised manuscript received 21 April 2003)

We investigate the entanglement properties of the two-mode coherent fields interacting with a two-level atom via the two-photon transition. We discuss the quantum entanglement between the two-mode coherent fields and the two-level atom by using the quantum reduced entropy and that between the two-mode coherent fields by using the quantum relative entropy. We also examine the influences of the initial states of the atom and the two-mode coherent fields on the quantum entanglement of the system. Our results show that three types of entangled states can be prepared via the two-mode coherent fields interacting with a two-level atom and choosing appropriately the initial-state parameters of the system.

Keywords: two-mode coherent fields, quantum entanglement, reduced entropy, relative entropy
PACC: 4250

1. Introduction

Quantum entanglement(QE) is one of the most striking features of quantum mechanics. The manipulation of entangled states that have nonclassical and nonlocal properties leads to a better understanding of basic quantum phenomena. On the other hand, QE is an essential ingredient and entangled state that constitute a valuable resource in quantum information processing,[1] such as quantum cryptography,[2] computer,[3] and teleportation.[4] Therefore, it is of great importance to quantify the entanglement to assess the efficacy of quantum information processing and to prepare quantum entangled states in well-controlled experimental conditions. In the latter case, the entanglement results from the interaction of a two-level atom with a cavity mode field. Interrupting the coherent interaction at selected times, one can generate atom-cavity entanglement,[5] exchange the atom and cavity states,[6] or realize a two-qubit quantum logics gate[7] for a basic unit of quantum information processing systems. These functions can be arranged to realize a quantum non-demolition measurement of a single photon[8] or to generate a three-particle entangled state.[9] Up to now, these experiments have involved a single cavity mode field interacting with one, two, or three atoms crossing it successively. Recently, unconditional quantum teleportation of an unknown coherent state has been realized experimentally by exploiting a two-mode squeezed vacuum state as an entanglement resource,[10] but the effect of environment on the two-mode fields has not been considered in the scheme. In another scheme,[11] the authors presented the realization of a more complex entanglement in cavity quantum electrodynamics, by involving in the experiment two independent cavity modes via emitting a single photon. These multi-mode entanglement opens interesting perspectives for quantum information processing and non-locality tests. In real experimental situations, due to the coupling to the environment, the entangled state of the two-mode fields inevitably loses its purity; it becomes mixed. This phenomenon-decoherence is the most dangerous obstacle for all entanglement manipulations.

In this paper, we investigate the entanglement properties of the two-mode coherent fields interacting

*Project supported by the National Natural Science Foundation of China (Grant No 19874020), the Natural Science Foundation of Hunan Province, China (Grant No 01JJY3030), and the Scientific Research Fund from the Education Bureau of Hunan Province, China(Grant No 01C260).

†Corresponding author.

with a two-level atom via the two-photon transition by regarding the atom as the environment. We study the QE between the two-mode coherent fields and the two-level atom by using the quantum reduced entropy and that between two-mode coherent fields by using the quantum relative entropy.[12] We examine the influences of the initial state of the atom and the two-mode coherent fields on the entanglement properties of the system. We find that three types of entangled states, such as a maximal-entangled state between the two-mode coherent fields and the atom, a maximal-entangled state between the two-mode coherent fields, and a sustained entangled state between the two-mode coherent fields, can be produced via choosing appropriately the parameters of the system. This provides a good way for preparing entangled states by using the two-mode coherent fields in the experiment.

2. The model and its solution

The system considered here consists of a two-level atom interacting with the two-mode coherent fields via the two-photon transition processes. The effective Hamiltonian in the rotating-wave approximation[13] is ($\hbar = 1$)

$$H = \omega_0 S_z + \sum_{j=1}^{2} \omega_j a_j^+ a_j + g(a_1^+ a_2^+ S_- + a_1 a_2 S_+), \quad (1)$$

where i ($i = z, +, -$)are the usual pseudo-spin operators of the two-level atom, $a_j^+(a_j)$ is the photon creation (annihilation) operator of the field mode of frequency w_j ($j \equiv 1, 2$) and g is the coupling constant between the atom and the two-mode coherent fields. For simplicity, we consider the two-photon resonance case, that is $\omega_0 = \omega_1 + \omega_2$. Using the standard techniques,[14] it can be shown that this Hamiltonian gives rise to the following time evolution operator in the interaction picture

$$T = \begin{bmatrix} \hat{u}_{11} & -\mathrm{i}\hat{u}_{21} \\ -\mathrm{i}\hat{u}_{12} & \hat{u}_{22} \end{bmatrix}, \quad (2)$$

where we have written the operators as

$$\hat{u}_{11} = \cos(\hat{A}gt), \qquad \hat{u}_{21} = a_1^+ a_2^+ \frac{\sin(\hat{A}gt)}{\hat{A}},$$

$$\hat{u}_{22} = \cos(\hat{B}gt), \qquad \hat{u}_{12} = a_1 a_2 \frac{\sin(\hat{B}gt)}{\hat{B}},$$

$$\hat{A} = \sqrt{(a_1 a_2 a_1^+ a_2^+)}, \quad \hat{B} = \sqrt{(a_1^+ a_2^+ a_1 a_2)}. \quad (3)$$

We consider that the two-level atom is in a coherent superposition state of the excited state$|\ e\rangle$ and the ground state $|\ g\rangle$ at time t=0,

$$|\psi_{\mathrm{A}}(0)\rangle = \cos(\theta/2)|e\rangle + \exp(\mathrm{i}\varphi)\sin(\theta/2)|g\rangle. \quad (4)$$

The field is in the two-mode coherent state

$$|\psi_{\mathrm{F}}(0)\rangle = |\alpha_1, \alpha_2\rangle = \sum_{n_1,n_2=0}^{\infty} F(n_1)F(n_2)|n_1, n_2\rangle,$$

$$F(n_j) = \exp\left(-\frac{\overline{n}_j}{2}\right)\frac{\alpha_j^{n_j}}{\sqrt{n_j!}}, \quad (5)$$

where $0 < \theta < \pi$ denotes the atomic distribution, $0 < \varphi < 2\pi$ is the phase of the atom dipole, $\alpha_j = \sqrt{\overline{n}_j}\exp(\mathrm{i}\psi_j)$, and $\overline{n}_j$ and ψ_j represent the initial average photon number and the direction angle of the excitation for j ($j \equiv 1, 2$)mode respectively.

The initial state of the system can be written as

$$|\psi_{\mathrm{FA}}(0)\rangle = |\psi_{\mathrm{F}}(0)\rangle \otimes |\psi_{\mathrm{A}}(0)\rangle. \quad (6)$$

At any time $t > 0$ the state vector of the system is given by

$$\begin{aligned} |\psi_{\mathrm{FA}}(t)\rangle =& U_{\mathrm{I}}(t)|\psi_{\mathrm{FA}}(0)\rangle \\ =& |D\rangle|e\rangle + |T|g\rangle, \end{aligned} \quad (7)$$

with

$$\begin{aligned} |D\rangle = \sum_{n_1,n_2=0}^{\infty} & F(n_1)F(n_2)(\cos(\theta/2) \\ & \times \exp(\mathrm{i}[n_1\psi_1 + n_2\psi_2]) \\ & \times \cos(gt\gamma_{n_1+1,n_2+1})|n_1, n_2\rangle \\ & - \mathrm{i}\sin(\theta/2)\exp(-\mathrm{i}[\varphi - n_1\psi_1 - n_2\psi_2]) \\ & \times \sin(gt\gamma_{n_1,n_2})|n_1 - 1, n_2 - 1\rangle), \end{aligned} \quad (8)$$

$$\begin{aligned} |T\rangle = \sum_{n_1,n_2=0}^{\infty} & F(n_1)F(n_2)(\cos(\theta/2) \\ & \times \exp(\mathrm{i}[n_1\psi_1 + n_2\psi_2]) \\ & \times \sin(gt\gamma_{n_1+1,n_2+1})|n_1 + 1, n_2 + 1\rangle \\ & + \mathrm{i}\sin(\theta/2)\exp(-\mathrm{i}[\varphi - n_1\psi_1 - n_2\psi_2]) \\ & \times \cos(gt\gamma_{n_1,n_2})|n_1, n_2\rangle), \end{aligned} \quad (9)$$

where $\gamma_{n_1,n_2} = \sqrt{n_1, n_2}$. At any time $t > 0$ the density operator for the system is given by

$$\rho_{\rm FA}(t) = |\psi_{\rm FA}(t)\rangle\langle\psi_{\rm FA}(t)| = \left[\begin{array}{cc} |D\rangle\langle D| & |D\rangle\langle T| \\ |T\rangle\langle D| & |T\rangle\langle T| \end{array}\right], \tag{10}$$

while the reduced two-mode coherent-field density operator for the system is given by

$$\begin{aligned}\rho_{\rm F}(t) =& {\rm Tr}_{\rm A}(\rho_{\rm FA}(t)) \\ =& |D\rangle\langle D| + |T\rangle\langle T|. \end{aligned} \tag{11}$$

The eigenvalues and the eigenstates of the reduced two-mode coherent-field density operator $\rho_{\rm F}(t)$ are given by

$$\begin{aligned}\lambda_{\rm F}^{\pm} =& \langle D|D\rangle \pm \exp(\mp\delta)|\langle D|T\rangle| \\ =& \langle T|T\rangle \pm \exp(\pm\delta)|\langle D|T\rangle|, \end{aligned} \tag{12}$$

$$\begin{aligned}|\psi_{\rm F}(t)\rangle =& \frac{1}{\sqrt{2\lambda_{\rm F}^{\pm}\cosh\delta}}(\exp[1/2({\rm i}\alpha \pm \delta)]|D\rangle \\ & \pm \exp[-1/2({\rm i}\alpha \pm \delta)]|T\rangle), \end{aligned} \tag{13}$$

where

$$\sinh(\delta) = \sinh^{-1}\left[\frac{1}{2|\langle D|T\rangle|}(\langle D|D\rangle - \langle T|T\rangle)\right], \tag{14}$$

$$\alpha = \tanh^{-1}\left[\frac{{\rm Im}\langle D|T\rangle}{{\rm Re}\langle D|T\rangle}\right]. \tag{15}$$

Similarly, we can obtain the reduced atom density operator for the system:

$$\rho_{\rm A}(t) = \left[\begin{array}{cc} \langle D|D\rangle & \langle T|D\rangle \\ \langle D|T\rangle & \langle T|T\rangle \end{array}\right]. \tag{16}$$

It turns out that the eigenvalues of $\rho_{\rm A}(t)$ are equal to those of $\rho_{\rm F}(t)$. That is $\lambda_{\rm F}^{\pm} = \lambda_{\rm A}^{\pm} = \lambda^{\pm}$.

The eigenstates of $\rho_{\rm A}(t)$ are given by

$$\begin{aligned}|\psi_{\rm A}^{\pm}(t)\rangle =& \frac{1}{\sqrt{2\lambda_{\rm A}^{\pm}\cosh\delta}}(\exp[1/2({\rm i}\alpha \pm \delta)]|e\rangle \\ & \pm \exp[-1/2({\rm i}\alpha \pm \delta)]|g\rangle). \end{aligned} \tag{17}$$

By using the Schmidt theorem,[15] the system state vector can be rewritten as

$$|\Psi_{\rm FA}(t)\rangle = \sqrt{\lambda^{+}}|\psi_{\rm F}^{+}\rangle|\phi_{\rm A}^{+}\rangle + \sqrt{\lambda^{-}}|\psi_{\rm F}^{-}\rangle|\phi_{\rm A}^{-}\rangle. \tag{18}$$

The properties of the QE between the two-mode coherent fields and the atom, and that between the two-mode coherent fields will be discussed in the next section by using above results.

3. The entanglement between the two-mode coherent fields and the two-level atom

The reduced entropy theory about interaction of the fields with the atom has been introduced by Knight *et al.*[15–17] They have shown that the reduced entropy is very useful and is a sensitive operational measure of the purity of the quantum state, which automatically includes all the moments of the density operator. The time evolution of the field (atomic) reduced entropy $S(\rho_{\rm F})$ reflects the time evolution of the degree of the entanglement between the field and the atom(DEFA). The higher the reduced entropy, the greater the DEFA is. The information concerning the field is inferred by the measurement of atomic properties. Following the work by Phoenix and Knight,[15] we can express the reduced entropy $S(\rho_{\rm F})$ of the two-mode coherent fields in terms of the eigenvalue $\lambda_{\rm F}^{\pm}$ of $\rho_{\rm F}(t)$ given by Eq.(12),

$$S(\rho_{\rm F}) = -\lambda_{\rm F}^{+}\log_2\lambda_{\rm F}^{+} - \lambda_{\rm F}^{-}\log_2\lambda_{\rm F}^{-}. \tag{19}$$

If $S(\rho_{\rm F})$ takes its minimum value to be zero, the two-mode coherent fields and the atom are disentangled; if $S(\rho_{\rm F})$ takes its maximum value to be 1, the two-mode coherent fields and the atom are in maximal entangled state.

Figures 1 and 2 present the numerical results for the time evolution of the reduced entropy $S(\rho_{\rm F})$ of the entanglement by dotted line, for the two cases $\overline{n}_1 = \overline{n}_2 = 20$, $\overline{n}_1 = \overline{n}_2 = 40$ and for different values of the atomic distribution angle θ and the relative phase $\beta = \varphi - (\psi_1 + \psi_2)$ between the two-mode coherent fields and the atom. It is shown that the reduced entropy of the entanglement is strongly dependent on the atomic initial state and the relative phase mentioned above.

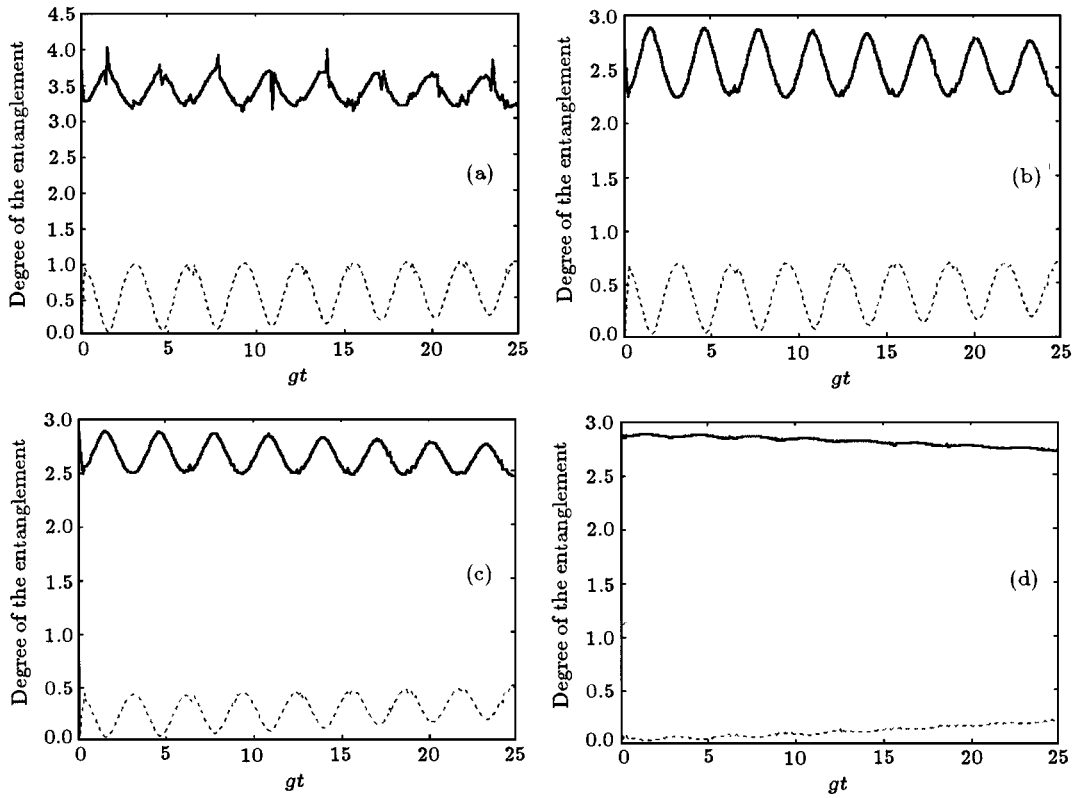


Fig.1. The time evolution of the degree of the entanglement. $S(\rho_{\rm F})$ is shown by the dotted line and $E_{\rm R}(\rho_{\rm F})$ by the solid line with $\overline{n}_1 = \overline{n}_2 = 20$; (a) $\theta = 0$, $\beta = 0$, (b) $\theta = \pi/2$, $\beta = \pi/2$, (c) $\theta = \pi/2$, $\beta = \pi/4$, (d) $\theta = \pi/2$, $\beta = 0$.

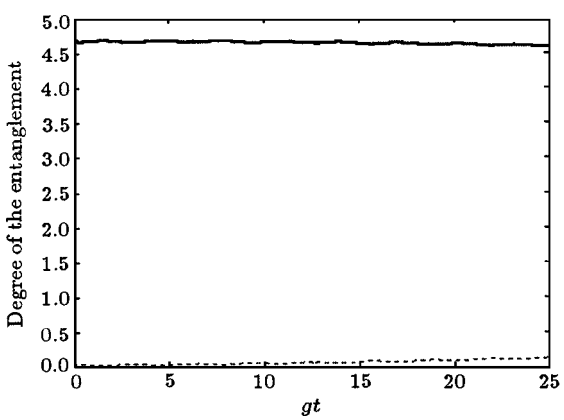


Fig.2. The time evolution of the degree of the entanglement. $S(\rho_{\rm F})$ is shown by the dotted line and $E_{\rm R}(\rho_{\rm F})$ by the solid line with $\overline{n}_1 = \overline{n}_2 = 40$, $\theta = \pi/2$, $\beta = 0$.

First, we examine the time evolution of DEFA when the atom is in an excited state ($\theta = 0$) as shown in Fig.1 (a). The $S(\rho_{\rm F})$ evolves with a period of $\pi/2$. At times $gt = \dfrac{2n-1}{2}\pi$, $S(\rho_{\rm F})$ takes its minimum value, which means that the two-mode coherent fields are disentangled from the atom. This point can be quantitatively interpreted by using Eqs.(12) and (18). Under a strong field (both $\overline{n}_1$ and $\overline{n}_2$ are very large), after a straightforward calculation, we can obtain $\lambda^+ \approx 1$ and $\lambda^- \approx 0$ in this case. The system state $|\psi_{\rm FA}\rangle$ can be expressed in the form

$$|\psi_{\rm FA}\rangle = |\psi_{\rm F}^+\rangle \otimes |\psi_{\rm A}^+\rangle. \tag{20}$$

Obviously, this is a disentangled direct product state. Therefore, $S(\rho_{\rm F})$ is equal to zero. When $gt = (2n-1)\pi$, the value of $S(\rho_{\rm F})$ tends to 1, which means that the DEAF is very large. This can also be explained from Eqs.(12) and (18). The state vector of the system can be written, by using the above-mentioned condition, as

$$|\Psi_{\rm FA}\rangle \approx \frac{1}{\sqrt{2}}(|\psi_{\rm F}^+\rangle|\psi_{\rm A}^+\rangle + |\psi_{\rm F}^-\rangle|\psi_{\rm A}^-\rangle). \tag{21}$$

This state is just the EPR (Einstein–Podolsky–Rosen) entanglement state and applied widely in quantum information (QI) processing.

Furthermore, we discuss the effects of atomic coherence on $S(\rho_{\rm F})$. In Figs 1(b)–(d), we take the atomic distribution angle $\theta = \pi/2$ and the relative phase $\beta = \pi/2$, $\pi/4$ and 0. It is obvious that the amplitude of vibration of the DEFA decreases monotonically with β. In Figs 1(b) and (c) when $\beta = \pi/2$ and $\pi/4$, respectively, it can be shown that the time

evolution of $S(\rho_{\rm F})$ are similar to Fig.1(a). While in Fig.1(d), when $\beta = 0$, it can be seen that the value of $S(\rho_{\rm F})$ becomes very small during the time evolution. Our results also show that when both $\overline{n}_1$ and $\overline{n}_2$ are very large (the strong field condition), $S(\rho_{\rm F})$ tends to its zero value in the time evolution process, which means that DEFA is very small(see Fig.2).

4. The entanglement between the two-mode coherent fields

The above results show that the Hamiltonian given by Eq.(1) leads to the entanglement between the two-mode coherent fields and the atom, so that the state of the two-mode coherent fields may evolve into a mixed state. In this case, the degree of the entanglement between the two-mode coherent fields cannot be measured by the reduced entropy. However, the relative entropy of the entanglement is a good measure for the degree of the entanglement of two subsystems in the mixed state, which is defined as[11]

$$E_{\rm R}(\rho) = \mathrm{main}_{\sigma\in D} S(\rho||\sigma), \tag{22}$$

where $S(\rho||\sigma) = \mathrm{Tr}[\rho(\log_2\rho - \log_2\sigma)]$ is the relative entropy. The minimum is taken over D, the set of all disentangled states. For a pure state, the relative entropy of the entanglement reduces to the reduced entropy of the entanglement, while for a mixed state, it is usually difficult to calculate the relative entropy of the entanglement except for some specific states. Recently, the following theorem on the relative entropy of the entanglement has been proved,[18,19] and it turns out to be quite suitable for the present analysis.

For a quantum state in two subsystems described by a density matrix

$$\rho = \sum_{n_1,n_2} a_{n_1,n_2} |\Phi_{n_1}\phi_{n_1}\rangle\langle\Phi_{n_2}\phi_{n_2}|, \tag{23}$$

the relative entropy of the entanglement is given by

$$E_{\rm R}(\rho) = -\sum_n a_{n,n}\log_2 a_{n,n} + \mathrm{Tr}(\rho\log_2\rho), \tag{24}$$

and the disentangled state ρ^* that minimizes the quantum relative entropy $S(\rho||\rho^*)$ is

$$\rho^* = \sum_n a_{n,n} |\Phi_n\phi_n\rangle\langle\Phi_n\phi_n|, \tag{25}$$

where $|\Phi_n\rangle$ and $|\phi_n\rangle$ are orthonormal states of each subsystem. It should be noted that the density operator of Eq.(11) can take the form of Eq.(25)

$$\rho_{\rm F} = \sum_n a_{n,n} |n,n\rangle\langle n,n|, \tag{26}$$

where $a_{n,n} = \langle n,n|\rho_{\rm F}|n,n\rangle$. Hence, the relative entropy of the entanglement of the state Eq.(11) is written as

$$\begin{aligned} E_{\rm R}(\rho_{\rm F}) =& -\sum_n a_{n,n}\log_2 a_{n,n} - S(\rho_{\rm F}) \\ =& -\sum_n a_{n,n}\log_2 a_{n,n} \\ &+ \lambda^+\log_2\lambda^+ + \lambda^-\log_2\lambda^-. \end{aligned} \tag{27}$$

Consequently, it is possible to compute numerically the relative entropy of the entanglement given by Eq.(27). The time evolutions of the relative entropy of the entanglement $E_{\rm R}(\rho_{\rm F})$ are shown by using the solid line in Figs 1 and 2, which reflect the time evolution of the degree of the entanglement between the two-mode coherent fields (DETMCF). The DETMCF increases monotonically with the relative entropy. Comparing the solid line with the dotted line in the Figs 1(a)–(d), we can find that the time evolution of the DETMCF is just the opposite to DEFA. That is, when the latter increases with time, the former decreases with time. This comes from the fact that DEFA can impair the DETMCF due to the atom interacting with the two-mode coherent fields. In particular, when $\beta = 0$, $\overline{n}_1 = \overline{n}_2$, and $gt = \dfrac{2n-1}{2}\pi$, it can be obtained that $\lambda^+ = 1$ and $\lambda^- = 0$. The states of the system can be written in a direct form as formula (21), which means that the two-mode coherent fields and the atom are disentangled. At the same time, Eq.(13) can be further reduced to

$$\begin{aligned} |\psi_{\rm F}\rangle =& \sqrt{2}\sum_{n=0}^{\infty} F(2n+1)F(2n+1)(-1)^{n+1} \\ &\times(\cos(\theta/2)|2n+1,2n+1\rangle \\ &-\mathrm{i}\sin(\theta/2)|2n,2n\rangle). \end{aligned} \tag{28}$$

Obviously, when $\theta = 0$ or π, the two-mode coherent fields are in the states with an odd or even photon number, respectively,

$$\begin{aligned} |\psi_{\rm F}\rangle =& \sqrt{2}\sum_{n=0}^{\infty} F(2n+1)F(2n+1)(-1)^{n+1} \\ &\times|2n+1,2n+1\rangle \quad (\theta=0), \\ |\psi_{\rm F}\rangle =& \mathrm{i}\sqrt{2}\sum_{n=0}^{\infty} F(2n+1)F(2n+1)(-1)^{n} \\ &\times|2n,2n\rangle \quad (\theta=\pi). \end{aligned} \tag{29}$$

When $\theta = \pi/2$, the maximal entangled state between the two-mode fields can be produced. That is

$$|\psi_{\rm F}\rangle = \sum_{n=0}^{\infty} F(2n+1)F(2n+1)(-1)^{n+1} \times (|2n+1, 2n+1\rangle - {\rm i}|2n, 2n\rangle). \quad (30)$$

The states given by Eqs.(29) and (30) have been applied widely in QI processing. Our result further shows that under the condition of a strong field (both $\overline{n}_1$ and $\overline{n}_2$ are very large) and when the atom is in a coherent trapping state, the maximal entangled state between the two-mode coherent fields, which is independent of the evolution time, can be obtained, as shown in Fig.2.

5. Conclusion

In conclusion, we have investigated the entangled states in the two-mode coherent fields interacting with a two-level atom via the two-photon transition. We discuss the quantum entanglement between the two-mode coherent fields and the two-level atom by using the quantum reduced entropy and that between the two-mode coherent fields by using the quantum relative entropy. Our results show that three types of entangled states can be prepared via the two-mode coherent fields interacting with a two-level atom. First, when the atom is initially in an excited state($\theta = 0$), by choosing the evolution time $gt = (2n-1)\pi$, the maximal entangled state between the two-mode coherent fields and the atom can be produced. Second, when the atom is initially in a coherent trapping state(namely $\theta = \pi/2$, $\beta = 0$) and at times $gt = \dfrac{2n-1}{2}\pi$, the two-mode coherent fields and the atom are disentangled, thus the maximal entangled state between the two-mode fields can be obtained. Finally, under the conditions of a strong field (both $\overline{n}_1$ and $\overline{n}_2$ are very large) and when the atom is initially in a coherent trapping state, the maximal entangled state between the two-mode coherent fields, which is independent of the evolution time, can be prepared. Our results are important for experimental realization of the preparation of entangled states.

References

[1] Bemett C H and Divincenzo D P 2000 *Nature* **404** 247
[2] Ekert A K 1992 *Phys. Rev. Lett.* **67** 661
[3] Deutsch D and Jozsa R 1992 *Proc. R. Soc. London* A **493** 553
[4] Bennett C H, Brassard G, Crepeau C, Jozsa R, Peres A and Wootters W 1993 *Phys. Rev. Lett.* **70** 1895
[5] Hagley E, Maitre X, Wunderlich C, Brune M, Raimond J M and Haroch S 1997 *Phys. Rev. Lett.* **79** 1
[6] Maitre X, Hagleg E, Nogues G, Wunderlich C, Goy P, Brune M, Raimond J M and Haroche S 1997 *Phys. Rev. Lett.* **79** 769
Ying W, Chu C M and Leung T P 1999 *Phys. Rev.* A **59** 3032
[7] Rauschenbeutel A, Nogues G, Osnaghi S, Bertet P, Brune M, Raimond J M and Haroche S 1999 *Phys. Rev. Lett.* **83** 5166
[8] Nogues G 1999 *Nature* **400** 239
[9] Rauschenbeutel A 2000 *Science* **288** 2024
Song K H 2000 *Acta Phys. Sin.* **49** 441 (in Chinese)
[10] Furusawa A, Sorensen J L, Braunstein S L, Fuchs C A, Kimble H J and Polzik E S 1998 *Science* **282** 706
[11] Rauschenbeutel A, Bertet P, Osnaghi S, Nogues G, Brune M, Raimond J M and Haroche S 2001 *Phys. Rev.* A **64** 50301
Liu J M, Li J and Guo G C 2002 *Chin. Phys.* **11** 339
[12] Monroe C, Meekhof D M, King B E, Itano W M and Wineland D J 1995 *Phys. Rev. Lett.* **75** 4714
[13] Cavers C M and Schumaker B L 1985 *Phys. Rev.* A **31** 3068
[14] Shen Y R 1984 *The Principles of Nonlinear Optics* (New York: Wiley) chap 10
[15] Phoenix S J D and Knight P L 1988 *Ann. Phys.* (*N Y*) **186** 381
[16] Phoenix S J D and Knight P L 1991 *Phys. Rev.* A **44** 6023
[17] Buzek V, Moya-Cess H and Knight P L 1992 *Phys. Rev.* A **45** 8190
[18] Rains E 1999 *Phys. Rev.* A **60** 179
[19] Wu S and Zhang Y 2000 *Preprint* quant-ph/0004018