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Dynamical properties of two-band superlattices with strong interband coupling in real space under the action of ac and dc–ac fields*

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Within a two-band tight-binding model driven by ac and dc–ac electric fields, using numerical methods, we investigate the dynamics of electrons and the quasi-energy spectrum of the system with strong interband coupling in real space. We find that when the bandwidth is suppressed to a value much smaller than the field frequency, the dynamical localization can exist in the system. The corresponding regions are found for the occurrence of dynamical localization in the parameter space.

Keywords: dynamical localization, quasi-energy spectrum, strong coupling
PACC: 7335, 7340L

1. Introduction

The single band approximation has provided a very simple and powerful framework for studying the spectral and dynamical properties of Bloch electrons in electric fields. Bloch oscillations[1] and Wannier–Stark ladders[2] are the most striking predictions within this framework. The observations of some theoretical predictions become possible,[3,4] due to the discovery of semiconductor superlattices[5] and atomic laser cooling technique.[6] Recently, there have also been many discussions using this approximation on the effect of dynamic localization induced by an ac electric field,[7] fractional Wannier–Stark ladders produced in a dc–ac field,[8] band collapse,[9] and band suppression.[10] Some of these new predictions have been observed in experiments.[11,12]

However, there are also experimental situations where the single band approximation fails and interband transition becomes important and even dominant, especially when a system is exposed to strong electric fields. A well-known example is the Landau–Zener tunnelling in static electric fields, which makes the Wannier–Stark ladders broaden in energy, and induces avoided crossings between the ladders from different bands. It is known that the dynamics of electrons is closely related to the energy spectrum or the quasi-energy spectrum. The well-known example is that the dynamical manifestation of Wannier–Stark ladders is the localization of Bloch electrons driven by dc electric fields. In the tight-binding model of a one-band system in an external dc–ac field $E(t) = E_0 + E_1 \cos(\omega t)$, one can see that the conditions of dynamical localization coincide with those of band collapse. The dynamical localization occurs when $\omega_{\rm B}/\omega \neq$ integer or $\omega_{\rm B}/\omega = m$(integer) and $J_m(edE_1/\omega) = 0$, where $\omega_{\rm B} = edE_0$ is the Bloch frequency, e the electron charge, d the lattice constant, and J_m the ordinary Bessel function of the order of m. When this happens, the energy (or quasi-energy) spectrum becomes nondispersional. The result should be still right in the tight-binding model of two-band systems.

In this paper, in real space, we investigate the dynamics of electrons and the quasi-energy spectrum of the system with strong interband coupling under the action of ac and dc–ac fields. We obtain the regions in parameter space, in which the band widths are suppressed to a value much smaller than the field frequency and dynamical localization that can exist

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there.

2. Models and methods

We use the standard tight-binding model of a two-band system in a dc–ac field $E(t) = E_0 + E_1\cos(\omega t)$, The model Hamiltonian[13] can be written as

$$\begin{aligned} H(t) = &\sum_n [(\Delta_a + neE(t)d)a_n^+ a_n \\ &+ (\Delta_b + neE(t)d)b_n^+ b_n \\ &- (W_a/4)(a_{n+1}^+ a_n + \text{h.c.}) \\ &+ (W_b/4)(b_{n+1}^+ b_n + \text{h.c.}) \\ &+ \mu E(t)(a_n^+ b_n + b_n^+ a_n)]. \end{aligned} \tag{1}$$

Here the integer n labels the lattice sites ($-N < n < N$), and the operators a and b refer to electrons in the lower and upper minibands, respectively. The first two terms describe the site energies of the Wannier states in the presence of an electric field, and $W_{a,b}$ are the widths of the isolated ($E(t) = 0$) minibands induced by the nearest-neighbour hopping: $\epsilon^{a,b}(k) = \Delta_{a,b} \mp (W_{a,b}/2)\cos(kd)$, where d is the lattice constant. The last term is the on-site electric dipole coupling between minibands; μ is the corresponding dipole moment. This Hamiltonian does neglect Coulomb interactions and electric dipole elements between Wannier states on different sites, but it contains the essential physics for the problem.[13–16] This model only includes the motion of the electrons along the SL axis. The reason of the simplification is that the electric field is applied along the SL axis, and the transverse motion of the electrons is that of the two-dimensional electron gas which is almost not affected by the electric field.

By expressing the particle state $|\psi(t)\rangle$ as a linear combination of Wannier states $|n_a\rangle$ and $|n_b\rangle$,

$$|\psi(t)\rangle = \sum_{n_a} C_{n_a}|n_a\rangle + \sum_{n_b} C_{n_b}|n_b\rangle, \tag{2}$$

where $C_{n_a}(t)$ and $C_{n_b}(t)$ are the time-dependent amplitudes $\langle n_a|\psi(t)\rangle$ and $\langle n_b|\psi(t)\rangle$, respectively, one obtains from Eq.(1) the following evolution equations for the amplitudes $C_{n_a}(t)$ and $C_{n_b}(t)$ (we set $\hbar = 1$ throughout this paper)

$$\begin{aligned} \mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}C_{n_a}(t) = &(\Delta_a + n_a edE(t))C_{n_a} \\ &- (W_a/4)(C_{n_a+1}(t) + C_{n_a-1}) \\ &+ \mu E(t)C_{n_b}, \end{aligned} \tag{3}$$

$$\begin{aligned} \mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}C_{n_b}(t) = &(\Delta_b + n_a edE(t))C_{n_b} \\ &+ (W_b/4)(C_{n_b+1}(t) + C_{n_b-1}) \\ &+ \mu E(t)C_{n_b}, \end{aligned} \tag{4}$$

with the initial condition

$$C_{n_a}(0) = \delta_{n_a,0}, \tag{5}$$

$$C_{n_b}(0) = 0. \tag{6}$$

We have considered a lattice of different sizes in solving Eqs.(3)–(6), and found no essential difference between N = a large number and $N \to \infty$. For example, solving Eqs.(3)–(6) with $N = 25$ and $N = 26$ will generate a same physical solution. In this paper Eqs.(3)–(6) ($-25 \le n \le 25$) are solved numerically by a Runge–Kutta integrator.

Our analysis in this paper is based on two observables: the probability propagator $\psi_n^a(t)(= |C_{n_a}(t)|^2)$ in band a and $\psi_n^b(t)(= |C_{n_b}(t)|^2)$ in band b. We have assumed the electron is at band a of site 0 initially. If the self-propagator $\psi_0^a(t)$ delays with time, we can conclude that an initially localized electron becomes delocalized. On the contrary, if it does not, we say we obtain a localized state. In addition, we can also obtain the property of tunnelling between the two bands of site 0 with the help of $\psi_0^b(t)$.

In order to obtain the regions in the space of parameters in which dynamical localization occurs, we need the help of the quasi-energy spectrum of the system. The quasi-energy spectrum can be calculated by using Floquet theory. The quasi-energy wavefunction of Eq.(1) is written as $|\psi(t)\rangle = \exp(-\mathrm{i}\epsilon t)|\phi(t)\rangle$ with $|\phi(t)\rangle = |\phi(t+T)\rangle$, where T is the period of the external field, ϵ is the quasi-energy, and $|\phi(t)\rangle$ satisfies

$$\left(H(t) - \mathrm{i}\frac{\partial}{\partial t}\right)|\phi(t) = \epsilon|\phi(t)\rangle. \tag{7}$$

For an integer matching ratio ($\omega_\mathrm{B}/\omega = p$)[8,14,17] the Brillouin zone of the quasi-energy is $-\omega/2 \le \epsilon \le \omega/2$. For a fractional matching ratio ($\omega_\mathrm{B}/\omega = p/q$) the Brillouin zone of the quasi-energy is compressed to $-\omega/2q \le \epsilon \le \omega/2q$.[17] For fixed matching ratio and widths of the isolated minibands, we can calculate the widths of the two quasi-energy bands under the action of ac and dc–ac fields in the space of the field and superlattice parameters. Thus, we can have the regions of parameters in which the widths of the two quasi-energy bands are suppressed to a value much smaller than the field frequency. We use the sets of parameters selected from the regions in solving Eqs.(3)–(6) to reach dynamical localization.

3. Results and discussions

In this section we search for the parameters of dynamical localization in the two-band system with a strong coupling under the action of an ac and a dc–ac fields, and display the dynamical properties of the system.

3.1.In the case of ac fields

Figure 1 shows the grey scale of the small range of quasi-energy band width W_d ($0 \leq W_d/\omega \leq 0.006$) as a function of Δ/ω ($= (\Delta_b - \Delta_a)/\omega$) and edE_1/ω under a pure ac field. For $W_d/\omega > 0.006$ the grey scale is of white colour. There we have taken $W_a/\omega = W_b/\omega = 0.2$, $\mu/ed = 0.5$. Due to the symmetry of the parameters, one knows that the two branches of the quasi-energy bands have the same width. Thus, we only need to plot the width of one branch of the quasi-energy bands in Fig.1. We note that, from Fig.1, there are some grey regions in the space of the parameters. The grey regions have very little dispersion, which may induce a dynamical localization. In order to display the effect, we choose the sets of parameters from the grey regions to calculate dynamics of the system by using Eqs.(3)–(6).

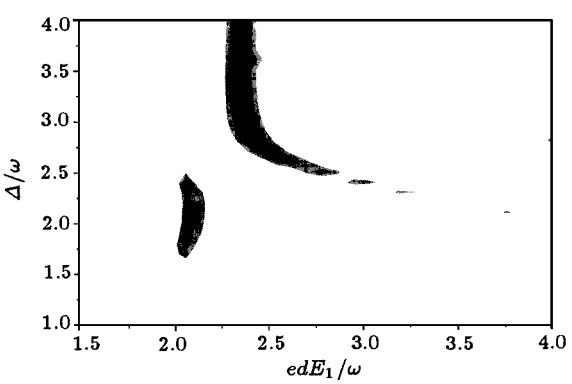


Fig.1. The grey scale of the small range of quasi-energy bandwidth W_d ($0 \leq W_d/\omega \leq 0.006$) as a function of Δ/ω and edE_1/ω under a pure ac field, with $W_a/\omega = W_b/\omega = 0.2$, $\mu/ed = 0.5$. For $W_d/\omega > 0.006$ the grey scale is white colour.

Figure 2(a) shows the dimensionless time ωt dependence of $\psi_0^a(t)$. Here the parameters are chosen from the grey regions in Fig.1, $\Delta/\omega = 2.35$ and $edE_1/\omega = 2.09$, the other parameters are the same as those in Fig.1. We see that the self-propagator $\psi_0^a(t)$ is oscillatory between 0.1 and 1 with time, and can quasi-periodically return to 1. Thus, with this set of parameters, dynamical localization takes place. Figure 2(b) shows the dimensionless time (ωt) dependence of $\psi_0^b(t)$, in which the superlattice and field parameters are the same as those in Fig.2(a). We see that the propagator $\psi_0^b(t)$ is oscillatory between 0 and 0.9 with time. Thus, the electron at site 0 undergoes an approximate Rabi oscillation between bands a and b. Similar situation happens when we choose $\Delta/\omega = 3.015$ and $edE_1/\omega = 2.35$ (the other parameters are the same as those in Fig.1). The self-propagator $\psi_0^a(t)$ is oscillatory between 0.68 and 1, and can quasi-periodically return to 1, as shown in Fig.3(a). Correspondingly, the propagator $\psi_0^b(t)$ oscillates between 0 and 0.32, as shown in Fig.3(b). Thus, the electron has a small probability to tunnel to other sites, but occupies band a with a larger probability than it does band b all the time. In this case the coupling between bands a and b is not taken to be an approximate Rabi oscillation. In order to display the delocalization of the system, we choose a set of parameters from the region out of the grey regions, $\Delta/\omega = 1.5$ and $edE_1/\omega = 2.5$, and the other parameters are the same as those in Fig.1. We can see clearly that $\psi_0^a(t)$ and $\psi_0^b(t)$ are oscillatory but their amplitudes decay rapidly, as shown in Figs.4(a) and 4(b). This is just the delocalization of the system.

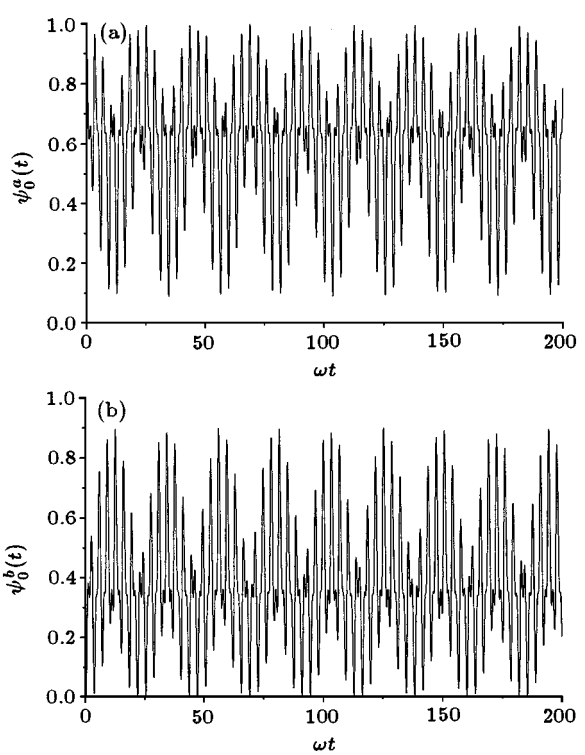


Fig.2. The dimensionless time (ωt) dependence of (a) $\psi_0^a(t)$, and (b) $\psi_0^b(t)$, with $\Delta/\omega = 2.35$ and $edE_1/\omega = 2.09$, the other parameters are the same as those in Fig.1.

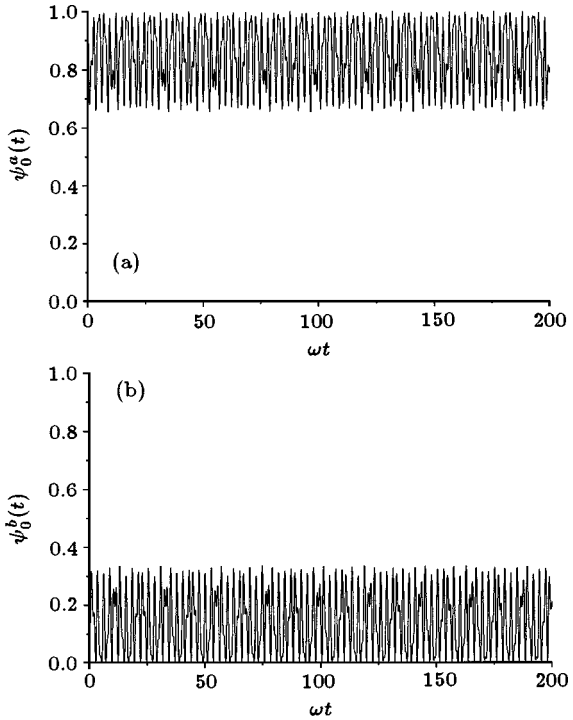


Fig.3. The dimensionless time (ωt) dependence of (a) $\psi_0^a(t)$, and (b) $\psi_0^b(t)$, with $\Delta/\omega = 3.015$ and $edE_1/\omega = 2.35$, the other parameters are the same as those in Fig.1.

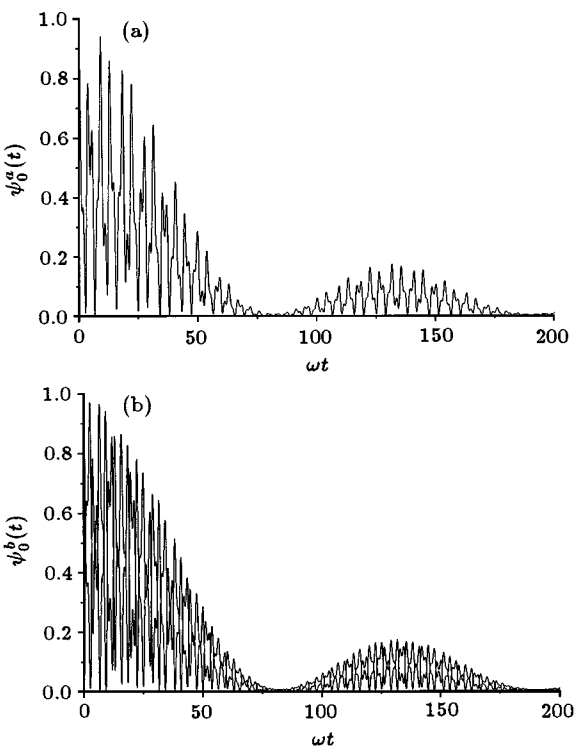


Fig.4. The dimensionless time (ωt) dependence of (a) $\psi_0^a(t)$, and (b) $\psi_0^b(t)$, with $\Delta/\omega = 1.5$ and $edE_1/\omega = 2.5$, the other parameters are the same as those in Fig.1.

3.2.In the case of dc–ac fields

Figure 5 shows the grey scale of the small range of quasi-energy band width W_d ($0 \leq W_d/\omega \leq 0.006$) as a function of Δ/ω and edE_1/ω under the action of a dc–ac field. For $W_d/\omega > 0.006$ the grey scale is of white colour, where we have taken $W_a/\omega = W_b/\omega = 0.8$, $\mu/ed = 0.5$, and $\omega_B/\omega = 1/3$. In this case the Brillouin zone of the quasi-energy is compressed to $0 \leq \epsilon \leq \omega/3$. Due to the symmetry of the parameters, one knows that the two branches of the quasi-energy bands have the same width. Thus, we only need to plot the width of one branch of the quasi-energy bands in Fig.5. We note that there are some grey regions in the space of the parameters. The grey regions have very little dispersion, which may induce a dynamical localization. In order to display the effect, we choose the sets of the parameters from the grey regions to calculate the dynamics of the system using Eqs.(3)–(6).

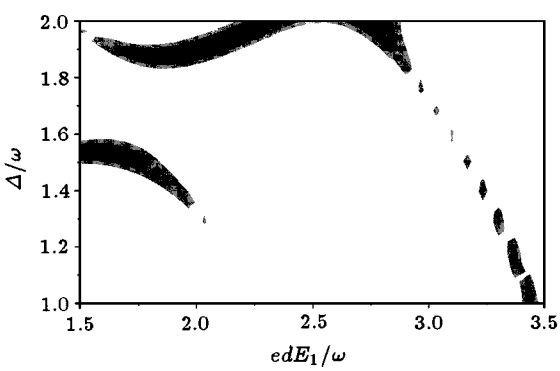


Fig.5. The grey scale of the small range of quasi-energy band widths W_d ($0 \leq W_d/\omega \leq 0.006$) as a function of Δ/ω and edE_1/ω under a dc–ac field, with $\omega_B/\omega = 1/3$, $W_a/\omega = W_b/\omega = 0.8$, and $\mu/ed = 0.5$. For $W_d/\omega > 0.006$ the grey scale is white colour.

Figure 6(a) shows the dimensionless time (ωt) dependence of $\psi_0^a(t)$, where the parameters are chosen from the grey regions in Fig.5, $\Delta/\omega = 1.9$ and $edE_1/\omega = 2.1$, the other parameters are the same as those in Fig.5. We see that the self-propagator $\psi_0^a(t)$ is oscillatory between 0.03 and 1 with time, and can quasi-periodically return to 1. Thus, under this set of parameters, dynamical localization can take place. Figure 6(b) shows the dimensionless time ωt dependence of $\psi_0^b(t)$, where the superlattice and field parameters are the same as those in Fig.6(a). We see that the propagator $\psi_0^b(t)$ is oscillatory between 0 and 0.58 with time. In order to display the delocalization of system, we choose a set of parameters from the region out of the grey regions, $\Delta/\omega = 1.2$ and $edE_1/\omega = 2.5$, the other parameters are the same as those in Fig.5.

We can see clearly that $\psi_0^a(t)$ and $\psi_0^b(t)$ are oscillatory but their amplitudes also decay rapidly, as shown in Figs.7(a) and 7(b). This is just the delocalization of the system.

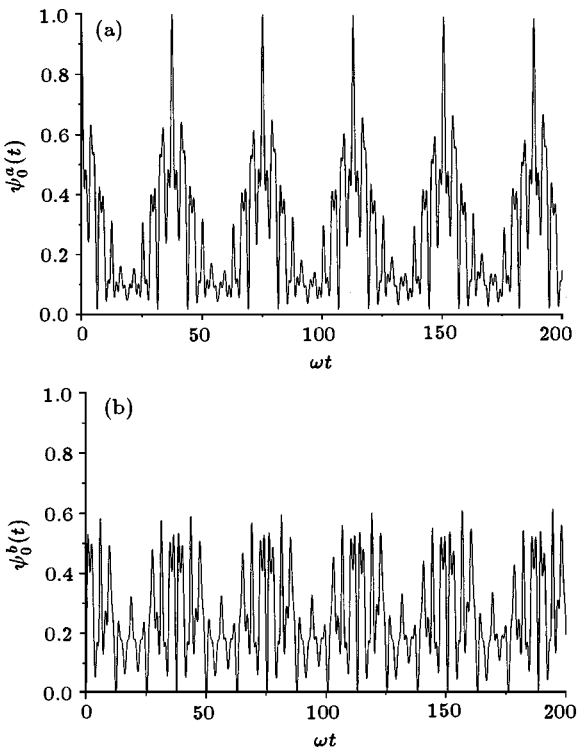


Fig.6. The dimensionless time (ωt) dependence of (a) $\psi_0^a(t)$, and (b)$\psi_0^b(t)$, with $\Delta/\omega = 1.9$ and $edE_1/\omega = 2.1$, the other parameters are the same as those in Fig.5.

According to the above results, we find that, under the action of ac and dc–ac fields, the dynamical localization of the two-band system can occur in some regions of the parameter space, in which the quasi-energy spectra have very little dispersion. Finally, we need to discuss the effect of the coupling strength μ of the two bands on the dynamical properties. For $\mu/ed \ll 1$, the dynamic localization occurs when $\omega_B/\omega \neq$ integer or $\omega_B/\omega = m$(integer) and $J_m(edE_1/\omega) = 0$, which is just the condition of dynamic localization in the tight-binding model of a one-band system. With increasing μ/ed the analytic condition of dynamic localization can hardly be obtained, but the numerical results show that the dynamic localization may occur in some regions in the parameter space. In the absence of any analytic condition of dynamic localization, it may be an effective method that one can obtain the regions of dynamic localization via calculating the energy (or quasi-energy) spectrum of the system.

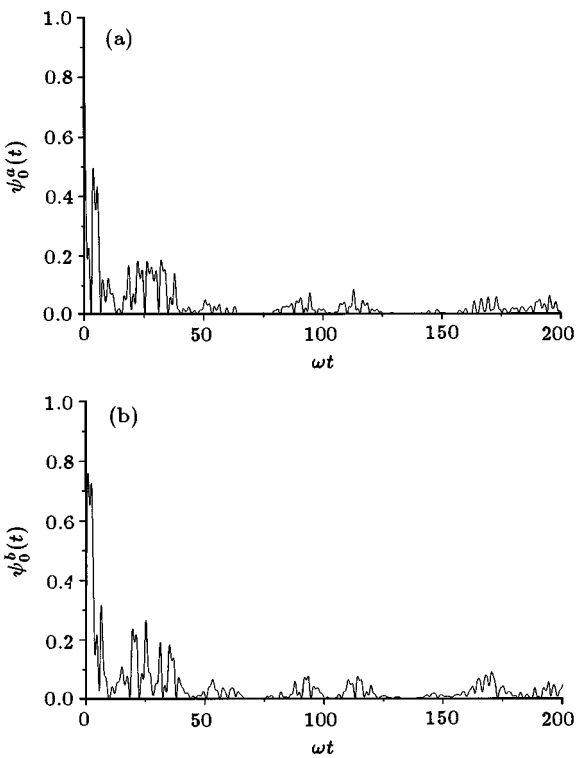


Fig.7. The dimensionless time (ωt) dependence of (a) $\psi_0^a(t)$, and (b)$\psi_0^b(t)$, with $\Delta/\omega = 1.2$ and $edE_1/\omega = 2.5$, the other parameters are the same as those in Fig.5.

In summary, within a two-band tight-binding model driven by ac and dc–ac electric fields, by using numerical methods, we have studied the quasi-energy spectrum and dynamical properties of the system with a strong interband coupling in real space. We find that when the band width is suppressed to a value much smaller than the field frequency, the dynamical localization can occur in the system. We have obtained some regions in the parameter space in which dynamical localization occurs.

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