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Propagation of ultrashort pulsed beams in dispersive media*

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Starting from the Rayleigh diffraction integral, the propagation equation of ultrashort pulsed beams in dispersive media is derived without making the paraxial approximation and slowly varying envelope approximation (SVEA). The spatiotemporal properties of ultrashort pulsed beams in dispersive media, such as spectrum redshifting, narrowing and pulse distortion are illustrated with pulsed Gaussian beams. It is stressed that the "antibeam" behaviour of ultrashort pulsed beams can be avoided, if a suitable truncation function is chosen.

Keywords: ultrashort pulsed beam, propagation, dispersive medium, "antibeam" behaviour
PACC: 4255B, 4280W

1. Introduction

Recently, much interest has been exhibited in ultrashort pulsed light beams due to rapid advances in femtosecond pulsed laser technologies, which have made ultrashort few-cycle, even single-cycle pulses available in laboratory.[1−5] The propagation of ultrashort pulsed beams in free space has been studied extensively.[6−15] Another interesting subject is the propagation of ultrashort pulsed beams in dispersive media.[16−21] In studying the far-field diffraction of pulsed beams in linear dispersive media by using the methods of angular spectrum representation and stationary phase, Agrawal pointed out that the pulse characteristics are affected considerably by the combination of diffraction and dispersion, and the group-velocity dispersion gives rise to pulse broadening.[18] Porras found a family of solutions of three-dimensional wave equation under the slowly evolving envelope approximation, and focused his attention on the dispersion-induced pulse-front bending and dispersion-diffraction coupled effects.[19] Very recently, it has been shown by Porras that the diffraction-free and dispersion-free localized pulsed beam transmission can be achieved if the transverse profile of the pulse is suitably selected and scaled.[20] The aim of the present paper is to study the propagation properties of ultrashort pulsed beams in dispersive media. Starting from the Rayleigh diffraction integral, the propagation equation of ultrashort pulsed beams in dispersive media is derived without making the paraxial approximation and slowly varying-envelope approximation (SVEA) in section 2. The spatiotemporal properties of ultrashort pulsed beams are illustrated with pulsed Gaussian beams propagating in dispersive media, and the problem how to avoid the "antibeam" behaviour is explained with numerical examples in section 3. Finally, section 4 summaries the main results.

2. Propagation equation of ultrashort pulsed beams in dispersive media

The propagation of ultrashort pulsed beams in linear lossless dispersive media can be treated in such a way as first to deal with the propagation of each Fourier monochromatic component of the pulses, then to perform the inverse Fourier transform, thus the propagation equation of ultrashort pulsed beams in the space–time domain can be obtained.

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The monochromatic component of pulses obeys the Rayleigh diffraction integral of the form[22]

$$u(\boldsymbol{r},\omega) = -\frac{1}{2\pi}\int\limits_{\Sigma} u^0(\boldsymbol{r}_0,\omega)\frac{\partial}{\partial z}\left(\frac{\mathrm{e}^{-\mathrm{i}ks}}{s}\right)\mathrm{d}^2\boldsymbol{r}_0, \quad (1)$$

where

$$s = |\boldsymbol{r}-\boldsymbol{r}_0| = \sqrt{(x-x_0)^2+(y-y_0)^2+z^2}, \quad (2)$$

$\boldsymbol{r}_0 = (x_0, y_0, 0)$ is a point at the incident plane, and $\boldsymbol{r} = (x, y, z)$ is a point at the output plane. $u^0(\boldsymbol{r}_0,\omega)$ is the Fourier monochromatic component of the pulse at the incident plane, k is the wavenumber, which satisfies the dispersion relation

$$k(\omega) = n(\omega)\omega/c. \quad (3)$$

Applying the formula

$$\frac{\partial}{\partial z}\left(\frac{\mathrm{e}^{-\mathrm{i}ks}}{s}\right) = \left(-\mathrm{i}k - \frac{1}{s}\right)\left(\frac{\mathrm{e}^{-\mathrm{i}ks}}{s}\right)\frac{\partial s}{\partial z}$$

and the assumption $kr \gg 1$, Eq.(1) becomes

$$u(\boldsymbol{r},\omega) = \frac{\mathrm{i}k}{2\pi}\int\limits_{\Sigma} u^0(\boldsymbol{r}_0,\omega)\frac{\mathrm{e}^{-\mathrm{i}ks}}{s}\frac{\partial s}{\partial z}\mathrm{d}^2\boldsymbol{r}_0. \quad (4)$$

The propagation distance s is expanded into a series, and the first and second terms are kept. Thus, we have

$$s \approx r + \frac{x_0^2+y_0^2-2(xx_0+yy_0)}{2r}, \quad (5)$$

where

$$r = \sqrt{x^2+y^2+z^2}. \quad (6)$$

It is noted that, as compared with Ref.[8], in which the far-field approximation for s is made, in our approach the additional term $(x_0^2+y_0^2)/(2r)$ is kept. There are somewhat resemblances of our approach with the Fresnel approximation, and the former is similar to the Fraunhofer approximation in the classic diffraction theory.

Replacing s in the exponential term in Eq.(4) by Eq.(5), and in other terms by r, Eq.(4) becomes

$$u(\boldsymbol{r},\omega) = \frac{\mathrm{i}k\cos\theta}{2\pi}\frac{\mathrm{e}^{-\mathrm{i}kr}}{r}\int\limits_{\Sigma} u^0(\boldsymbol{r}_0,\omega) \times \exp\left[-\frac{\mathrm{i}k}{2r}(x_0^2+y_0^2-2xx_0-2yy_0)\right]\mathrm{d}^2\boldsymbol{r}_0, \quad (7)$$

where

$$\cos\theta = z/r \quad (8)$$

with θ being the diffraction angle.

Consider that the initial pulsed light beam at the plane z=0 has a Gaussian shape, i.e.,

$$u^0(\boldsymbol{r}_0,\omega) = f(\omega)\exp\left(-\frac{\rho_0^2}{2w_0^2}\right), \quad (9)$$

where $\rho_0^2 = x_0^2+y_0^2$, $f(\omega)$ denotes the initial spectrum on the axis, and w_0 is the waist width. In consideration of the frequency dependence of the waist width,[9,18] we have

$$w_0 = \sqrt{l/k}, \quad (10)$$

where l is the constant diffraction length, independent of frequency ω.

Assume that the size of the diffraction aperture at the incident plane is much larger than the beam waist width, so that the integration in Eq.(7) can be performed from $-\infty$ to $+\infty$. The substitution from Eqs.(9) and (10) into Eq.(7) yields

$$u(\boldsymbol{r},\omega) = \frac{\mathrm{i}l\cos\theta}{r+\mathrm{i}l}f(\omega)\exp(-\mathrm{i}kr) \times \exp\left[-\frac{klr\sin^2\theta}{2(r+\mathrm{i}l)}\right]. \quad (11)$$

Eq.(11) is the propagation equation of each Fourier component of ultrashort pulsed beams having a Gaussian-shape at the plane z=0, which is valid if

$$r^3 \gg \frac{k}{8}[x_0^2+y_0^2-2(xx_0+yy_0)]^2_{\max}. \quad (12)$$

Inequality (12) is only a sufficient but not necessary condition, because according to the stationary phase principle,[23] Eq.(12) is still applicable even when r is relatively small.

It is readily seen that Eq.(11) simplifies to

$$u(\boldsymbol{r},\omega) = \frac{\mathrm{i}l\cos\theta}{r}f(\omega)\exp(-\mathrm{i}kr)\exp\left(-\frac{kl\sin^2\theta}{2}\right) \quad (13)$$

if the far-field approximation $r \gg l$ is made, and Eq.(13) is consistent with Eq.(7) in Ref.[8] and Eq.(6) in Ref.[18]. On the other hand, under the paraxial approximation, Eq.(11) reduces to a Gaussian form

$$u(\boldsymbol{r},\omega) = \frac{\mathrm{i}l}{z+\mathrm{i}l}f(\omega)\exp(-\mathrm{i}kz)\exp\left[-\frac{\mathrm{i}k\rho^2}{2(z+\mathrm{i}l)}\right], \quad (14)$$

where $\rho^2 = x^2+y^2$. Eq.(14) is just the monochromatic component of Eq.(14) in Ref.[11].

For pulses wider than a few cycles whose carrier frequency is far from the resonant frequency of the media, according to Ref.[18] the propagation constant

can be expanded into a Taylor series around the carrier frequency ω_0, namely

$$k(\omega)=\beta_0+\beta_1(\omega-\omega_0)+\frac{1}{2}\beta_2(\omega-\omega_0)^2+\cdots, \quad (15)$$

where $\beta_m=(\mathrm{d}^m\beta/\mathrm{d}\omega^m)|_{\omega=\omega_0}$, m=0,1,2,..., $\beta_1=1/\nu_\mathrm{g}$ being inversely related to the group velocity, and β_2 being the group velocity dispersion parameter.

The substitution of Eq.(15) into Eq.(11) yields

$$\begin{aligned}u(\boldsymbol{r},\omega)=&\frac{\mathrm{i}l\cos\theta}{r+\mathrm{i}l}f(\omega)\\&\times\exp\left\{-\mathrm{i}r\left[\beta_0+\beta_1(\omega-\omega_0)\right.\right.\\&\left.\left.+\frac{1}{2}\beta_2(\omega-\omega_0)^2\right]\right\}\\&\times\exp\left\{-\frac{lr\sin^2\theta}{2(r+\mathrm{i}l)}\left[\beta_0+\beta_1(\omega-\omega_0)\right.\right.\\&\left.\left.+\frac{1}{2}\beta_2(\omega-\omega_0)^2\right]\right\}. \quad (16)\end{aligned}$$

Making use of the inverse Fourier transform,[18] it turns out from Eq.(16) that

$$\begin{aligned}u(\boldsymbol{r},t)=&\frac{\mathrm{i}l\cos\theta}{\sqrt{2\pi}(r+\mathrm{i}l)}\exp(\mathrm{i}\omega_0t-\mathrm{i}r\beta_0)\\&\times\exp\left[-\frac{lr\sin^2\theta\beta_0}{2(r+\mathrm{i}l)}\right]\int f(\Delta\omega+\omega_0)\\&\times\exp\left[-\mathrm{i}r\left(\beta_1\Delta\omega+\frac{1}{2}\beta_2\Delta\omega^2\right)\right]\\&\times\exp\left[-\frac{lr\sin^2\theta}{2(r+\mathrm{i}l)}\left(\beta_1\Delta\omega+\frac{1}{2}\beta_2\Delta\omega^2\right)\right]\\&\times\exp(\mathrm{i}\Delta\omega t)\mathrm{d}\Delta\omega, \quad (17)\end{aligned}$$

where $\Delta\omega=\omega-\omega_0$. Eq.(17) is the propagation equation of ultrashort pulsed beams in dispersive media, provided that $u^0(\boldsymbol{r}_0,\omega)$ has a Gaussian form. As compared with Ref.[19], in our derivation of Eqs.(11) and (17), the paraxial approximation and SVEA are not used. Therefore, Eqs. (11) and (17) are applicable for relatively large diffraction angles. However, the so-called "antibeam" behaviour may appear, which will be discussed later.

3. Spatiotemporal properties of ultrashort pulsed beams propagating in dispersive media

From Eq.(16) it is seen that if[18]

$$k(\omega)=\beta_0+\beta_1(\omega-\omega_0)+\frac{1}{2}\beta_2(\omega-\omega_0)^2+\cdots<0, \quad (18)$$

$u(\boldsymbol{r},\omega)$ increases with increasing diffraction angle θ, which leads to the "antibeam" behaviour. To avoid the unwanted behaviour, it is necessary to let $f(\omega)$=0 for $k(\omega)<0$.[19] In particular, the condition Eq.(18) means $\omega<0$ in free-space propagation, this is the reason why the analytical signal complex representation can eliminate the singularity in the transversal pulse profile.[11,12]

Consider a pulsed Gaussian beam whose initial spectrum at r=0 takes the form

$$f(\omega)=\frac{T}{\sqrt{2}a_\mathrm{g}}\exp\left[-\frac{T^2(\omega-\omega_0)^2}{4a_\mathrm{g}^2}\right]h(\omega-\omega_\mathrm{T}), \quad (19)$$

where $T=mT_0$ denotes the pulse duration, T_0 is the oscillating period corresponding to the carrier frequency ω_0, m is the number of the oscillating periods within the pulse duration, $a_\mathrm{g}=\sqrt{2\ln 2}$. $h(\omega-\omega_\mathrm{T})$ is the Heaviside step function, introduced to avoid "antibeam" behaviour, where ω_T is determined by

$$\begin{aligned}k(\omega)=&\beta_0+\beta_1(\omega-\omega_0)+\frac{1}{2}\beta_2(\omega-\omega_0)^2\\\approx&\beta_0+\beta_1(\omega-\omega_0)<0 \quad (20)\end{aligned}$$

and is given by

$$\omega_\mathrm{T}=\omega_0-\beta_0/\beta_1. \quad (21)$$

On substituting Eq.(19) into Eq.(17) we obtain

$$\begin{aligned}u(\boldsymbol{r},t)=&\frac{\mathrm{i}l\cos\theta T}{2\sqrt{\pi}a_\mathrm{g}(r+\mathrm{i}l)}\exp(\mathrm{i}\omega_0t-\mathrm{i}r\beta_0)\\&\times\exp\left[-\frac{lr\sin^2\theta\beta_0}{2(r+\mathrm{i}l)}\right]\\&\times\int_{-\beta_0/\beta_1}^{+\infty}\exp\left[-\frac{T^2\Delta\omega^2}{4a_\mathrm{g}^2}\right]\\&\times\exp\left[-\mathrm{i}r\left(\beta_1\Delta\omega+\frac{1}{2}\beta_2\Delta\omega^2\right)\right]\\&\times\exp\left[-\frac{lr\sin^2\theta}{2(r+\mathrm{i}l)}\left(\beta_1\Delta\omega+\frac{1}{2}\beta_2\Delta\omega^2\right)\right]\\&\times\exp(\mathrm{i}\Delta\omega t)\mathrm{d}\Delta\omega. \quad (22)\end{aligned}$$

The corresponding Fourier spectrum reads

$$\begin{aligned}u(\boldsymbol{r},t)=&\frac{\mathrm{i}l\cos\theta T}{\sqrt{2}a_\mathrm{g}(r+\mathrm{i}l)}h(\omega-\omega_\mathrm{T})\exp(-\mathrm{i}r\beta_0)\\&\times\exp\left[-\frac{lr\sin^2\theta\beta_0}{2(r+\mathrm{i}l)}\right]\exp\left[-\frac{T^2(\omega-\omega_0)^2}{4a_\mathrm{g}^2}\right]\\&\times\exp\left\{-\mathrm{i}r\left[(\beta_1(\omega-\omega_0)+\frac{1}{2}\beta_2(\omega-\omega^2)^2\right]\right\}\\&\times\exp\left\{-\frac{lr\sin^2\theta}{2(r+\mathrm{i}l)}\left[\beta_1(\omega-\omega_0)\right.\right.\\&\left.\left.+\frac{1}{2}\beta_2(\omega-\omega_0)^2\right]\right\}. \quad (23)\end{aligned}$$

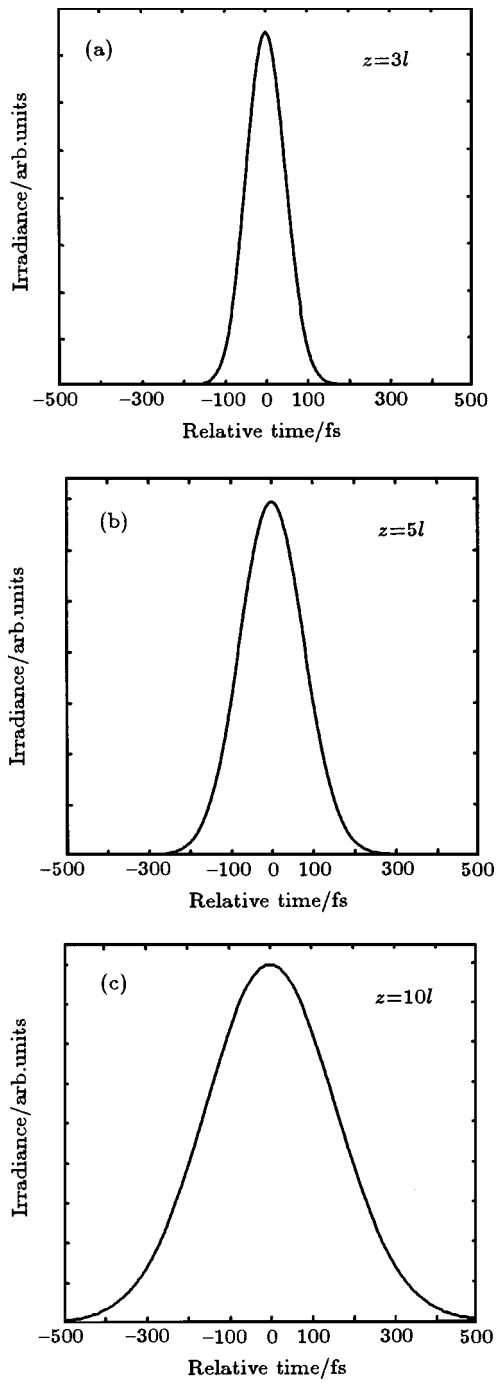


Fig.1. On-axis pulse forms of a pulsed Gaussian beam. (a) $z=3l$; (b) $z=5l$; (c) $z=10l$. The calculation parameters are seen in the text.

Numerical calculations were performed using Eqs.(22) and (23). The on-axis pulse forms of a pulsed Gaussian beam for several values of $z=3l$, $5l$, $10l$ are shown in Fig.1. The calculation parameters are $\theta = 0°$, l=10mm, ω_0=1.9fs^{-1}, m=5, β_0=9193mm^{-1}, β_1=4881mm^{-1}fs, and β_2=21.78mm^{-1}fs^2. As can be seen, the pulse broadening appears, but its shape remains nearly unchanged, as the pulse propagates along the axis. Fig.2 gives power spectra of a pulsed Gaussian beam for several values of the diffraction angle $\theta = 0°$, 4.8°, 5.0°, 5.2°, 5.4°. The calculation parameters are l=10mm, ω_0=1.9fs^{-1}, m=5, $z = 4l$, β_0=9193mm^{-1}, β_1=4881mm^{-1}fs, β_2=21.78mm^{-1}fs^2. From Fig.2 we see that there is a frequency shift towards the lower frequency up to the diffraction angle $\theta = 4.8°$. For larger values of θ, the spectrum approaches ω_T, and the spectral distortion and narrowing appear. The off-axis pulse forms of a pulsed Gaussian beam for several values of $\theta = 0°$, 4.8°, 5.0°, 5.4°, 6.0°, 35° are depicted in Fig.3 (solid line). The calculation parameters are the same as those in Fig.2. The pulse form remains unchanged for the diffraction angle $\theta \leq 4.8°$. However, for $4.8° < \theta \leq 6°$, the pulse distortion takes place, and for larger values of $\theta > 6°$, the pulse broadens notably. The above qualitative results hold true also for negative dispersive media and are confirmed by numerical calculations.

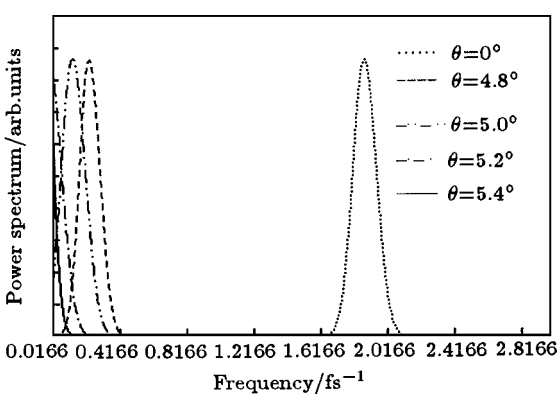


Fig.2. Power spectra of a pulsed Gaussian beam for several values of θ. The calculation parameters are seen in the text.

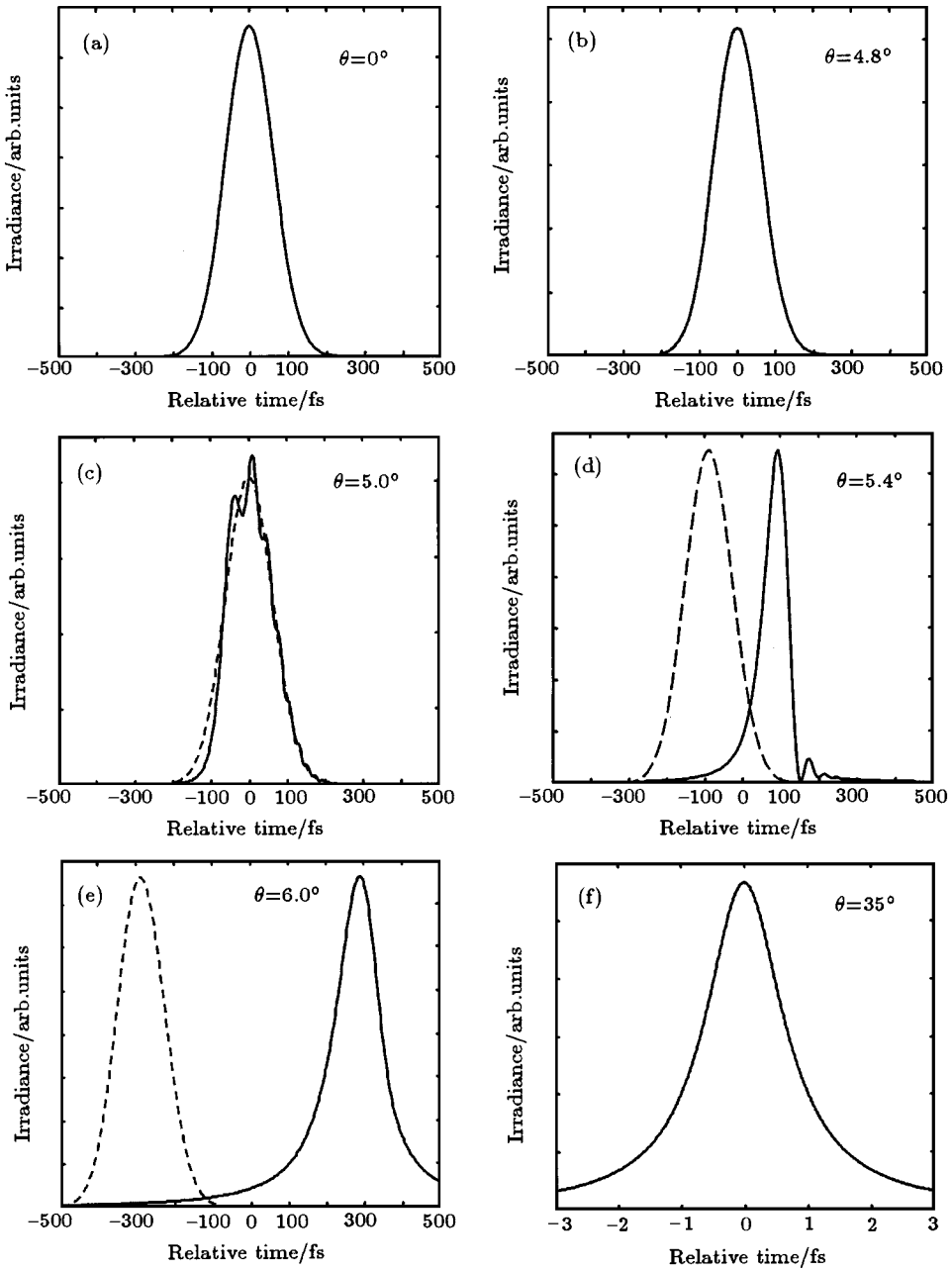


Fig.3. Off-axis pulse forms of a pulsed Gaussian beam for several values of θ. (a) $\theta = 0°$; (b) $\theta = 4.8°$; (c) $\theta = 5.0°$; (d) $\theta = 5.4°$; (e) $\theta = 6.0°$; (f) $\theta = 35°$. For comparison, the solid lines in (d) and (e) are multiplied by a factor of 50.8 and 7.13×10^{18}, respectively. "—" using Eq.(22), "---" using Eq.(24).

If the truncation function $h(\omega-\omega_{\mathrm{T}})$ in Eq.(19) is not used, thus the lower limit of the integration in Eq.(22) is expanded to $-\infty$, the straightforward integral of Eq.(22) yields

$$u(\boldsymbol{r},t) = \frac{il\cos\theta T}{2a_{\mathrm{g}}(r+il)}\exp(\mathrm{i}\omega_0 t - \mathrm{i}rb_0)\left[\frac{1}{T^2/(4a_{\mathrm{g}}^2)+\mathrm{i}r\beta_2/2+lr\beta_2\sin^2\theta/(4r+4il)}\right]^{1/2}$$
$$\exp\left[-\frac{lr\beta_0\sin^2\theta}{2(r+il)}\right]\exp\left\{-\frac{[t-r\beta_1+\mathrm{i}lr\beta_1\sin^2\theta/(2r+2il)]^2}{4[T^2/(4a_{\mathrm{g}}^2)+\mathrm{i}r\beta_2/2+lr\beta_2\sin^2\theta/(4r+4il)]}\right\}. \tag{24}$$

The analytical expression (24) indicates that the pulse retains the Gaussian shape during propagation, although it experiences time shift, becomes chirped, and its width changes. The corresponding calculation results are shown in Figs.3(a)–(e) (dashed line), from which we see that for the diffraction angle of

$\theta < 5.0°$ the dashed line coincides with the solid line, whereas a remarkable difference between the dashed and solid lines appears if $\theta \geq 5.0°$. In addition, it is found that for $\theta \geq 8°$ Eq.(24) diverges due to the "antibeam" behaviour discussed above, so that in the case of Fig.3(f) Eq.(24) is not used. The "antibeam" behaviour introduced by Eq.(24) is evident with increasing diffraction angle, as shown in Fig.4. According to Eq.(22), the maximum irradiance (solid line) decreases monotonically as the diffraction angle increases. However, the maximum irradiance in accordance with Eq.(24)(dashed line) first decreases with increasing θ for $\theta < 5.0°$, then increases with increasing θ remarkably for $\theta \geq 5.0°$.

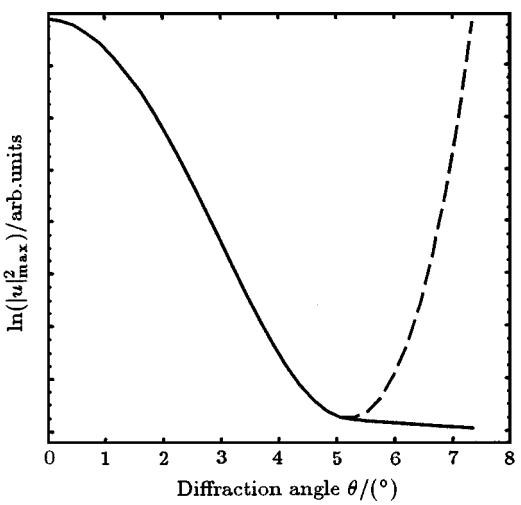


Fig.4. Maximum irradiance $\ln(|u|^2_{\max})$ (ln-natural logarithm) versus diffraction angle θ. "—" using Eq.(22), "- - -" using Eq.(24).

Similar to Ref.[19], the valid condition for Eq.(24) can be derived as follows. Defining the redshift of a pulse as

$$\delta = \frac{\int_{-\infty}^{+\infty} (\omega - \omega_0)|u(\boldsymbol{r},\omega)|^2 \mathrm{d}\omega}{\int_{-\infty}^{+\infty} |u(\boldsymbol{r},\omega)|^2 \mathrm{d}\omega}, \tag{25}$$

using

$$f(\omega) = \frac{T}{\sqrt{2}a_\mathrm{g}} \exp\left[-\frac{T^2(\omega-\omega_0)^2}{4a_\mathrm{g}^2} \right] \tag{26}$$

(untruncated pulsed Gaussian spectrum)

and

$$\exp\left[-\frac{lr^2\sin^2\theta}{r^2+l^2}\left(\beta_1\Delta\omega + \frac{1}{2}\beta_2\Delta\omega^2\right)\right] \approx 1 - \frac{lr^2\sin^2\theta}{r^2+l^2}\beta_1\Delta\omega, \tag{27}$$

the substitution of Eq.(16) into Eq.(25) gives

$$\delta = -\frac{lr^2\sin^2\theta\beta_1 a_\mathrm{g}^2}{(r^2+l^2)T^2}. \tag{28}$$

For pulsed Gaussian spectrum with N cycles the root-mean-square bandwidth reads as

$$\Delta\omega_{\mathrm{rms}} = \sqrt{\int(\omega-\omega_0)^2|f(\omega)|^2\mathrm{d}\omega \Big/ \int |f(\omega)|^2\mathrm{d}\omega} = \frac{a_\mathrm{g}}{T} \tag{29}$$

and $|\delta|$ should satisfy the condition

$$|\delta| < \omega_0 - \omega_\mathrm{T} - \Delta\omega_{\mathrm{rms}}. \tag{30}$$

From Eqs.(21) and (28)–(30) we obtain

$$\sin^2\theta < \frac{(T^2\beta_0 - a_\mathrm{g}\beta_1 T)(z^2+l^2)}{lz^2\beta_1^2 a_\mathrm{g}^2 + \beta_0 T^2 l^2 - a_\mathrm{g}\beta_1 T l^2}. \tag{31}$$

Inequality (31) implies that for the diffraction angle θ satisfying Eq.(31), Eq.(24) is valid; the results are consistent with Eq.(22), and there is no "antibeam" behaviour. However, for the relatively large diffraction angle θ, which violates inequality (31), Eq.(24) is not applicable. For our numerical calculation example of Fig.3, from Eq.(31) we obtain $\theta < 5.0°$.

4. Concluding remarks

In this study, by using the Rayleigh diffraction integral the propagation equation of ultrashort pulsed beams in dispersive media has been derived, where neither the paraxial approximation nor the SVEA is used. The spatiotemporal properties of ultrashort pulsed beams propagating in dispersive media have been analysed with pulsed Gaussian beams. It has been shown that a closed-form propagation Eq. (24) of pulsed Gaussian beams in dispersive media can be derived, but the "antibeam" behaviour may appear for relatively large diffraction angles. To avoid it, a suitable truncation has to be introduced. The valid condition for the propagation equation has been derived. For the relatively large diffraction angle, the propagation Eq. (22) has to be used.

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