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Sub-strategy updating evolution in minority game*

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In this paper, we propose and study a new evolution model of minority game. Any strategy in minority game can be regarded as composed of sub-strategies corresponding to different histories. Based on the evolution model proposed by Li–Riolo–Savit, in which those agents that perform poorly may update their strategies randomly. This paper presents a new evolution model in which poor agents update their strategies by changing only a part of sub-strategy sets with low success rate. Simulation result shows that the new model with sub-strategy-set updating evolution mechanism may approach its steady state more quickly than the Li–Riolo–Savit model. In the steady state of the new model, stronger adaptive cooperation among agents will appear, implying that the social resource can be allocated more rationally and utilized more effectively compared with the Li–Riolo–Savit model.

Keywords: minority game, sub-strategy-set, adaptation, evolution model
PACC: 0175, 0565

1. Introduction

In many biological and social systems, it can often be seen that individuals compete for two limited resources with similar size. In such systems, the minority side always benefits. For example, in the stock market, no matter buyer or seller for one stock, the side with less population will certainly benefit. The vehicles have to choose one way to go when coming to a Y-junction. It is apparent that the vehicle choosing the spur track with sparse vehicle population will move fast. Hungry wolves hunt in packs. The wolf group with too crowded population will be more difficult to find food. Hence, we can see widely that successful agents are always those who are different furthest from other competitors. Minority group is always in the ascendant.

To understand the phenomena that minority wins, Challet and Zhang first proposed in 1997, and studied by other authors later[1−5] the minority game (MG) model which consists of N (odd) agents playing a game as follows. At each time step of the game, each of the N agents joins one of two groups, labelled 0 and 1. Each agent that is in the minority group at that time step is awarded a point, while each agent belonging to the majority group obtains nothing. An agent chooses which group to join based on the prediction of his/her strategy. The strategy uses information from the historical record of which group was the minority group, expressed as a function of time. A strategy of memory m is a table of two columns and 2^m rows. The left column contains all possible different m-bit histories, while each entry in the right column is the choice 0 or 1 corresponding to different histories. For a given m, there are totally 2^m possible different histories, thus there are totally 2^k ($k = 2^m$) different strategies. At the beginning of the game, each agent draws randomly out S strategies from a strategy pool as his/her own strategies, repetition being permitted. At each time step, each agent adds a virtual point to those strategies which predict winning side correctly, then after, each agent will choose the strategy with the highest virtual score to make decision. If there are several strategies with the same highest virtual score, then he will choose one randomly from them.

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Table 1. An example of two strategies hold by an agent in a minority game with m=3 and S=2.

History	Strategy 1	Strategy 2
000	0	1
001	1	1
010	1	0
011	0	0
100	0	0
101	1	1
110	0	1
111	1	0

Table 1 is an example of two strategies hold by an agent for minority game with m=3, S=2. History is composed of a 3-bit binary sequence which is the time record of the recent three winning groups. In the right two strategy columns, "0" ("1") means choice to join "0" ("1") group. From this table, we can easily see that the strategy in a minority game is actually composed of decisions under different histories. Decisions under different histories are independent of each other. Hence, what is the choice under a given history for a strategy can be regarded as a sub-strategy. A strategy can be regarded as composed of 2^m sub-strategies corresponding to 2^m different histories which are independent of each other.

In Fig.1, the upper curve shows the normalized variance σ^2/N as a function of memory m in a minority game of N=101, and S=2. Here, the standard deviation σ is defined as

$$\sigma = \sqrt{\frac{\sum_{t=1}^{T}(A_t - \overline{A})^2}{T}}, \qquad (1)$$

where A_t is the number of agents who join A group at time step t and $\overline{A}$ is the average number of agents who join A group during time period from 1 to T.

The upper curve of Fig.1 shows that the variance σ^2/N is rather big for small m, and reduces quickly as m increases. There is a minimum in the variance at $m = m_c$=6. As m increases beyond 6, the variance slowly increases. For large m, the curve approaches a horizontal line with the variance value for the random choice game (RCG). Since the small variance means that more agents may win and the loss of the social resource is less, the minority game shows that in a society consisting of agents competing for limited resources, some adaptive cooperation mechanisms may still appear although each agent is selfish and pursues his own maximal profit.

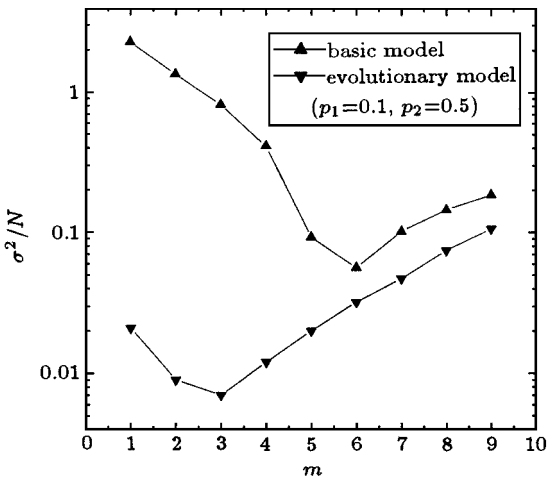


Fig.1. The normalized variance σ^2/N as a function of m for N=101, and S=2. The upper curve is for the basic MG model, showing the average results of 32 independent runs of 10000 time steps after 10000 time steps transient. The lower curve is for the evolutionary model with p_1=0.1 and p_2=0.5, showing the average results of 8 independent runs for last 50 generations of 200 generations. Each generation contains 10000 time steps.

In recent years, the study of complex adaptive systems has become a growing area of research. The focus problems are how to establish the physics model of financial market based on the minority game and the statistical properties of real price data, and how to consider the effects of heterogeneous agents competing for limited resource, the information transmission, imitation among agents, self-segregation versus clustering, and genetic evolution, etc.[6−14]

In the basic minority game, after each agent has initially selected randomly his own strategies, all the strategies will not be changed. However, when Challet and Zhang proposed their minority game in 1997, the evolution concept had been introduced. In the evolutionary MG model, the poorest agent in a certain time interval may be replaced by a new agent, who is the "clone" of the most successful agent, that is to say, the new agent has same strategies with the most successful one, but simultaneously permitting one strategy within them updated randomly with a small probability.[1,2] They found that cooperation among agents may increase by introducing evolution and mutation.

In 2000, Li–Riolo–Savit[15] proposed an evolutionary model in which the poor agents may update their strategies after each generation ended. In their model, when each generation consisting of 10000 time steps ended, a part of worst performing agents may change

randomly the whole of their strategies. Simulation result of this model shows that evolution in this way may enhance cooperation among agents greatly.

We study here a new type of evolutionary minority game. Since a strategy can be regarded as composed of 2^m sub-strategies, each agent can grade all his sub-strategies corresponding to different histories. Hence, each sub-strategy can be ranked according to its success rate. In our model, the poorest agent waiting for adaptation and evolution is not necessary to change his whole strategies. He can keep a part of sub-strategies with a fixed high success rate, but only change another part of sub-strategies with a low success rate. The simulation result shows that the new model may evolve faster into a steady state compared with the Li–Riolo–Savit model, which means that social resource can be utilized more effectively.

2. Definition of the model

We discuss a minority game with N agents, each agent having memory m and S strategies. Thus, each agent holds $k = 2^m$ sub-strategy pairs. Each pair contains S sub-strategies. Different pair is corresponding to different history. Suppose a generation consists of 10000 time steps. At the beginning of each generation, the wealth of each agent is set zero. According to the rule of minority game, at each time step, all the agents joining the minority (winning) group will have one more point to his wealth and each agent uses his strategy with highest virtual score to make decision. If he wins, he adds one more point to the sub-strategy pair corresponding to the current history. The success rate $\eta(j)$ of the sub-strategy pair $j = 1, \cdots, k$ for a generation is defined as

$$\eta(j) = t_{\text{win}}(j)/t(j), \tag{2}$$

where $t_{\text{win}}(j)$ is the number of time steps when the sub-strategy pair succeeds, and $t(j)$ is the number of all the time steps in this generation belonging to a given history. If the history corresponding to a given sub-strategy pair never appears in one generation, then the success rate of this sub-strategy pair is defined as 1.

At the end of each generation, all the agents can be ranked according to their wealth, namely, the accumulated points. A "poor" agent is defined as one whose wealth is in the lowest percentile p_1. Each agent will rank all his sub-strategy pairs according to their success rate. For S=2, all the sub-strategy pairs can have four possible different forms: (0,0), (1,1), (0,1), (1,0). Two figures in each bracket represent the choices of two strategies, respectively. It can be noted that in the first two forms, the choices of two sub-strategies are the same, but in the last two forms, the choices are different. In our model, at the end of each generation, a poor agent whose wealth is in the lowest percentile p_1 will change his sub-strategy pairs with a success rate in the lowest percentile p_2 by random updating. Each agent after sub-strategy pair updating will set the virtual score of his strategies as zero, and the rest of the agents without updating will keep the virtual scores of their strategies unchanged.

For S=2, the choices of the agents who hold sub-strategy pair (0,0) or (1,1) keep fixed in the corresponding history case. However, for the agents who hold (0,1) or (1,0) sub-strategy pairs, there appears a period-two dynamic process, that is, when the same history case appears repeatedly, the agent will be apt to adopt the sub-strategy that made correct prediction in the last same history case. Because most agents select the group which was the winning group in the last history case, the group now becomes the underdog. Hence, in average, the period-two dynamic process is harmful for the success rate of the agents. The success rate is reduced by this process.

3. Simulation result

Let us study the model in which the poor agent may update a part of his sub-strategy pairs. The lower curve in Fig.1 shows the normalized variance as a function of memory m for the evolutionary MG model with parameters p_1=0.1, p_2=0.5 and S=2. Compared with the result of the basic MG model, it can be found that the variance of the evolutionary model is less than that of the basic model, especially for small m in efficient phase of the MG model. It is remarkable that the critical point of the minimal variance which locates at m=6 for the basic model has been moved to m=3 for the new model. This means that the phase transition from an efficient phase to a non-efficient phase will appear earlier as memory increases for the new model. In the evolutionary model, the memory region where the market principle is efficient shrinks, however, the non-efficient region is expanded from $m \geq 6$ to $m \geq 3$.

It is interesting to observe the critical point of phase transition of the new model. Figure 2 shows the normalized variance as a function of generation number for the evolutionary model at the critical point

m=3. Different curves in the figure correspond to different parameters. It can be found that the variances for the different parameters will approach asymptotically to the stable value. However, the speed approaching the final steady state is different, it is faster for small p_2 (=0.5) than for large p_2 (= 1.0) when p_1 value keeps the same. The speed is also high for larger p_1 when keeping p_2 the same.

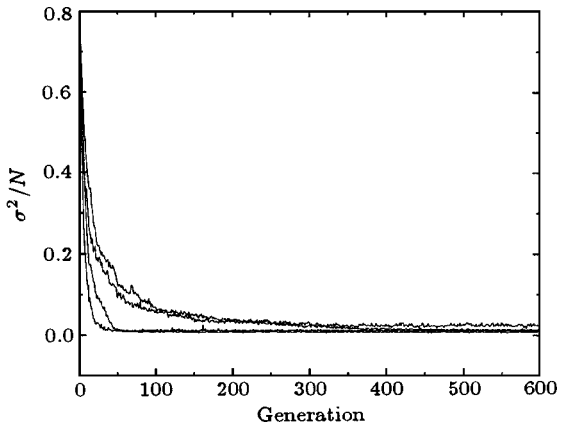


Fig.2. σ^2/N as a function of generation number for m=3, N=101 and S=2. Each curve shows the average result over 8 independent runs. The parameter (p_1, p_2) = (0.1, 1.0), (0.2, 1.0), (0.1, 0.5) and (0.2, 0.5) for the four curves from top to bottom, respectively.

In Fig.3, we plot the average Hamming distance between strategies held by an agent as a function of generation number. The Hamming distance of a strategy pair held by agent j, is defined as

$$D_{\rm h}(j) = \sum_{i=1}^{2^m} |T_j^{[1]}(i) - T_j^{[2]}(i)|, \qquad (3)$$

where $T_j^{[1]}$, $T_j^{[2]}$ are the sub-strategies corresponding to the ith history held by the agent j, for the first and the second strategies, respectively. Hamming distance shows the correlation of two strategies.[16] Larger Hamming distance means a smaller correlation between two strategies.

For the evolutionary model with different parameters, Fig.3 shows that the average Hamming distance falls quickly in the initial decades of generations, then afterwards entering an oscillation phase. Reducing of the Hamming distance means decreasing difference and increasing similarity between the sub-strategies. For the fixed p_1 (p_2), the time average value of the Hamming distance in steady oscillating state decreases as p_2 (p_1) reduces.

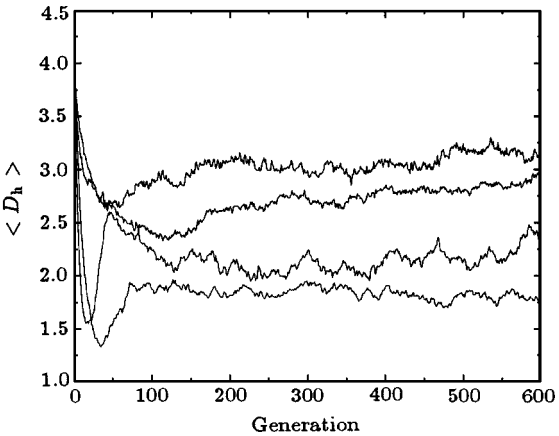


Fig.3. $< D_{\rm h} >$ as a function of generation number for m=3, N=101 and S=2. Four curves from top to bottom show the averaged results over 8 independent runs for (p_1, p_2) = (0.2, 1.0), (0.1, 1.0), (0.2, 0.5), (0.1, 0.5), respectively.

To explore the evolution effect in the situation without adaptation, we also study the special case in which each agent holds only one strategy (S=1). Figure 4 shows the simulation result of the N=101 and S=1 model with p_1=0.1 and p_2=0.5. Figures 4(a) and (b) show the σ^2/N as a function of m in the first generation and in the 200-th generation, respectively. It can be seen that the distribution in m of the variance in the 200-th generation is clearly lower than in the first generation. This result is quite different from the Li–Riolo–Savit model.[15]

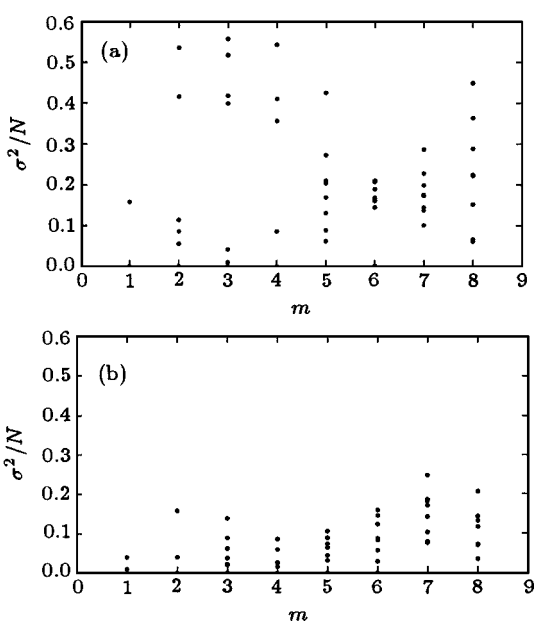


Fig.4. σ^2/N as a function of m for N=101, S=1, p_1=0.1 and p_2=0.5. The eight different points for a fixed m correspond to eight different initial configurations, (a) in the first generation; (b) in the 200-th generation.

4. Discussion

The evolution mechanism in our new model exists in stochastic updating of the sub-strategies held by poor agents. Compared with the basic MG model, the new model shows the further reducing of the system variation, which indicates the better cooperation among the agents due to the new metabolic evolution mechanism. Compared with the Li–Riolo–Savit model, the new model presents a larger speed approaching to the steady state, which means that the system can approach faster to an ideal state showing the best global collaboration favourable for the rational location of social resource.

Let us analyse the main reason why the variance can be reduced in the new model. First, the poor agent holds two strategies with a larger Hamming distance in general. More differences appearing in sub-strategy pairs means that the harmful period-two process will appear in more possible history cases, hence leading to increasing loss possibility for an agent. The updating of part sub-strategy pairs may have a chance to generate two new strategies with less Hamming distance, which will improve the behaviour of the agents and increase his success rate. Secondly, updating stochastically strategies held by poor agents at every generation and the evolution in many generations will lead to a homogeneous dispersive distribution in the whole strategy space from a concentrative distribution in initial stage for the strategies held by agents. The dispersive distribution of the strategies in the strategy space is favourable to the improvement of the global behaviour of the system.

For the model without adaptation in which each agent holds only one strategy, to permit the poor agents updating stochastically his strategies will introduce adaptive cooperation among agents. Hence, to permit poor agents updating his sub-strategy with a lowest success rate will enhance cooperation among agents furthermore and leads to an allocation of the social resource with more rationality.

5. Conclusion

For the minority game, we introduce the concept of sub-strategy. Each strategy can be looked as a combination composed of 2^m sub-strategies corresponding to different histories. Each sub-strategy can be ranked according to its success rate during the corresponding history. Li–Riolo–Savit proposed the evolutionary model in which the poor agent can update his strategy in the whole. We proposed a new model in which poor agent updates not his strategy in the whole, but only sub-strategies with a low success rate composing his strategy. The evolution mechanism of the new model exists in the updating process of the sub-strategies in which the sub-strategy with a high success rate can be kept and that with low success rate falls into disuse. The simulation result shows that compared with the Li–Riolo–Savit model, the new model adopting sub-strategy updating may approach to the steady state faster, introduce stronger adaptation effect, evolve more rapidly into the best cooperation among agents, and lead to the more rational allocation of the social resource.

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