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Atomic emission and cavity field spectra of the Jaynes–Cummings model*

Li Fu-Li(李福利)†, Gao Shao-Yan(高韶燕), and Zhao Yong-Tao(赵永涛)

Department of Applied Physics, Xi'an Jiaotong University, Xi'an 710049, China

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Atomic emission and cavity field spectra of the Jaynes–Cummings model are analytically compared. We show that the two spectra are in general different, except for the special case where the interaction is resonant, the cavity is initially empty and the retarded time of one-photon emission is considered in calculations of the spectra. In this case, the two spectra are the same. We also show that this result comes from the fact that the two-time autocorrelation function of the cavity field is not directly proportional to that of the atom.

Keywords: resonance fluorescence spectrum, two-time correlation function
PACC: 4250, 3250

1. Introduction

The spectral properties of light emission of excited atoms or molecules contain much information on the interaction between matter and radiation. For stationary processes, the light emission spectrum can be calculated according to the Wiener–Khintchine theorem. However, for nonstationary cases, the spectrum which is calculated from the Wiener–Khintchine theorem becomes unphysical.[1] In order to rectify this and directly connect experimental observations with theoretical calculations of the spectrum for nonstationary processes, Eberly and Wódkiewicz[1] introduced the time-dependent physical spectrum of light emission based on the counting rate of a photodetector

$$\begin{aligned} &S_f(\omega,t,\Gamma)\\ =&2\Gamma\int_0^t \mathrm{d}t_1\int_0^t \mathrm{d}t_2 \mathrm{e}^{-(\Gamma-\mathrm{i}\omega)(t-t_1)-(\Gamma+\mathrm{i}\omega)(t-t_2)}\\ &\times\langle\hat{\boldsymbol{E}}^{(-)}(t_1)\cdot\hat{\boldsymbol{E}}^{(+)}(t_2)\rangle, \end{aligned} \tag{1}$$

where t is the time at which the measurement is conducted, Γ characterizes the passband width of the filter, and $\hat{\boldsymbol{E}}^{(+)}$ and $\hat{\boldsymbol{E}}^{(-)}$ are positive and negative frequency operators of the electric field, respectively. Since the retarded positive frequency operator of the electric field emitted or scattered by atoms is related to the atomic polarization operator $\hat{P}^{(+)}(t)$ in the far zone limit in free space by the relation[2]

$$\hat{\boldsymbol{E}}^{(+)}(r,t)=-\frac{\omega_0^2}{c^2 r}\hat{\boldsymbol{n}}\times(\hat{\boldsymbol{n}}\times\hat{\boldsymbol{d}})\hat{P}^{(+)}\left(t-\frac{r}{c}\right), \tag{2}$$

the autocorrelation function of the electric field in Eq.(1) can be replaced by the atomic autocorrelation function $\langle\hat{P}^{(-)}(t_1)\hat{P}^{(+)}(t_2)\rangle$. Substituting Eq.(2) into Eq.(1), we obtain

$$\begin{aligned} &S_{\mathrm{a}}(\omega,t,\Gamma)\\ =&2\Gamma\int_0^t \mathrm{d}t_1\int_0^t \mathrm{d}t_2 \mathrm{e}^{-(\Gamma-\mathrm{i}\omega)(t-t_1)-(\Gamma+\mathrm{i}\omega)(t-t_2)}\\ &\times\langle\hat{P}^{(-)}(t_1)\cdot\hat{P}^{(+)}(t_2)\rangle. \end{aligned} \tag{3}$$

Obviously, if Eq.(2) holds, then Eq.(3) is proportional to Eq.(1) and both are equivalent. Equation (3) is often used in theoretical calculations of the atomic resonance fluorescence spectrum for transient or stationary processes.[1–3]

The Jaynes–Cummings model (JCM)[4] describes the interaction of a single two-level atom with a single quantized mode of the electromagnetic field in an ideal cavity. This model has been realized experimentally in a superconductive cavity of very high quality factor.[5] Because any loss processes are not included

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†Corresponding author: E-mail: flli@mail.xjtu.edu.cn

in the JCM, the dynamics of both the atom and field is obviously nonstationary. For this case, if we employ Eqs.(1) and (3), two kinds of spectra appear. According to Eq.(1), we have the cavity field spectrum

$$\begin{aligned} & S_f(\omega,t,\Gamma) \\ =& 2\Gamma\int_0^t \mathrm{d}t_1\int_0^t \mathrm{d}t_2 \mathrm{e}^{-(\Gamma-\mathrm{i}\omega)(t-t_1)-(\Gamma+\mathrm{i}\omega)(t-t_2)} \\ & \times\langle\hat{a}^+(t_1)\hat{a}(t_2)\rangle, \end{aligned} \tag{4}$$

where $\hat{a}^\dagger(\hat{a})$ is the creation(annihilation) operator for the cavity mode. According to Eq.(3), we have the atomic emission spectrum

$$\begin{aligned} & S_{\mathrm{a}}(\omega,t,\Gamma) \\ =& 2\Gamma\int_0^t \mathrm{d}t_1\int_0^t \mathrm{d}t_2 \mathrm{e}^{-(\Gamma-\mathrm{i}\omega)(t-t_1)-(\Gamma+\mathrm{i}\omega)(t-t_2)} \\ & \times\langle\hat{\sigma}^\dagger(t_1)\hat{\sigma}(t_2)\rangle, \end{aligned} \tag{5}$$

where $\hat{\sigma}$ and $\hat{\sigma}^\dagger$ are the atomic level descending and lifting operators given by $|g\rangle\langle e|$ and $|e\rangle\langle g|$, respectively, with the lower level $|g\rangle$ and the upper level $|e\rangle$. Spectral properties of the JCM have been extensively investigated on the basis of Eqs.(4) and (5).[6−20] Some interesting results, such as vacuum Rabi splitting, have been revealed.[6−8] In the JCM, the atom and cavity field are isolated from the surroundings and any loss processes are not considered. In order to measure the two spectra given by Eqs.(4) and (5), we may suppose that at one of the cavity ends there is a small hole from which the cavity field may leak out. We also assume that on the side of the cavity there is a small window from which the fluorescent field emitted by the atom into the background modes may escape. Of course, we assume that, compared with the Rabi frequency of the atom, the leakage rate of photons is very small. So, the influence of the photon leakage processes on the dynamics of the system is insignificant and can be neglected. In this case, Eq.(4) is the transmission spectrum of the cavity field, and Eq.(5) is the spectrum of the resonance fluorescence radiated by the atom out of the side of the cavity.[8,17] As we see, unlike the proportionality of Eqs.(1) and (3), Eqs.(4) and (5) correspond to different physical measurement situations. In general, they should be different. However, previous studies showed numerically that the two spectra look like the same under certain conditions.[6−18] To some extent, there is some ambiguity on the two spectra. In this paper, we will analytically compare the two kinds of spectra under various situations. The conditions under which the spectra are the same or different are given.

2. Atomic emission spectrum and Cavity field spectrum

In the rotating-wave approximation, the Hamiltonian of the JCM takes the form

$$\hat{H}=\frac{1}{2}\hbar\omega_{\mathrm{a}}\hat{\sigma}_z+\hbar\omega_{\mathrm{c}}\hat{a}^\dagger\hat{a}+\hbar\lambda_{\mathrm{c}}(\hat{a}^\dagger\hat{\sigma}+\hat{\sigma}^\dagger\hat{a}), \tag{6}$$

where ω_{a} is the frequency of the atomic transition, ω_{c} the frequency of the cavity eigenmode, σ_z the atomic level population inversion given by $|e\rangle\langle e|-|g\rangle\langle g|$ and λ_{c} the atom-field coupling constant. It is easy to find eigenenergies and corresponding eigenstates of Eq.(6), which are given by

$$|\varphi_0\rangle=|0,g\rangle \tag{7}$$

with eigenenergy $E_0=-\frac{1}{2}\hbar\omega_{\mathrm{a}}$,

$$|\varphi_{n+}\rangle=\sin\theta_n|n-1,e\rangle+\cos\theta_n|n,g\rangle \tag{8}$$

with eigenenergy $E_{n+}=\frac{1}{2}\hbar[(2n-1)\omega_{\mathrm{c}}+\sqrt{\Delta^2+4\lambda_{\mathrm{c}}^2 n}]$ and

$$|\varphi_{n-}\rangle=\cos\theta_n|n-1,e\rangle-\sin\theta_n|n,g\rangle \tag{9}$$

with eigenenergy

$$E_{n-}=\frac{1}{2}\hbar[(2n-1)\omega_{\mathrm{c}}-\sqrt{\Delta^2+4\lambda_{\mathrm{c}}^2 n}].$$

In Eqs.(8) and (9), $n=1,2,3,\ldots$ and

$$\sin\theta_n=\frac{2\lambda_{\mathrm{c}}\sqrt{n}}{[4\lambda_{\mathrm{c}}^2 n+(\sqrt{\Delta^2+4\lambda_{\mathrm{c}}^2 n}-\Delta)^2]^{\frac{1}{2}}}, \tag{10}$$

$$\cos\theta_n=\frac{\sqrt{\Delta^2+4\lambda_{\mathrm{c}}^2 n}-\Delta}{[4\lambda_{\mathrm{c}}^2 n+(\sqrt{\Delta^2+4\lambda_{\mathrm{c}}^2 n}-\Delta)^2]^{\frac{1}{2}}}, \tag{11}$$

where the detuning parameter $\Delta=\omega_{\mathrm{a}}-\omega_{\mathrm{c}}$. The state vector $|n-1,e\rangle=|n-1\rangle\otimes|e\rangle$ represents the atom in the upper state with $n-1$ photons in the cavity, and $|n,g\rangle=|n\rangle\otimes|g\rangle$ represents the atom in the ground state with n photons in the cavity.

Suppose that at the initial time $t=0$ the atom is in the upper level $|e\rangle$ and the photon distribution probability is P_n. From Eqs.(4) and (5), we can find the atomic emission spectrum

$$\begin{aligned} S_{\mathrm{a}}(\omega,t,\Gamma)=& 2\Gamma\sum_n P_n\{|(\sin^2\theta_{n+1}\cos\theta_n)A_n \\ & +(\cos^2\theta_{n+1}\cos\theta_n)B_n|^2 \\ & +|(\sin^2\theta_{n+1}\sin\theta_n)C_n \\ & +(\cos^2\theta_{n+1}\sin\theta_n)D_n|^2\}, \end{aligned} \tag{12}$$

and the cavity field spectrum

$$\begin{aligned} S_f(\omega,t,\Gamma) =& 2\Gamma\sum_n P_n\{|[\sin\theta_{n+1}(\sqrt{n+1}\cos\theta_{n+1}\cos\theta_n+\sqrt{n}\sin\theta_{n+1}\sin\theta_n)]A_n \\ &+[\cos\theta_{n+1}(-\sqrt{n+1}\sin\theta_{n+1}\cos\theta_n+\sqrt{n}\cos\theta_{n+1}\sin\theta_n)]B_n|^2 \\ &+|[\sin\theta_{n+1}(-\sqrt{n+1}\cos\theta_{n+1}\sin\theta_n+\sqrt{n}\sin\theta_{n+1}\cos\theta_n)]C_n \\ &+[\cos\theta_{n+1}(\sqrt{n+1}\sin\theta_{n+1}\sin\theta_n+\sqrt{n}\cos\theta_{n+1}\cos\theta_n)]D_n|^2\}, \end{aligned} \tag{13}$$

where

$$A_n = \frac{\mathrm{e}^{-\mathrm{i}[(\omega-\omega_\mathrm{c})-\frac{1}{2}(\sqrt{\Delta^2+4\lambda_\mathrm{c}^2(n+1)}-\sqrt{\Delta^2+4\lambda_\mathrm{c}^2 n})]t}-\mathrm{e}^{-\Gamma t}}{\Gamma-\mathrm{i}[(\omega-\omega_\mathrm{c})-\frac{1}{2}(\sqrt{\Delta^2+4\lambda_\mathrm{c}^2(n+1)}-\sqrt{\Delta^2+4\lambda_\mathrm{c}^2 n})]}, \tag{14}$$

$$B_n = \frac{\mathrm{e}^{-\mathrm{i}[(\omega-\omega_\mathrm{c})+\frac{1}{2}(\sqrt{\Delta^2+4\lambda_\mathrm{c}^2(n+1)}+\sqrt{\Delta^2+4\lambda_\mathrm{c}^2 n})]t}-\mathrm{e}^{-\Gamma t}}{\Gamma-\mathrm{i}[(\omega-\omega_\mathrm{c})+\frac{1}{2}(\sqrt{\Delta^2+4\lambda_\mathrm{c}^2(n+1)}+\sqrt{\Delta^2+4\lambda_\mathrm{c}^2 n})]}, \tag{15}$$

$$C_n = \frac{\mathrm{e}^{-\mathrm{i}[(\omega-\omega_\mathrm{c})-\frac{1}{2}(\sqrt{\Delta^2+4\lambda_\mathrm{c}^2(n+1)}+\sqrt{\Delta^2+4\lambda_\mathrm{c}^2 n})]t}-\mathrm{e}^{-\Gamma t}}{\Gamma-\mathrm{i}[(\omega-\omega_\mathrm{c})-\frac{1}{2}(\sqrt{\Delta^2+4\lambda_\mathrm{c}^2(n+1)}+\sqrt{\Delta^2+4\lambda_\mathrm{c}^2 n})]}, \tag{16}$$

$$D_n = \frac{\mathrm{e}^{-\mathrm{i}[(\omega-\omega_\mathrm{c})+\frac{1}{2}(\sqrt{\Delta^2+4\lambda_\mathrm{c}^2(n+1)}-\sqrt{\Delta^2+4\lambda_\mathrm{c}^2 n})]t}-\mathrm{e}^{-\Gamma t}}{\Gamma-\mathrm{i}[(\omega-\omega_\mathrm{c})+\frac{1}{2}(\sqrt{\Delta^2+4\lambda_\mathrm{c}^2(n+1)}-\sqrt{\Delta^2+4\lambda_\mathrm{c}^2 n})]}. \tag{17}$$

3. Discussions

From Eqs.(12)–(17), it is clear that when the cavity field is initially in a Fock state $|n\rangle$, the peak positions of the two spectra are determined by Eqs.(14)–(17). Therefore, the cavity field and atomic emission spectra should have the same peak structure. However, Eqs.(12) and (13) also show that the amplitudes of corresponding peaks of the two spectra may be different. As a result, both spectra may be different. A detailed discussion is as follows.

At first, let us consider the case in which the cavity field is initially in the vacuum state. Thus, $P_n = \delta_{n0}$ and from Eqs.(10) and (11), $\cos\theta_0 = 1.0$, and $\sin\theta_0 = 0.0$. In the case of resonance, $\Delta = 0.0$, and $\cos\theta_1 = \sin\theta_1 = 1/\sqrt{2}$, and from Eqs.(12) and (13) we have $S_\mathrm{a}(\omega,t,\Gamma) = \Gamma|A_0+B_0|^2/2$ and $S_f(\omega,t,\Gamma) = \Gamma|A_0-B_0|^2/2$. These results show that both spectra have two peaks at $\omega = \omega_\mathrm{c} \pm \lambda_\mathrm{c}$ with the same amplitude. This is the well known vacuum Rabi splitting.[6,7] In this case, the difference between the two spectra is only the sign of the peak-overlap term, which results from the retarded photon emission of the atom (see later discussions). In the off-resonance case, i.e. $\Delta \neq 0.0$, Eqs.(12) and (13) show that two peaks at $\omega = \omega_\mathrm{c} + \frac{1}{2}(\Delta \pm \sqrt{\Delta^2+4\lambda_\mathrm{c}^2})$ could exist. From Eqs.(10) and (11), $\sin\theta_1 \approx 1.0$ and $\cos\theta_1 \approx 0.0$ when $\Delta \gg 2\lambda_\mathrm{c}$. In this case, the "Rayleigh scattering" peak at $\omega = \omega_\mathrm{c}$ disappears, and at the same time the "resonance fluorescence" peak at $\omega \approx \omega_\mathrm{c} + \Delta$ is greatly enhanced in the atomic emission spectrum. This result is the same as that of the numerical calculation given in Ref.[6]. However, Eq.(13) shows that in the cavity field spectrum the heights of the two peaks are always the same and decrease together as Δ increases. In Fig.1, the cavity field and atomic emission spectra are shown for different values of the detuning parameter. We see that the two spectra have different behaviours with respect to the detuning parameter.

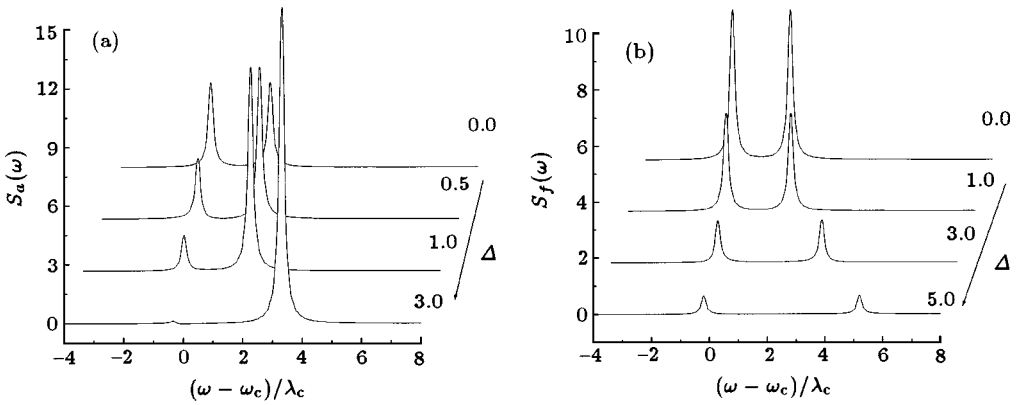


Fig.1. (a) Atomic emission spectra; (b) Cavity field spectra. The field is initially in the vacuum. Each spectrum has a different value of the detuning parameter $\Delta = \omega_a - \omega_c$, which is marked on the corresponding line. The measurement time t is $40\lambda_c^{-1}$ and the resolution of the spectrometer Γ is $0.1\lambda_c$.

Now let us suppose that the cavity field is in a Fock state $|n\rangle$ at the initial time. In this case, Eqs.(12) and (13) show that the two spectra could have two central peaks at $\omega = \omega_c \pm \frac{1}{2}(\sqrt{\Delta^2 + 4\lambda_c^2(n+1)} - \sqrt{\Delta^2 + 4\lambda_c^2 n})$ and two side-band peaks at $\omega = \omega_c \pm \frac{1}{2}(\sqrt{\Delta^2 + 4\lambda_c^2(n+1)} + \sqrt{\Delta^2 + 4\lambda_c^2 n})$. Obviously, as the initial number of photons in the cavity increases the two central peaks merge together and the two side-band peaks separate further and further. In the case of resonance, $\Delta = 0.0$, and $\cos\theta_n = \sin\theta_n = 1/\sqrt{2}$, Eq.(12) shows that the central and side-band peaks have the same amplitude. Thus, as the initial number of photons increases, the atomic emission spectrum gradually develops from two peaks, to four peaks and finally to a triplet of peaks. However, Eq.(13) shows that the amplitudes of the central peaks of the cavity field spectrum are proportional to the initial number of photons and become larger while the amplitudes of the side-band peaks are inversely proportional to the initial number of photons and become smaller with increasing the initial number of photons. Therefore, when the initial photon number is very large, the side-band peaks disappear and the central peaks merge together; and thus the cavity field spectrum approaches a single sharp central peak. In the off-resonance case, from Eqs.(10) and (11), when $\Delta \gg 2\lambda_c\sqrt{n}$, $\sin\theta_n \approx 1.0$ and $\cos\theta_n \approx 0.0$, Eq.(12) shows that in the atomic emission spectrum the central peaks disappear and the left side-band peak is depressed but the right one enhanced. However, in this case, Eq.(13) show different behaviour in the cavity field spectrum. The right central peak is enhanced and all other peaks are depressed. In Figs.2 and 3, the atomic emission and cavity field spectra are shown for different values of the initial photon number and the detuning parameter.

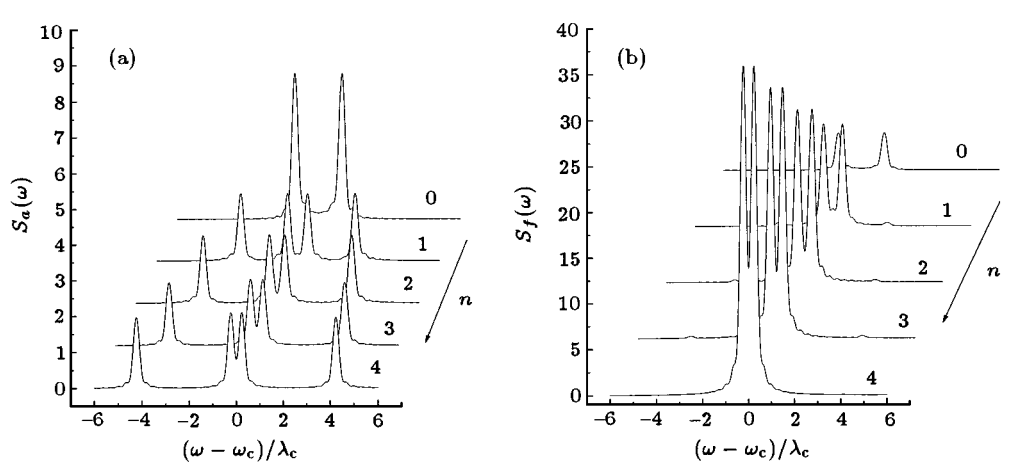


Fig.2. (a) Atomic emission spectra; (b) Cavity field spectra. The field is initially in a Fock state $|n\rangle$. The detuning parameter $\Delta = 0.0$ and other parameters are the same as in Fig.1.

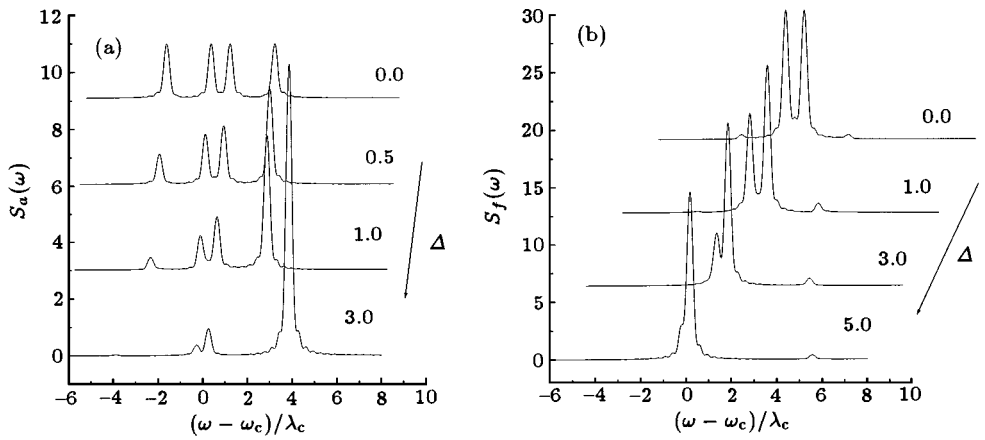


Fig.3. (a) Atomic emission spectra; (b) Cavity field spectra. The field is initially in a Fock state $|n = 1\rangle$. The detuning parameters $\Delta = \omega_a - \omega_c$ for each spectrum are given in the figure. Other parameters are the same as in Fig.1.

When the cavity field is in a coherent state at the initial time, the atomic level population inversion and mean photon number in the cavity can display collapse and revivals.[21] Eqs. (12) and (13) depend on the time of measurement t. In Fig.4, we show the two spectra at several typical times corresponding to the beginning of the first steady period, and the middle points of the first and the second revival periods of the atomic level population inversion. We see that the atomic emission and cavity field spectra have different shapes for the same value of t.

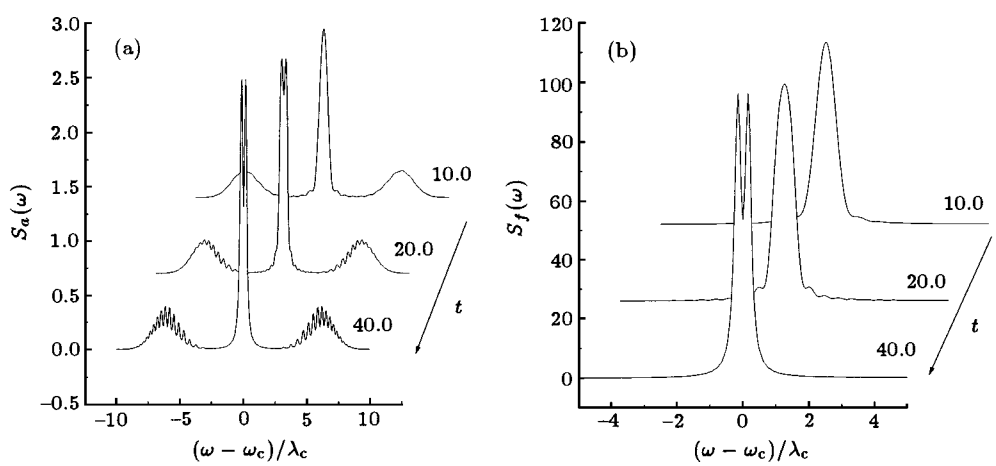


Fig.4. (a) Atomic emission spectra; (b) Cavity field spectra. The field is initially in the coherent state $|\alpha = 3.0\rangle$. The detuning parameter $\Delta = 0.0$ and other parameters are the same as in Fig.1.

The difference between the cavity field and atomic emission spectra mentioned above can be understood in the following way. In the interaction picture, the autocorrelation function of the field $\langle \hat{a}^\dagger(t_1)\hat{a}(t_2)\rangle$ can be written as

$$\begin{aligned}\langle\hat{a}^{\dagger}(t_1)\hat{a}(t_2)\rangle =&\langle\hat{a}^{\dagger}(0)\hat{U}^{\dagger}(\hat{a}^{\dagger}\hat{a}+1,t_1)\hat{U}(\hat{a}^{\dagger}\hat{a},t_1)\hat{U}^{\dagger}(\hat{a}^{\dagger}\hat{a},t_2)\hat{U}(\hat{a}^{\dagger}\hat{a}+1,t_2)\hat{a}(0)\rangle\\ &-\mathrm{i}\lambda_{\mathrm{c}}\Big\langle\hat{a}^{\dagger}(0)\hat{U}^{\dagger}(\hat{a}^{\dagger}\hat{a}+1,t_1)\hat{U}(\hat{a}^{\dagger}\hat{a},t_1)\hat{\sigma}(t_2)\hat{U}^{\dagger}(\hat{a}^{\dagger}\hat{a},t_2)\frac{\sin\left(\sqrt{\left(\frac{\Delta}{2}\right)^2+\lambda_{\mathrm{c}}^2\hat{a}\hat{a}^{\dagger}}t_2\right)}{\sqrt{\left(\frac{\Delta}{2}\right)^2+\lambda_{\mathrm{c}}^2\hat{a}\hat{a}^{\dagger}}}\Big\rangle\\ &+\mathrm{i}\lambda_{\mathrm{c}}\Big\langle\frac{\sin\left(\sqrt{\left(\frac{\Delta}{2}\right)^2+\lambda_{\mathrm{c}}^2\hat{a}\hat{a}^{\dagger}}t_1\right)}{\sqrt{\left(\frac{\Delta}{2}\right)^2+\lambda_{\mathrm{c}}^2\hat{a}\hat{a}^{\dagger}}}\hat{U}(\hat{a}^{\dagger}\hat{a},t_1)\hat{\sigma}^{\dagger}(t_1)\hat{U}^{\dagger}(\hat{a}^{\dagger}\hat{a},t_2)\hat{U}(\hat{a}^{\dagger}\hat{a}+1,t_2)\hat{a}(0)\Big\rangle\\ &+\lambda_{\mathrm{c}}^2\Big\langle\frac{\sin\left(\sqrt{\left(\frac{\Delta}{2}\right)^2+\lambda_{\mathrm{c}}^2\hat{a}\hat{a}^{\dagger}}t_1\right)}{\sqrt{(\frac{\Delta}{2})^2+\lambda_{\mathrm{c}}^2\hat{a}\hat{a}^{\dagger}}}\hat{U}(\hat{a}^{\dagger}\hat{a},t_1)\hat{\sigma}^{\dagger}(t_1)\hat{\sigma}(t_2)\hat{U}^{\dagger}(\hat{a}^{\dagger}\hat{a},t_2)\frac{\sin\left(\sqrt{(\frac{\Delta}{2})^2+\lambda_{\mathrm{c}}^2\hat{a}\hat{a}^{\dagger}}t_2\right)}{\sqrt{(\frac{\Delta}{2})^2+\lambda_{\mathrm{c}}^2\hat{a}\hat{a}^{\dagger}}}\Big\rangle,\end{aligned} \tag{18}$$

where

$$\begin{aligned}\hat{U}(\hat{a}^{\dagger}\hat{a},t)=&\Bigg[\cos\left(\sqrt{(\frac{\Delta}{2})^2+\lambda_{\mathrm{c}}^2\hat{a}^{\dagger}\hat{a}}t\right)\\ &+\mathrm{i}\frac{\Delta}{2}\frac{\sin(\sqrt{\left(\frac{\Delta}{2}\right)^2+\lambda_{\mathrm{c}}^2\hat{a}^{\dagger}\hat{a}}t)}{\sqrt{\left(\frac{\Delta}{2}\right)^2+\lambda_{\mathrm{c}}^2\hat{a}^{\dagger}\hat{a}}}\Bigg]|g\rangle\langle g|\\ &+\Bigg[\cos\left(\sqrt{\left(\frac{\Delta}{2}\right)^2+\lambda_{\mathrm{c}}^2\hat{a}\hat{a}^{\dagger}}t\right)\\ &-\mathrm{i}\frac{\Delta}{2}\frac{\sin(\sqrt{\left(\frac{\Delta}{2}\right)^2+\lambda_{\mathrm{c}}^2\hat{a}\hat{a}^{\dagger}}t)}{\sqrt{\left(\frac{\Delta}{2}\right)^2+\lambda_{\mathrm{c}}^2\hat{a}\hat{a}^{\dagger}}}\Bigg]|e\rangle\langle e|\\ &-\mathrm{i}\lambda_{\mathrm{c}}\frac{\sin(\sqrt{\left(\frac{\Delta}{2}\right)^2+\lambda_{\mathrm{c}}^2\hat{a}^{\dagger}\hat{a}}t)}{\sqrt{\left(\frac{\Delta}{2}\right)^2+\lambda_{\mathrm{c}}^2\hat{a}^{\dagger}\hat{a}}}\hat{a}^{\dagger}|g\rangle\langle e|\\ &-\mathrm{i}\lambda_{\mathrm{c}}\frac{\sin(\sqrt{\left(\frac{\Delta}{2}\right)^2+\lambda_{\mathrm{c}}^2\hat{a}\hat{a}^{\dagger}}t)}{\sqrt{\left(\frac{\Delta}{2}\right)^2+\lambda_{\mathrm{c}}^2\hat{a}\hat{a}^{\dagger}}}\hat{a}|e\rangle\langle g|\end{aligned} \tag{19}$$

is the evolution operator of the JCM. In Eq.(18), the first three terms are directly related to the initial number of photons in the cavity through $\hat{a}^{\dagger}(0)$ and $\hat{a}(0)$, and become zero when photons are absent at the initial time. The last term is related to the atomic autocorrelation function $\langle\hat{\sigma}^{\dagger}(t_1)\hat{\sigma}(t_2)\rangle$. As we can see, the correlation function $\langle\hat{a}^{\dagger}(t_1)\hat{a}(t_2)\rangle$ contains not only atomic emission information which is represented by the last term of Eq.(18), but also the initial field information. Therefore, Eq.(18) shows that in the JCM $\langle\hat{a}^{\dagger}(t_1)\hat{a}(t_2)\rangle$ is in general not proportional to $\langle\hat{\sigma}^{\dagger}(t_1)\hat{\sigma}(t_2)\rangle$. When the interaction is in resonance and the initial photon number is zero, we have $\langle\hat{\sigma}^{\dagger}(t_1)\hat{\sigma}(t_2)\rangle = \cos\lambda_{\mathrm{c}}t_1\cos\lambda_{\mathrm{c}}t_2$ and $\langle\hat{a}^{\dagger}(t_1)\hat{a}(t_2)\rangle = \sin\lambda_{\mathrm{c}}t_1\sin\lambda_{\mathrm{c}}t_2$. Considering the retarded time $\pi/2\lambda_{\mathrm{c}}$ of one-photon emission from the excited atom, we have $\langle\hat{a}^{\dagger}(t_1+\pi/2\lambda_{\mathrm{c}})\hat{a}(t_2+\pi/2\lambda_{\mathrm{c}})\rangle = \langle\hat{\sigma}^{\dagger}(t_1)\hat{\sigma}(t_2)\rangle$. Therefore, in this case, the atomic emission and cavity field spectra are the same.

4. Conclusions

In conclusion, we have shown that the positions of corresponding peaks of the atomic emission and cavity field spectra of the JCM are the same, but the heights of the peaks of the two spectra vary in different ways with the detuning parameter and the initial number of photons in the cavity. As a result, we see that the two spectra are in general different. For the special case where the interaction is resonant and the cavity is initially empty, the previous studies showed that the two spectra could be the same. However, we have found that this situation occurs only when the retarded time of one-photon emission is considered in calculations of the atomic emission spectrum. We also show that, unlike the atomic resonance fluorescence in

free space, the autocorrelation function of the cavity field is in general not proportional to that of the atom in the JCM. It is the lack of this proportionality that results in the difference between the atomic emission and cavity field spectra.

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